Time value of money.
FV, PV, annuities, perpetuities.

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Outline

- Time value of money
- FV, PV, yield
- Single payment security
  - Example: zero coupon bond
- Multiple payment security
  - Annuities and perpetuities

Future Value, FV

- Timeline
- Investing for a single period
  - $FV = PV \times (1+r)$
- Investing for multiple periods
  - Interest on interest
  - Compounding
    - $FV = PV \times (1+r)^t$
- Simple interest
Present Value, PV

- Single-period case:
  - How much do you need to invest today, with an interest rate of \( r \), to have \( FV \) next year?
  - \( PV = FV \frac{1}{1+r} \)

- Multiple-period case:
  - \( PV = FV \frac{1}{(1+r)^t} \)

- Discount factor (= present value factor):
  - \( \frac{1}{(1+r)^t} \)

PV, FV, r, t are tied together.
If you know 3, you can find the last one

- What is the yield, \( r \)?
  - Suppose you know the present value, and the (future) value at a given future time:
    What is the yield (or the return)?

- How many periods, \( t \)?
  - Suppose you have a given amount of money, you know the interest rate, and you know the future value that you need:
    How many periods does it take?

Zero Coupon Bond

- Where do zero coupon bonds come from?
  - Issued in primary markets (Treasury notes)
  - Stripping of coupon bonds (example of “financial engineering”)

- Pricing of zeros by arbitrage
  (Extremely important pricing principle!)
**Multiple Payments**

- **Timeline**
- **Future value of stream of cash flows** \( C(0), C(1), \ldots, C(T) \):
  \[
  FV(T) = C(0)(1 + r)^T + C(1)(1 + r)^{T-1} + \ldots + C(T)
  \]
- **Present value**:
  \[
  PV = C(0) + C(1) \frac{1}{(1 + r)} + \ldots + C(T) \frac{1}{(1 + r)^T}
  \]
- **Yield**: IRR (discussed in later class)

**Perpetuities**

- **Perpetuities**
  - **Definition**: Pays a fixed cashflow, \( C \), every period forever
  - **Example**: consol bond
  - **Pricing**: \( PV = \frac{C}{r} \)
- **Example**:
  - Suppose that maintenance of your grave costs $100 every year, forever.
  - The interest rate is 5% per year.
  - How much money should you leave the trustee of your grave?

**Annuities**

- **Annuities**:
  - **Definition**: Pays a fixed cashflow, \( C \), for \( T \) periods
  - **Price**:
    \[
    PV = \frac{C}{r} \left( \frac{1 - \frac{1}{(1 + r)^T}}{r} \right)
    \]
  - **What is the future value?**
- **Example: Which car you afford?**
  - You have no money
  - You can borrow at an interest rate of 1% per month
  - You can pay $632 per month
  - You want to have paid the loan in full in 48 months
Relationship between interest rate and price

- For perpetuities?
- For other bonds?
- Later in class: duration.
- For stocks?

- Does Alan Greenspan matter?