

Portfolio Selection with Two Risky Securities.

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Outline

- Portfolio: expected return and SD
- Diversification
- Investment opportunity set
- Investor preference: risk-return tradeoff
- Optimal portfolio choice with 2 risky assets

Portfolio Expected Return and Standard Deviation

- The expected return on the portfolio is:

$$E(R_p) = \sum_{i=1}^N \omega_i E(R_i)$$

- With 2 securities, the portfolio variance is:

$$\sigma_p^2 = \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\omega_1 \omega_2 \rho_{12} \sigma_1 \sigma_2$$

- The standard deviation is:

$$\sigma_p = \sqrt{\sigma_p^2}$$

Diversification with 2 assets:

Example

- Suppose we have two assets, US and JP, with:

	mean	volatility
US	13.6%	15.4%
JP	15.0%	23.0%

and with correlation 27%.

- If an investor holds 60% in the US and 40% in JP what is the mean and volatility of the portfolio?
- 'volatility' is another word for 'standard deviation'

Diversification with 2 assets:

Example

- Portfolio mean:

$$E(R_p) = 0.6 \cdot 0.136 + 0.4 \cdot 0.150 = 14.2\%$$

- Portfolio variance:

$$\begin{aligned} \text{var}(R_p) &= (0.6)^2 \cdot (0.154)^2 + (0.4)^2 \cdot (0.230)^2 \\ &\quad + 2 \cdot 0.6 \cdot 0.4 \cdot 0.27 \cdot 0.154 \cdot 0.230 \\ &= 0.022 \end{aligned}$$

$$\sigma_p = 14.7\%$$

- This portfolio has higher expected return and lower risk than the US market alone!

Risk and Return with Varying Weights

- Let ω be the weight in the US, and $1-\omega$ the weight in JP.

- The expected return of the portfolio is:

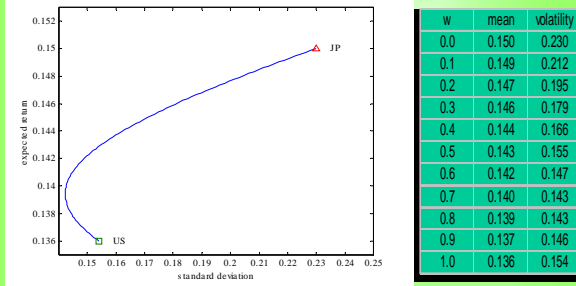
$$E(r_p) = \omega \cdot 0.136 + (1-\omega) \cdot 0.150$$

- The variance of the portfolio return is:

$$\begin{aligned} \text{var}(r_p) &= \omega^2 \cdot (0.154)^2 + (1-\omega)^2 \cdot (0.230)^2 \\ &\quad + 2 \cdot \omega \cdot (1-\omega) \cdot 0.27 \cdot 0.154 \cdot 0.230 \end{aligned}$$

- What happens when we vary ω ?

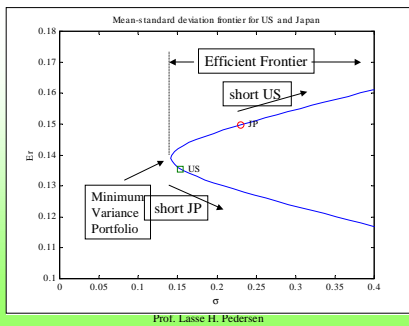
Varying the Portfolio Weights gives: The Investment Opportunity Set



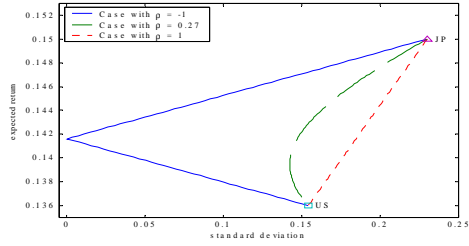
Portfolio Terminology

- The investment opportunity set consists of all available risk-return combinations.
- An efficient portfolio is a portfolio that has the highest possible expected return for a given standard deviation
- The efficient frontier is the set of efficient portfolios. It is the upper portion of the minimum variance frontier starting at the minimum variance portfolio.
- The minimum variance portfolio (mvp) is the portfolios that provides the lowest variance (standard deviation) among all possible portfolios of risky assets.

Portfolio Terminology



Investment Opportunity Set with Varying Correlations



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Optimal Portfolio Choice with 2 Risky Assets

- Any (mean-variance) investor should choose an efficient portfolio to benefit from diversification.
- The specific choice depends on the investor's risk aversion
- A more risk-averse investor should choose a portfolio with
 - lower risk
 - lower expected return

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