I consider an infinite-period race where players choose between low- and high-variance motion technologies. I provide sufficient conditions under which, in equilibrium, the leader chooses a safe technology and the laggard a risky one, thus formalizing the sports intuition that the laggard has nothing to lose. Various examples and empirical implications are presented.

1. Introduction

Designing an R&D strategy in a competitive environment is a complex problem. One of the elements of this problem, which has been extensively dealt with in the literature, is the choice of the amount of R&D effort [see Reinganum (1989) and references therein]. A different but related decision concerns the type of R&D projects to choose.

Consider the example of research in mainframe computers.\(^1\) The mobility of researchers across firms is very low in the industry. And given the specialized nature of R&D, it is quite expensive to vary the number of researchers on short notice. For these reasons, the R&D resources available to a firm are nearly constant over time. The firm can however determine how to invest its limited R&D resources, focusing more on marginal improvements or on developing a new chip architecture. This suggests a game where the decision of how to spend the R&D resources is more important than the decision of how much to spend: given the high adjustment costs in personnel recruitment, firms take their R&D budget as fixed, at least in the short run.

In this paper, I consider a game where firms compete in R&D. I assume that firms have fixed R&D budgets and that their strategies

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1. This example is adapted from Khanna and Iansiti (1997).

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consist of choosing one of two alternative paths to improve their performance, one with a higher variance than the other. I also assume that a firm’s payoff is a function of the quality difference between its product and the rival’s. The example of mainframe computers seems broadly consistent with this assumption. Although there are several measures of performance—including module gate density, cycle time, and gate delay—empirical observation reveals a high degree of correlation among these measures, so that we can talk about “speed” as a single performance target. And demand is typically a function of relative performance.

My main goal is to find the firm’s optimal strategy (high or low variance) as a function of its relative positioning. Conventional wisdom from sports competition provides an indication of what the answer might be. As a football match nears the end, the trailing team has a big incentive to adopt a high-variance strategy, like passing on a fourth down (American football) or using a goalie as a striker (association football).

Unlike sports, business situations are typically open-ended. Accordingly, I consider an infinite-period race. Despite the change in the structure of the game, I obtain a similar result to the finite-period case. Specifically, I provide sufficient conditions under which, in equilibrium, the leader chooses a safe path and the laggard a risky one, thus formalizing the sports intuition that the laggard has nothing to lose. In addition to developing these theoretical results, I propose some illustrative examples and derive a number of testable empirical implications.

Previous papers addressed the strategic choice of variance in R&D races. A series of authors have analyzed static models to compare private and social incentives for risktaking in R&D. See Bagwell and Staiger (1990), Bhattacharya and Mookherjee (1986), Cabral (1994), Dasgupta and Maskin (1987), Klette and de Meza (1986), and Vickers (1985). Given the static nature of their models, they do not address the relative incentives of leader and follower. Rosen (1991) and Khanna and Iansiti (1997) explicitly consider asymmetries across firms, but still in a static framework. Aron and Lazear (1990) consider a finite-stage model of entry into a new market in which firms learn from each other’s experience. They show that, in equilibrium, the initial market laggard enters a new, uncertain market, and then the current market leader follows when the rival’s entry is successful.

2. In a related paper (Cabral, 2002), I consider the case when each firm chooses the covariance of its R&D outcome with respect to the rival’s.
In this paper, I explicitly consider an infinite-period model, as opposed to a static or finite-period one. This seems a more realistic approach than the previous literature, as most real-world business situations evolve over an infinite number of periods. Moreover, it is not obvious what the extension of the sports intuition is in the context of an infinite-period model. In fact, one might argue that the force of reasonings like “nothing to lose” depends crucially on the finiteness of the game. My results help to clarify what the meaning of such intuitions is in an infinite-horizon context. Moreover, I present a series of examples and empirical implications that point to possible areas of empirical research.

The paper is structured as follows. In Section 2, I introduce the model, and the main results are presented in Section 3. Section 4 derives a series of empirical implications, and final remarks are collected in Section 5.

2. Model

Consider an infinite-period game with two players. In each period, the state of the game is summarized by an integer \( z \in \mathbb{Z} \). Short-run payoffs are summarized by the functions \( p_i(z) \), \( i = 1, 2 \). I assume that payoff functions are monotonic and symmetric, i.e., \( p_1(z) \) is increasing in \( z \) and \( p_2(z) = p_1(-z) \) [and thus \( p_2(z) \) is decreasing in \( z \)]. By an abuse of notation that simplifies the analysis, I denote by \( p(n) \) the payoff for a player who is “ahead” in state \( z = n \geq 0 \) (the “leader”); the payoff for the rival player (the “laggard”) is therefore \( p(-n) \).

One useful way of thinking about the model is that two firms attempt to move up a quality ladder (or down a cost ladder) by exerting R&D effort. In each period, payoffs are determined by the difference in quality levels, \( n = q_i - q_j \). Motion across states is therefore determined by the firms’ success in moving up the ladder.

3. Judd (1985) and Hoernig (1999) present dynamic models of R&D competition. Judd (1985) examines a patent race and is interested in comparing private and social incentives for risktaking in R&D. Hoernig’s (1999) focus is closer to my paper’s. However, he assumes that laggards have the option of leapfrogging the leader in one go, a possibility that I do not consider. Moreover, he (and Judd) use numerical solutions, whereas my equilibrium characterization is analytical.

4. As in repeated-game theory, “infinite” does not need to be interpreted literally. The crucial feature is that there is uncertainty about the end period. Such a situation may be modeled as an infinite game, where the discount factor reflects the conditional probability of the game ending after each period.

5. Notice that symmetry does not imply any particular relation between \( p_1(z) \) and \( p_2(z) \). The only constraint is that, as we move from state \( z \) to state \(-z\) (the symmetric state), payoff levels are interchanged: \( p_2(z) = p_1(-z) \) and \( p_1(z) = p_2(-z) \).

6. In the Appendix, I provide an example satisfying the assumption that payoffs depend on the difference between quality levels.
A crucial feature of the model is that players must choose between two available paths, \(a\) and \(b\). In fact, I will assume that this is the only decision each player makes in each period. If we interpret the model as one of R&D competition, then this amounts to assuming that the R&D budget is fixed and that the only choice is between different research paths. Path \(a\) allows players to move up one step with probability one. Path \(b\) allows players to move up two steps with probability \(\frac{1}{2}\) and zero with probability \(\frac{1}{2}\). That is, path \(b\) is a mean-preserving spread of path \(a\).

A Markov strategy for player \(i\) is map \(x_i(n)\), giving the probability of choosing path \(a\) in state \(n\). A pair of strategies \(x_i(n)\), together with the (common) discount factor \(\delta\), induce value functions \(v_i(n)\). I treat value functions in terms of average period payoff, so \(v_i(n) = (1 - \delta)p_i(n) + \delta v_i^+\), where \(v_i^+\) is player \(i\)'s expected continuation value. Moreover, I restrict to symmetric equilibria. For simplicity, if with some abuse of notation, I denote strategies and value functions by \(x(n)\) and \(v(n)\), respectively.

3. Analysis of Equilibrium

Do players prefer a safe technology (technology \(a\)) or a risky one (technology \(b\)), knowing that the expected motion is the same for both (in terms of the number of steps taken)? My first result relates concavity of the payoff function to the optimal choice when the discount factor is small.\(^7\)

**Proposition 1** (Nothing to Lose): Suppose that \(p(n)\) is strictly concave if \(n > 0\) and strictly convex if \(n < 0\). There exists a \(\delta\) such that, for \(\delta < \delta\), in equilibrium \(x(n) = 1\) if \(n > 1\) and \(x(n) = 0\) if \(n < -1\).

Proposition 1 is very close to the sports intuition that laggards have little to lose. Typically, the payoff for a loser in sports is approximately the same regardless of the severity of the loss. That is, in terms of the final outcome, the laggard’s payoff function is convex around the point of minimum loss. Now suppose that we are close to the end of a match or race. That means that the value function is close to the final value function, and convexity implies that the laggard is better with a mean-preserving spread of the motion across states (by Jensen’s inequality). The assumption that we are close to the end of the match or race is similar to the assumption in Proposition 1 that \(\delta\) is close to zero.

\(^7\) All proofs are included in the Appendix.
In the business world, most situations of dynamic competition evolve over a large number of periods. Moreover, the discount factor is typically greater than zero. How much of the intuition from sports remains valid in this context?

In what follows, I will use the term “best response” with reference to state \( n \)’s behavioral strategy. Specifically, player \( i \)'s best response function in state \( n \) gives the value \( x_i(x_j) \) that maximizes \( v_i(n) \) given the value \( x_j \) and a set of continuation value functions.

**Proposition 2** (Things Will Get Better): \( x(-n) = 0 \) is a strictly best response with respect to \( x(n) = 1 \) if and only if \( p(-n) < v(-n) \).

This result establishes the precise correspondence between the static intuition for risk-loving strategies (“nothing to lose”) and the dynamic intuition (“things will get better”). The situation when things will get better corresponds to \( p(-n) < v(-n) \), that is, current payoff is less than current average discounted payoff. A player in state \(-n\) knows that he is in a particularly bad state. Choosing a high-risk path is then the best way to move away from the current state (given that the rival chooses a low-risk path).

Proposition 2 extends to infinite games the sports intuition that a laggard has nothing to lose. Specifically, if the laggard’s current payoff is lower than his equilibrium average discounted value, then he is better off by choosing a risky path when the leader chooses a safe one. The essence of the proof is that \( p(-n) < v(-n) \) implies that \( v(-n) \) is convex. And convexity of \( v(-n) \) implies that a risky path, by increasing variability of motion across states, increases discounted value (as seen in the proof of Proposition 1).

In order to derive more complete results, some constraint must be imposed on the payoff function. Specifically, I make the following assumption:

**Assumption 1:** There exist finite \( \underline{n} \), \( \overline{n} \) (\( \underline{n} < \overline{n} \) and \( \underline{n} < -\overline{n} \)) such that \( p(n) = \overline{p} \) for \( n > \overline{n} \), \( p(n) = \underline{p} \) for \( n < \underline{n} \), and \( \underline{p} < p(n) < \overline{p} \) for \( \underline{n} < n < \overline{n} \).

I can now state that, if different strategies are chosen, then the leader will choose the safe one and the laggard the risky one.

**Proposition 3:** Suppose that players choose different paths in state \( n > \overline{n} \) (with probability one). Then it must be that the leader chooses the safe path and the laggard the risky one.

A numerical example will prove useful in interpreting Propositions 2–3 and their difference with respect to Proposition 1. Consider
TABLE I.
LEADERSHIP AND RISK-TAKING BEHAVIOR

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(n)</td>
<td>.5</td>
<td>.61</td>
<td>.73</td>
<td>.86</td>
</tr>
<tr>
<td>p(n + 1) - p(n)</td>
<td>.11</td>
<td>.12</td>
<td>.13</td>
<td>.14</td>
</tr>
<tr>
<td>(\hat{x}(n))</td>
<td>[0,1]</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>v(n)</td>
<td>.5000</td>
<td>.5878</td>
<td>.6733</td>
<td>.7528</td>
</tr>
<tr>
<td>v(n + 1) - v(n)</td>
<td>.0878</td>
<td>.0855</td>
<td>.0795</td>
<td>.0683</td>
</tr>
<tr>
<td>p(n) - v(n)</td>
<td>0</td>
<td>.0222</td>
<td>.0567</td>
<td>.1072</td>
</tr>
</tbody>
</table>

At state \(n = 2\), the leader’s payoff function is convex. However, if \(\delta = 0.9\), the value function is concave and the equilibrium calls for a safe path to be chosen: \(\hat{x}(2) = 1\). Notice also that \(p(2) > v(2)\).

the values of \(p(n)\) given in Table I. Suppose moreover that \(p(n) = 1\) for \(n > 3\), and that \(p(-n) = 1 - p(n)\). As can be seen from the third row in Table I, the payoff function is convex for \(1 \leq n \leq 3\). By the reasoning in Proposition 1, if the discount factor is very small, then in state \(n = 2\) the leader should pick the risky path and the follower the safe one.

The same is not true, however, for higher values of the discount factor. Table I presents equilibrium values for the case when \(\delta = 0.9\). Although \(p(n)\) is convex for \(1 \leq n \leq 3\), the leader picks the safe path at \(n = 2\), whereas the laggard picks the risky one. Notice that, in state 2, \(p(n) > v(n)\), consistently with Proposition 2. Notice also that \(v(n)\) is concave for the leader even though \(p(n)\) is convex. In fact, it is concavity of the value function, not concavity of the payoff function, that determines the choice of a safe path (if the discount factor is very small, then concavity of the payoff function implies concavity of the value function, which is the essence of the proof of Proposition 1).

The numerical results for a high value of \(\delta\) also suggest a possible extension of Proposition 3. As the future becomes relatively more important, the expected discounted payoff (the value function) will converge to the average of all “connected” states (that is, connected by the equilibrium motion). This implies that, in the limit, \(p(n) > v(n)\) if \(n\) is large enough and \(p(n)\) is increasing (a weaker assumption than Assumption 1).

8. No claim is made that this is the unique equilibrium. Notice however that the equilibrium strategies for the low-\(\delta\) case cannot be equilibrium strategies for the high-\(\delta\) case. In fact, since the two players choose opposite pure strategies both in the low-\(\delta\) and in the high-\(\delta\) case, it follows that the value functions are the same in both cases. From Table I, we see that the equilibrium value function is concave for the leader, which is inconsistent with the hypothesis that the leader follows a risky path.
4. Implications and Evidence

As suggested in the introduction, there are situations when the manager’s main decision is not how much to invest in R&D but rather how to spend an existing R&D budget. I considered the case when (1) market performance is a function of relative position in the technical performance ladder (e.g., microprocessor speed); (2) movements up the ladder are a function of R&D decisions. The main managerial implication of my model is that the firm’s optimal decision is to choose safe R&D projects when it is ahead in the R&D race and risky ones when it falls behind. Notice, however, that I have considered alternative paths with the same expected value. If path $a$ had a very low expected motion with respect to path $b$, then both leader and laggard would prefer the high-variance strategy. Conversely, if path $a$ had a very high expected motion with respect to path $b$, then both leader and laggard would prefer the low-variance strategy.

In the remainder of this section, I derive a series of additional examples and implications regarding the strategic choice of variance.

4.1 The Race for Synthetic Human Insulin

Research in pharmaceuticals shares several of the features of my model. The race to develop human insulin is particularly illustrative. The race took place from 1976 to 1978 and consisted of taking four different steps: (1) isolation, (2) conversion, (3) cloning, and (4) expression [in (a) animal and (b) human form].

It was a four-way race. One team, led by scientists Goodman and Rutter of the University of California at San Francisco, adopted a step-by-step approach. They took the lead in the fall of 1976 when they completed stage 1. In January 1977, they completed stage 2 and thus extended the lead with respect to the three rivals.

The second competitor was Genentech, led by Boyer, one of scientists who in November 1973 discovered recombinant DNA technology (gene splicing). Genentech took a different approach from the UCSF team. They decided to attempt to synthesize somatostatin. The result was that they managed to complete steps 1 and 2 in one go, in the spring of 1977.

The third competitor was a team of Harvard scientists (working at an MIT lab). Initially, this team took a similar path to that of the USCF team. By the time they reached stage 2 they were already trailing behind Genentech and the UCSF team. In fact, in May 1977 the

9. For a more detailed account of the events described here, see Hall (1987) and Brandenburger et al. (1992).
UCSF team completed stage 3, followed in August 1977 by Genentech. However, a bold move by the Harvard team allowed them to complete steps 3 and 4(a) in one go. The Harvard team thus caught up with its rivals. In fact, by mid-1978 all three teams were ready to work on the final step: the expression of insulin in human form. The winner—by a few days, as it turned out—was Genentech, which completed stage 4(b) in the early hours of August 24, 1978, and announced its results in September 6, 1978.

While the three American competitors were racing for the discovery of synthetic human insulin, a European lab, Novo Industri, worked on the transformation of porcine insulin into human insulin. Basically, the idea was to replace the amino acid in porcine insulin that differs from the human amino acid. Novo Industri succeeded in doing so in 1978, at about the same time as Genentech. Eventually, Novo Industri reached the European market (1982), whereas Eli Lilly, which in the meantime signed an agreement with Genentech, reached the American market (1983). It was a race with two winners.

The race to develop human insulin illustrates that there are usually alternative paths to achieve the same goal. Moreover, it suggests that different paths may imply slower or faster progress along the different stages: compare the UCSF team’s step-by-step approach with Genentech’s 1-and-2 or the Harvard team’s 3-and-4a approach. Finally, it confirms the intuition that it is the laggards that have a greater incentive to follow the riskier alternatives.

### 4.2 Variability of Firm Growth and Rate of Return

Proposition 2 predicts that the laggard should have a greater variance in quality level \( q \) than the leader. (Recall that \( n = q_i - q_j \).) Bowman (1980) presents evidence that “business risk and return are negatively correlated across companies within industries.” In a later paper (1982), he explains this by showing that “troubled firms take more risks,” which seems broadly consistent with Proposition 2. Notice, however, that the prediction that troubled firms take more risks may result from limited-liability considerations rather than the positioning of laggards with respect to leaders.

Suppose that firm size is proportional to the value of \( q \), that is, firms with better products are proportionately bigger (under the interpretation that \( q \) measures product quality). Evans (1987) shows that

10. “The Harvard researchers realized they could splice their insulin gene into a special location within the plasmid pBR322 known as the Pst site. . . . If the Harvard researchers spliced the gene for insulin within this larger penicillinase gene, perhaps the bacteria would not only make the hormone but skip it” (which effectively corresponds to steps 3 and 4a) (Hall, p. 188).
the variance of firm growth rates decreases increasing size, a result which, under the above assumption, is consistent with Proposition 2.

4.3 Mutual-Fund Managers’ Behavior

Brown et al. (1996) and Chevalier and Ellison (1997) document that underperforming mutual-fund managers tend to choose riskier portfolios than managers who beat the market. Moreover, Chevalier and Ellison (1997) show that the relation between performance and inflow of new investments is, roughly speaking, convex for underperforming managers and concave for overperforming ones. If the inflow of funds is a good indicator of the manager’s value function, then the results of Chevalier and Ellison (1997) are consistent with Propositions 1 and 2.

4.4 Product Introductions in the Ready-to-Eat Breakfast-Cereal Industry

The RTE breakfast cereal industry is dominated by Kellogg, with a market share of more than 40%, followed by General Mills and, at a much greater distance, Post (recently merged with Nabisco), Ralston, and Quaker. New-product introductions play an important role in the industry. From 1980 to 1996, more than 150 new brands were introduced. Kellogg led the market in number of new products introduced: from 1980 to 1996, more than 30% of the new products were Kellogg’s. However, anecdotal evidence suggests that smaller firms were responsible for the most radical innovations, both technological (e.g., Post’s Blueberry Morning) and other (Post’s and Ralston’s brands), a fact that is broadly consistent with Proposition 2.

A possible indicator of the degree of innovativeness in new product introduction is the percentage of new products that are tried locally but never make it at the national level. The idea is that a firm trying very innovative products is more likely to have bad draws, which correspond to discontinued products. In the period 1980–1991, 18% of the products introduced at some level were discontinued, whereas only 8% of Kellogg’s new products were discontinued.11

4.5 Patents

Patent data provide a promising area of application of my results. In fact, a firm that is engaged in a high-risk R&D strategy is likely to file patents that have a short list of citations (laggards, according to Proposition 2).

11. Source: my calculations are based on data kindly provided by Aviv Nevo, to whom I am grateful.
5. Concluding Remarks

I have developed a model of dynamic competition that isolates one aspect of the strategic choices faced by a firm. There are obviously other elements that one should take into consideration when analyzing real-world R&D competition. For example, in her study of innovation in the photolithographic alignment equipment industry, Henderson (1993) shows that organizational factors can be as important as the strategic factors considered in the previous sections. Specifically, leaders (incumbent firms) fail to perform as well as laggards (entrants) with respect to radical innovation not only because incumbents invest relatively less in radical innovation, but also because “the research efforts of incumbents seeking to exploit radical innovation are significantly less productive than those of entrants.” The results in this paper imply that industry laggards are in the main responsible for radical industry innovation (path b in my model). However, as the above example suggests, there may be other explanations for this pattern.

Appendix

Example Satisfying the Assumption Regarding \( p(n) \): Suppose that each consumer receives utility \( u = \max\{x_1 q_1, x_2 q_2\} + x_0 \), where \( x_i \) is the quantity of good \( i \), \( q_i \) is the quality of good \( i \), and \( x_0 \) denotes other goods. Suppose that each consumer buys at most one unit from each firm \( (x_i \in [0, 1]) \) and is subject to a budget constraint such that it can only spend \( y \). Finally, assume that marginal cost is constant and equal across firms (with no further loss of generality, assume marginal cost is zero). If firms simultaneously set prices and consumers then choose \( x_0, x_1, x_2 \), then in equilibrium consumers buy from the firm with higher quality (say, firm \( i \)). The price paid is given by \( \min\{q_i - q_j, y\} \). The profit received by the firm with higher quality is \( p(n) = q_i - q_j = n \), if \( n < y \), and \( p(n) = y \), if \( n > y \). The firm with lower quality receives \( p(-n) = 0 \).

Proof of Proposition 1. If \( \delta \) is close to zero, then the payoff function \( p(n) \) provides a good first-order approximation to the value function. In particular, \( v(n) \) is strictly concave (convex) if \( p(n) \) is strictly concave (convex).

The value function is given by

\[
\begin{align*}
v(n) &= (1 - \delta)p(n) + \delta[\phi''v(n - 2) + \phi'v(n - 1) + \phi^2v(n) \\
&\quad + \phi'v(n + 1) + \phi''v(n + 2)],
\end{align*}
\]
where
\[
\phi'' \equiv [1 - x(-n)] \frac{1}{2} [1 - x(n)] \frac{1}{2},
\]
\[
\phi' \equiv [1 - x(-n)] \frac{1}{2} x(n) + x(-n) [1 - x(n)] \frac{1}{2},
\]
\[
\phi^0 \equiv 1 - 2 \phi' - 2 \phi''.
\]
Taking the derivative with respect to \(x(n)\) and collecting terms, we get
\[
\frac{\partial v(n)}{\partial x(n)} = \frac{1}{4} [1 - x(-n)] [2v(n - 1) - v(n - 2) - v(n)]
\]
\[
+ \frac{1}{4} [1 - x(-n)] [2v(n + 1) - v(n + 2) - v(n)]
\]
\[
+ \frac{1}{2} x(-n) [2v(n) - v(n + 1) - v(n - 1)]
\]
\[
> 0,
\]
where the inequality follows from repeated application of concavity of \(v(n)\). This implies that \(x(n) = 1\) is a dominant strategy. A similar argument implies that \(x(-n) = 0\) is also a dominant strategy. \(\square\)

**Proof of Proposition 2.** Suppose that \(p(-n) < v(-n)\) and that the leader chooses \(x(n) = 1\). The laggard’s value function is given by
\[
v(-n) = (1-\delta) p(-n) + \delta [\phi v(-n-1) + \phi v(-n+1) + (1-2\phi) v(-n)],
\]
where \(\phi \equiv \frac{1}{2} [1 - x(-n)]\). Suppose that \(x(-n) = 1\). Then \(\phi = 0\), and (1) implies that \(v(n) = p(n)\), which contradicts the starting hypothesis. Suppose therefore that \(x(-n) \neq 1\). Then \(\phi \neq 0\), and (1) implies that
\[
v(-n) - \frac{1}{2} v(-n - 1) - \frac{1}{2} v(-n + 1) = \frac{1 - \delta}{\delta} [p(-n) - v(-n)].
\]
This implies that \(v(-n)\) is locally convex, since \(p(-n) < v(-n)\). The optimal \(x(-n)\) is therefore the one that maximizes \(\phi\), that is, \(x(-n) = 0\). By the same token, if \(p(n) > v(n)\), then \(x(-n) = 0\) minimizes the value function, and so it is not optimal. \(\square\)

**Proof of Proposition 3.** For \(n > \bar{n}\), we must have \(p < v(-n), v(n) < \bar{p}\). Indeed, suppose the first inequality is not true and \(v(-n') = p\), \(v(-n' + 1) > p\). This is only possible if \(x(n') = x(-n') = 1\). But clearly this is not an equilibrium, for in state \(n'\) the player receiving a payoff of \(p\) would increase his or her value by choosing a different \(x(-n')\). By the same token, \(v(n') = \bar{p}\) would be impossible in equilibrium. The result then follows from Proposition 2. \(\square\)
References


