The Term Structure of Money Market Spreads During the Financial Crisis[*]

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Abstract

I estimate a no-arbitrage model of the term structure of money market spreads during the financial crisis to attribute movements in spreads to credit and liquidity factors. The model restrictions imply that longer-term spreads are linear, risk-adjusted expectations of future short-term spreads. This linear representation of spreads can be partitioned into two components: one related to time-varying expectations of spreads, and another to time-variation in risk premia. Estimation highlights the importance of time-variation in risk premia. Up to 50% of the variation of spreads is explained by time-varying risk premia, and risk premia have significant predictive power for spreads.

Keywords: Interest rate spreads, credit risk, liquidity risk, risk premia, term structure

JEL Classification Codes: E430, E440, G100, G120, G180

Interest rate spreads are a common measure of financial market stress, and the recent financial crisis saw an unprecedented increase in both the level and volatility of spreads in a variety of markets. One particular money market spread receiving attention has been the spread between LIBOR and OIS rates of comparable maturity, the LOIS spread. LIBOR rates are interest rates for unsecured, longer-term interbank lending, while OIS rates are a measure of secured, short-term interbank lending, often used as proxy for expectations of future Federal Reserve policy. Figure 1 plots LOIS spreads at the one-, three-, and twelve-month maturities. Before the onset of the crisis, these spreads were low and exhibited very little time variation. However, August of 2007 saw a sharp increase in the LOIS spreads and they have fluctuated well above historical averages since then, rising to over 300 bps during the panic of 2008.

The purpose of this paper is to decompose the observed interest rate spreads into two distinct yet interrelated factors: credit and liquidity. By credit, I am referring to the perceived increase in default probabilities of financial institutions who participate in the LIBOR survey, since counterparties want to be compensated for any losses that might occur in the event of default. By liquidity, I am referring to the premium needed to entice investors for illiquidity, the fear that assets in their portfolios might not be able to be traded easily and without significant price impact on other assets. To do so, I estimate a fully-specified model of the LOIS term structure using the no-arbitrage, affine models of Duffie and Singleton (1999) and Ang and Piazzesi (2003). These models build on the theory of the Expectations Hypothesis (EH), in which longer-term interest rates are expectations of future short-term interest rates plus a constant term premium. My model predicts that LOIS spreads are linear, *risk-adjusted* expected values of future short-term spreads:

$$z_t^{(n)} = a_n + b_n^{\mathrm{T}} X_t, \tag{1}$$

where $z_t^{(n)}$ is the LOIS spread for maturity n, X_t is the vector of state variables, and b_n is a vector of response coefficients of the LOIS spread to the state variables. The model assumes that the state vector X_t follows a vector autoregression (VAR), and thus the term structure in equation (1) reduces to a VAR with non-linear, cross-equation restrictions imposed by no-arbitrage.

The model delivers closed-form solutions for LOIS spreads as a function of the state vector X_t, and I can identify how much of the spreads is due to each of the variables included in X_t. Specifically, I include three different variables in X_t: a benchmark interbank interest rate, a proxy for credit, and a proxy for liquidity, all three of which are entirely *observable*. This is in contrast to Dai and Singleton (2002), among others, which uses only latent variables in the state vector. Ang and Piazzesi (2003), Piazzesi (2005), Ang, Piazzesi, and Wei (2006), and Ang, Dong, and Piazzesi (2007) incorporate observable macroeconomic variables to replace latent factors and obtain a better fit of yield curve dynamics.

A critical contribution of the model comes from a partition of the response coefficients b_n from equation (1) that separates LOIS spreads into two components. The first is related to time-varying expectations of the future spread between LIBOR and the Federal Funds rate, while the second is due to time-variation in risk premia caused by changes in risk attitudes of investors. Since the model provides closed-form solutions for the separate components, I can identify how much of the movements in spreads is directly attributable to each. Combining the VAR estimation with estimates of risk premia, LOIS spreads react most sensitively to movements in the credit factor. A substantial proportion of the volatility of spreads can be explained by risk premia (up to 50% for the twelve-month LOIS), and the response of spreads to shocks to X_t is most sensitive to credit risk premia. To test the model, I examine the behavior of relative excess returns between LIBOR and OIS, which my model predicts are attributable solely to time-variation in risk premia. Regressions of relative excess returns on the state vector X_t result in a significant correlations of both the

credit and liquidity factors with returns, highlighting the importance of time-varying risk premia in explaining LOIS spreads.

This paper relates to recent empirical work that has decomposed the increase in spreads such as Taylor and Williams (2008), McAndrews et.al. (2008), and Schwarz (2009). These papers use ordinary least squares (OLS) regressions to attribute the rise in spreads to the same two factors I concentrate on, credit and liquidity. While the coefficient estimates derived from OLS regressions of LOIS spreads on my measures of credit and liquidity are comparable to the response coefficients in equation (1), there is no way to decompose the OLS coefficient estimates to identify time-variation in risk premia. Another recent literature has developed that incorporates these affine pricing tools in structural general equilibrium models, including Bekaert et.al. (2005), Rudebusch and Wu (2007), Gallmeyer et.al. (2007), and Rudebusch and Swanson (2009). A full general equilibrium model would develop a structural relationship between the macroeconomy, interest rate spreads, and risk premia. However, the question I am asking is only related to an empirical explanation of the term structure of LOIS spreads as a function of credit and liquidity.

The model has potential implications for how policy can respond to money market spreads. This is of interest since most of the non-traditional policy actions in 2007 and early 2008 by the Federal Reserve promoted liquidity injections. The results from my model suggest that coincident policy aimed at credit (i.e. capital requirements, leverage ratios, etc.) might have decreased spreads much more by driving down risk premia. Comparing the linear equations for spreads from my model to OLS regression results suggests that the no-arbitrage restrictions imposed on the estimation of the response coefficients b_n dampen the effect of the liquidity factor relative to the OLS results, while the results for the interest rate and credit factors are similar across the two specifications. The predictive power of the model and the fact that it allows exact identification of time-varying risk premia due to no-arbitrage provides policymakers with another tool when analyzing how monetary policy

should react to movements in financial markets going forward.

The setup of this paper is as follows. Section 1 describes LIBOR and OIS contracts. Section 2 outlines the model and motivates the affine model used in this analysis as an alternative to common intuition underlying models of interbank rates. Section 3 details the estimation procedure and results. Section 4 describes the behavior of LOIS spreads predicted by the model. Section 5 discusses the importance of risk premia. Section 6 concludes.

1 LIBOR and Overnight Index Swaps

This section provides details on the LOIS interest rate spreads by describing the LIBOR and OIS contracts and explaining their behavior during the recent financial crisis.

1.1 LIBOR Interest Rates

LIBOR stands for the London Interbank Offered Rate published by the British Banker's Association. LIBOR indicates the average rate that a participating institution can obtain *unsecured* funding for a given period of time in a given currency in the London money market. The rates are calculated based on the trimmed, arithmetic mean of the middle two quartiles of rate submissions from a panel of the largest, most active banks in each currency. In the case of the U.S. LIBOR, the panel consists of fifteen banks. These rates are a benchmark for a wide range of financial instruments including futures, swaps, variable rate mortgages, and even currencies.

Each participating bank is asked to base its quoted rate on the following question: "At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to 11 a.m. London time?" An important distinction is that this is an offered rate, not a bid rate, for a loan contract. Actual

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transactions may not occur at this offered rate, but LIBOR rates do reflect the true cost of borrowing given the sophisticated methods each participating bank has at its disposal to ascertain risks in the underlying financial markets when it chooses to enter financial contracts.[1]

Figure 2 plots the weekly averages of the daily Federal Funds rate along with weekly averages of the daily one-, three-, and twelve-month LIBOR rates. Before the onset of the financial crisis, LIBOR rates closely tracked the Federal Funds rate. Yet with the crisis came a decoupling of LIBOR rates from the Federal Funds rate, with LIBOR rates fluctuating above the Federal Funds rate. While comparing these two rates is interesting, the Federal Funds rate is an overnight rate, while LIBOR is a term rate. Therefore, the next section will discuss an interest rate with comparable maturity to the LIBOR rate that captures movements in the Federal Funds rate.

1.2   Overnight Index Swaps

An overnight indexed swap (OIS) is a fixed/floating interest rate swap where the floating rate is determined by the geometric average of a published overnight index rate over each time interval of the contract period. The two counterparties of an OIS contract agree to exchange, at maturity, the difference between interest accrued at the agreed fixed rate and the floating rate on the notional amount of the contract. The party paying the fixed OIS rate is, in essence, borrowing cash from the lender that receives the fixed OIS rate. No principal is exchanged at the beginning of the contract. In contrast to a plain vanilla swap, there are

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[1]Controversy has been raised over the reliability of the offering rates that LIBOR banks were posting during the crisis. Institutions were thought to be quoting lower rates at which they could take on interbank loans in an effort to disguise any default risk they thought was present in their respective institution. However, this paper looks at spreads between LIBOR and comparable interest rates, and thus the spreads reported in this paper are consistent with a lower bound on spreads that might have been reported if offered rates had been higher during the crisis. In addition, the only LIBOR rates proven to be fraudulent at the time of this draft are denominated in Euros. The results for Term Fed Funds provide additional evidence regarding the prevalence of risk premia in these short-term money markets.

no intermediate interest payments. In the case of the United States, the floating rate of the OIS contract is tied to the Federal Funds rate. The fixed rate of an OIS in the United States is meant the capture the expected Federal Funds rate over the term of the swap plus any potential risk premia.

It is useful to understand the mechanics of how an OIS operates. Assume that the time interval is weekly, where w reflects the number of weeks in the contract of the swap. Let N denote the notional amount of the OIS, i_{fixed} the fixed rate, i_{float} the floating rate, $i_{FF,t}$ the Federal Funds rate at time t, and T the maturity date. Table 1 shows the payments made during the duration of the swap, where the floating rate of the swap is computed using the formula below:

$$i_{float} = \frac{52}{w} \left[\prod_{i=t}^{T-1} \left(1 + \frac{i_{FF,i}}{52} \right) - 1 \right]. \tag{2}$$

As a simple example, assume that a one-month OIS contract is signed on $t = 01/01/2009$ to mature at time $T = 02/01/2009$. This implies that there are four relevant Federal Funds rates that will be used to compute the floating rate given by equation (2) at time T. On 01/01/2009, two parties agree to exchange $N = \$1000$ at a fixed rate of $i_{fixed} = 5.05\%$. Party A is the receiving party of the swap, which means they receive the fixed rate i_{fixed} and pay the floating rate i_{float}. Party B maintains the opposite position, receiving the floating rate and paying the fixed rate. At time T, when computing the floating rate, the following stream of Federal Funds rates are observed: $i_{FF,1/1} = 5.00\%$, $i_{FF,1/8} = 5.05\%$, $i_{FF,1/15} = 5.01\%$, and $i_{FF,1/22} = 5.01\%$. Given this information, the floating rate is computed as:

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\begin{aligned}
i_{float} &= \frac{52}{4} \times \left[\left(1 + \frac{0.0500}{52} \right) \left(1 + \frac{0.0505}{52} \right) \left(1 + \frac{0.0501}{52} \right) \left(1 + \frac{0.0501}{52} \right) - 1 \right] &(3) \\
&= 13 \times [(1.00096)(1.00097)(1.00096)(1.00096) - 1] &(4) \\
&= 13 \times 0.00386 &(5) \\
&= 5.02\%. &(6)
\end{aligned}
$$

Therefore, the net payment to Party A (since he/she paid the lower interest rate of $i_{float} = 5.02\%$) will be $1000 $\times 0.03\% \times \frac{1}{13}$ = $2.31. The net payment is much smaller than the notional amount of the OIS. This reflects that the fixed rate was a good approximation of the expected Federal Funds rate over the term of the contract, and also that the OIS contract is not contaminated with as high a level of default or liquidity characteristics as LIBOR.

Figure 3 shows how OIS fixed rates have moved during the financial crisis, and compares them to movements in the underlying Federal Funds rate. The figure plots weekly averages of the daily Federal Funds rate and weekly averages of the daily one-, three-, and twelve-month OIS rates based on an average of OIS rates quoted each day. Even with the onset of the crisis, OIS rates remained very close to the Federal Funds rate. Longer-term OIS rates differ from short-term OIS rates due to the longer length of the contract and more uncertainty in movements of the underlying Federal Funds rate.

1.3   Motivation for a Model of LOIS Spreads

Suppose we specify a model for the term structure of unsecured, longer-term interbank interest rates. With the yield curve derived from this model, we can price a variety of derivatives, including forward contracts and swaps. Let $i_t^{(1)}$ denote the short-term interbank rate at time t, $i_t^{(n)}$ the longer-term interbank rate at time t that matures at time $t + n$, and $f_t^{(n-1)\to(n)}$ the forward rate at time t for a contract that starts in time $t + n - 1$ and ends at time $t + n$. The common way to derive forward rates is by using current, short-term interbank rates using the following formula:

$$f_t^{(n-1)\to(n)} = \mathbb{E}_t[i_{t+n-1}^{(1)}] + c. \tag{7}$$

Equation (7) states that the forward rate is the expected future short rate $\mathbb{E}_t[i^{(1)}_{t+n-1}]$ plus a constant term premium c. Therefore, any movements in the short end of the interbank yield curve will directly impact forward rates.

Interest rate swaps on interbank interest rates are essentially bundles of forward contracts on the underlying short-term interbank interest rate. Therefore, the yield curve for interbank rates will imply a series of forward rates, and bundling these forward rates provides us with swap rates. Figure 4 provides a diagram of an unsecured, longer-term interest rate $i^{(n)}_t$ (e.g. the LIBOR rate), and the intervals of short-term interest rates between the start time t and the maturity date $t+T$ from which the swap rate (e.g the OIS rate) will be calculated. With the entire yield curve for unsecured interbank rates, swap rates can be computed using the short-term interest rate from that yield curve.

Using Figure 4, we can back out that the longer-term rate $i^{(n)}_t$ is approximately equal to the probability of default (ignoring liquidity effects) over the time interval $[t,T]$, while the swap rate is the geometric average of the probability of default in the intervals $[t,t+1]$, $[t+1,t+2],\ldots,[T-1,T]$ generated by rolling over each of the spot rates at each time interval. If it's the case that longer-term rates are high due to the probability of default (i.e. default rates are higher over longer horizons), then forward contracts on the short-term rate must be high as well. This is because any default priced into the unsecured, interbank yield curve will be picked up by the short rate $i^{(1)}_t$, and thus forward rates will reflect it, as well, as seen in equation (7). Therefore, spreads between unsecured, longer-term interbank rates like LIBOR and their swap counterparts like OIS would be low, a prediction rejected by the data during the financial crisis.

Why might LOIS spreads be so high? First, the unsecured nature of LIBOR loans implies that the quoted borrowing rates provided by LIBOR-participating institutions must take into account the lack of collateral posted at the start of the contract. A second reason is because the probability of default (and/or liquidity pressures) between, for example

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$[T-1, T]$, is much higher than the probability of default between $[t, t+1]$, and the geo-metric averaging performed in computing swap rates creates a large difference between the probability of default between $[t, T]$ and the average of the smaller time intervals. Lastly, the parties involved in short- vs. longer-term contracts might differ. High-default institutions could show up for longer-term loans, while low-default institutions show up for short-term loans.[2] This implies that the OIS is a derivative that has as the basis a short-term rate for low-default institutions, while the longer-term LIBOR rate is high for high-default institutions. Therefore, taking a geometric average of the short-term rate when computing the forward contracts that make up the OIS rate provides a lower rate than the longer-term LIBOR rate due to the default adjustment. In the next section, I will provide a model that reconciles spreads between LIBOR and OIS rates using an adjusted short rate for LIBOR loans that takes account of credit and liquidity factors.

2 Model of the Term Structure of Money Market Spreads

I describe a model of the LOIS term structure during the financial crisis that incorporates movements in credit and liquidity factors and relies on both the defaultable bond work of Duffie and Singleton (1999) and the no-arbitrage, discrete time affine models of Ang and Piazzesi (2003). LIBOR and OIS will each be modeled as zero-coupon bonds, and the assumption of no-arbitrage provides closed-form solutions for both prices and yields of these bonds.

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[2]Indeed, while only 15 institutions participate in the LIBOR survey, over 30 participate in the OIS market. Therefore, the average probability of default across all of the institutions in the OIS market could be far below that for the LIBOR market.

2.1 VAR Dynamics

As in Ang and Piazzesi (2003), suppose X_t is a 3-dimensional vector of state variables driving the economy, and assume that X_t follows a Gaussian VAR(1) process:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{8}$$

where $\varepsilon_t \sim$ iid $N(0, I)$, Σ is assumed to be lower-triangular, and t is at a weekly frequency. In this analysis, X_t consists of (in order) an interest rate factor r_t, a credit factor C_t, and a liquidity factor L_t, all of which are observable variables. ε_t is the vector of shocks driving the system in equation (8), and the system is Cholesky factorized in order to separately identify the impact of shocks to the system.

2.2 The LOIS Term Structure

Let $i_t^{L,(n)}$ denote the n-period LIBOR rate, and $i_t^{O,(n)}$ the n-period OIS rate. I make the following assumptions on the short-term interest rates:

$$i_t^{O,(1)} = r_t \tag{9}$$

$$i_t^{L,(1)} = i_t^{O,(1)} + \gamma_t. \tag{10}$$

The adjustment term γ_t is linear X_t:

$$\gamma_t = \gamma_0 + \gamma_1^{\mathrm{T}} X_t, \tag{11}$$

where $\gamma_1^{\mathrm{T}} = (0, \gamma_{1,1}, \gamma_{1,2})$ picks up movements in the credit and liquidity factors. Equation (10) states that the short-term rate for LIBOR is equal to the short-term rate for OIS plus γ_t, which depends on the credit and liquidity factors. Here, I assume that the Federal Funds

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rate is the benchmark, safest short-term rate for the interbank market.

2.2.1 Market Prices of Risk

In order to derive the LOIS term structure under no-arbitrage, I posit a particular pricing kernel for the pricing of LIBOR and OIS securities, which will be given by the following convenient form:

$$m_{t+1} = \exp(-\text{shortrate}_t) \exp(-0.5\lambda_t^T \lambda_t - \lambda_t^T \varepsilon_{t+1}), \tag{12}$$

where the market price of risk λ_t is linear in the state vector:

$$\lambda_t = l_0 + l_1 X_t. \tag{13}$$

Taking logs and rewriting equation (12) implies that

$$\ln(m_{t+1}) = -\text{shortrate}_t - 0.5\lambda_t^T \lambda_t - \lambda_t^T \varepsilon_{t+1}. \tag{14}$$

The log pricing kernel in equation (14) is conditionally normally distributed, with $\mathbb{E}_t[\ln(m_{t+1})] = -\text{shortrate}_t$. Therefore, the pricing kernel (12) is conditionally log-normally distributed, with $\mathbb{E}_t[m_{t+1}] = \exp(-\text{shortrate}_t)$. A log-normally-distributed pricing kernel of a similar form can be derived from the utility maximization problem of a representative consumer with CRRA utility and log-normally-distributed consumption growth. To see this, recall that the nominal pricing kernel for such a representative agent is given by the following formula:

$$\frac{m_{t+1}}{\Pi_{t+1}} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}, \tag{15}$$

where β is the discount factor, C_t is consumption at date t, γ is the coefficient of relative

risk aversion, and Π_{t+1} is the gross rate of inflation between periods t and $t+1$. Assuming $\ln\left(\frac{C_{t+1}}{C_t}\right) \sim N(\mu_{c,t}, \sigma_{c,t}^2)$ and with assumptions on the processes for $\mu_{c,t}$ and $\sigma_{c,t}^2$, we can obtain pricing kernels similar to equation (12).[3]

The pricing kernel has two components. First, there is standard discounting given by the term $\exp(-\text{shortrate}_t)$, where $-\text{shortrate}_t$ will be given by equation (9) for OIS rates and equation (10) for LIBOR rates. Second, there is an added term $\exp(-0.5\lambda_t^T\lambda_t - \lambda_t^T\varepsilon_{t+1})$ that incorporates the risk premia λ_t. This term does not differ across LIBOR and OIS, which implies that the uncertainty embedded within the pricing kernel does not differ across the two securities. In the absence of risk premia in which $\lambda_t = 0 \; \forall t$, investors are risk neutral and the Expectations Hypothesis would hold, implying that longer-term interest rates equal the expectations of future short-term rates not adjusted for time-varying risk. Risk premia in this model are linear, where l_0 captures the constant risk premia and l_1 captures any time-variation in risk premia.

2.2.2 The Difference Between γ_t and λ_t

Two key features of the model are equation (11) for γ_t and equation (13) for λ_t. Both are linear functions of the state vector X_t, but their roles in the model are distinct. γ_t is motivated by the continuous-time models of Duffie and Singleton (1999), which specify affine, no-arbitrage models for unsecured, longer-term zero-coupon bonds. The authors show how affine assumptions on the default and/or liquidity processes of these assets imply closed form solutions for both prices and yields. Assume that, conditional on no default before time t, the probability of default between s and $s+1$ for $s > t$ is given by c_s. In addition, assume that the liquidity of the asset between s and $s+1$ be given by l_s. If the processes for c and l are specified as affine and exogenous, then unsecured, longer-term interest rates

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[3]For an overview of the consumption-based pricing kernels, see Campbell, Lo, and MacKinlay (1997) and Cochrane (2005).

can be derived using an adjusted short-term interest rate, an *implicit* rate composed of the true short-term interest rate and adjustments for the default and liquidity processes.[4] In the model, the wedge between the LIBOR short rate $i_t^{L,(1)}$ and the true interbank short rate $i_t^{O,(1)} = r_t$ is γ_t, which is affine in X_t and deterministic. Any difference between the LIBOR short rate and the true interbank short rate is due to a constant plus movements in the credit and liquidity factors.

λ_t represents the risk premia in the model. In order to make assets attractive to risk averse investors, the assets are priced with a premia given by λ_t that is attached to unexpected movements in the state vector X_t. In this model, λ_t is a three-dimensional vector that measures interest rate, credit, and liquidity risks. Although λ_t is known at time t, its effect on pricing is not realized until time $t+1$. This can be seen through the pricing kernel equation (12) where λ_t is attached to ε_{t+1}, the shocks to the state vector X_{t+1}. If $\lambda_t = 0$, then the pricing kernel would collapse to $\exp(-\text{shortrate}_t)$ and all pricing would be done as if investors were risk neutral. Note, however, that the short rate term still depends on γ_t. γ_t can be non-zero for both risk averse and risk neutral investors, whereas λ_t can be non-zero only for risk averse investors.

2.2.3 No-Arbitrage Pricing Functions

For a zero-coupon bond at time t with maturity n and price $P_t^{(n)}$, the assumption of no-arbitrage is equivalent to the following:

$$P_t^{(n)} = \mathbb{E}_t[m_{t+1} P_{t+1}^{(n-1)}], \tag{16}$$

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[4]The continuous-time-to-discrete-time conversion relies on several technical conditions outlined in Duffie and Singleton (1999).

where $P_t^{(1)} = \mathbb{E}_t[m_{t+1} \times 1] = \exp(-\text{shortrate}_t)$. Applying this to the pricing functions for both LIBOR and OIS implies that:

$$P_t^{L,(n)} = \mathbb{E}_t[\exp(-i_t^{L,(1)})\exp(-0.5\lambda_t^{\mathrm{T}}\lambda_t - \lambda_t^{\mathrm{T}}\varepsilon_{t+1})P_{t+1}^{L,(n-1)}] \tag{17}$$

$$P_t^{O,(n)} = \mathbb{E}_t[\exp(-i_t^{O,(1)})\exp(-0.5\lambda_t^{\mathrm{T}}\lambda_t - \lambda_t^{\mathrm{T}}\varepsilon_{t+1})P_{t+1}^{O,(n-1)}], \tag{18}$$

where $P_t^{L,(n)}$ is the price of the n-period LIBOR and $P_t^{O,(n)}$ is the price of the n-period OIS. I substitute for the pricing kernel given by equation (12) with the appropriate short rate for each security. I conjecture that the pricing functions are exponential affine in the state vector:

$$P_t^{L,(n)} = \exp(A_{L,n} + B_{L,n}^{\mathrm{T}}X_t) \tag{19}$$

$$P_t^{O,(n)} = \exp(A_{O,n} + B_{O,n}^{\mathrm{T}}X_t). \tag{20}$$

Using the conjecture on LIBOR given by equation (19) and the assumptions on the short rates given by equations (9) and (10) leads to the following solution for the recursive LIBOR coefficients:

$$A_{L,n+1} = A_{L,n} + B_{L,n}^{\mathrm{T}}(\mu - \Sigma l_0) + 0.5B_{L,n}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{L,n} - \gamma_0 \tag{21}$$

$$B_{L,n+1}^{\mathrm{T}} = B_{L,n}^{\mathrm{T}}(\Phi - \Sigma l_1) - (1, \gamma_{1,1}, \gamma_{1,2}), \tag{22}$$

where $A_{L,1} = -\gamma_0$ and $B_{L,1}^{\mathrm{T}} = -(1, \gamma_{1,1}, \gamma_{1,2})^{\mathrm{T}}$. Using a similar conjecture on OIS, the recursive coefficients for OIS are given by:

$$A_{O,n+1} = A_{O,n} + B_{O,n}^{\mathrm{T}}(\mu - \Sigma l_0) + 0.5B_{O,n}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{O,n} \tag{23}$$

$$B_{O,n+1}^{\mathrm{T}} = B_{O,n}^{\mathrm{T}}(\Phi - \Sigma l_1) - (1, 0, 0), \tag{24}$$

where $A_{O,1} = 0$ and $B_{O,1}^T = -(1,0,0)^T$.

Using these recursive coefficients, the continuously-compounded yields for LIBOR and OIS can be written as:

$$i_t^{L,(n)} = a_{L,n} + b_{L,n}^T X_t \tag{25}$$

$$i_t^{O,(n)} = a_{O,n} + b_{O,n}^T X_t, \tag{26}$$

where $a_{L,n} = -A_{L,n}/n$ and $a_{O,n} = -A_{O,n}/n$ are the constants, and $b_{L,n}^T = -B_{L,n}^T/n$ and $b_{O,n}^T = -B_{O,n}^T/n$ are the response coefficients for the LIBOR and OIS yields, respectively.

2.3 LOIS Spreads

Given the closed form solutions (25) and (26), define the LOIS spread for maturity n as:

$$z_t^{(n)} = i_t^{L,(n)} - i_t^{O,(n)} \tag{27}$$

$$= (a_{L,n} + b_{L,n}^T X_t) - (a_{O,n} + b_{O,n}^T X_t) \tag{28}$$

$$= a_n + b_n^T X_t, \tag{29}$$

where $a_n \equiv a_{L,n} - a_{O,n}$ and $b_n^T \equiv b_{L,n}^T - b_{O,n}^T$. What is of interest in the analysis in this paper is the response coefficient vector b_n, since it determines how spreads respond to movements in the underlying factors of the economy. While the model is linear, these coefficients b_n have risk premia built into them, and I will examine how behavior of this risk premia generates movements in spreads.

3 Data and Econometric Methodology

This section describes the data and estimation procedure used for the model specified in Section 2, as well as any robustness exercises.

3.1 Data

The sample period used for estimation is from 1/1/2007 - 6/18/2009, which encompasses the lead-up to and duration of the financial crisis. For the estimation, I use weekly averages of daily data for one-, three-, six-, nine-, and twelve-month maturities of LIBOR $i_t^{L,(n)}$ and OIS rates $i_t^{O,(n)}$. I use these to compute the LOIS spreads $z_t^{(n)}$ to be used in the estimation, which are plotted in Figure 1. In order to estimate the short rate parameters γ_0 and γ_1, I use data on the one-week LIBOR rate. For robustness, rather than using LIBOR rates as a measure of unsecured, longer-term interbank borrowing, I estimate the model using term Federal Funds rates. These are rates on longer-term interbank borrowing in the Federal Funds market, as opposed to the LIBOR market. Any issues about how the LIBOR survey is conducted and how this would bias the LIBOR rates reported by the BBA is not present in these term Federal Funds rates. Estimation results for the term Federal Funds rates in place of LIBOR is reported in the appendix in Table 12. The vector of economic factors X_t includes weekly averages of daily data on the Federal Funds rate r_t, the credit factor C_t, and the liquidity factor L_t. In Figure 5, I plot the Federal Funds rate, along with the credit and liquidity factors described below.

3.1.1 The Credit Factor

For the credit factor, I use the spread between the three-month LIBOR and the three-month interbank repurchase agreement rates. Repurchase agreements (REPOs) are *secured* financial contracts that allow the borrower to use financial securities as collateral for cash loans

at a fixed interest rate, in contrast to LIBOR contracts which are *unsecured*. Here, I have used REPOs in which essentially default-free U.S. Treasury securities are posted as collateral, which allows me to identify these interbank REPOs as safe, collateralized interbank loans. Therefore, the spread between LIBOR and REPO rates is a measure of the credit in the interbank market. In addition, since parties involved in LIBOR and REPO contracts are similar, any variation in liquidity would be common across the two securities and removed once examining the spread.

In addition, I perform robustness exercises using other measures of the credit factor, including: (i) the one-month LIBOR-REPO spread, (ii) the median five-year credit default swap (CDS) rate for LIBOR institutions, (iii) the mean of all three of the credit measures, and (iv) the first principal component of all three of these measures. The results of the robustness across different measures of this credit factor are reported in the appendix in Tables 11 and 12.

CDS are essentially insurance policies on corporate bonds. The buyer of a CDS pays a periodic fee to the seller in exchange for a promise of payment, in the event of bankruptcy or default of the institution that issued the bond, equal to the difference between the par and market values of the bond. The CDS rates in the data are reported in terms of the premium to be paid. For example, suppose the five-year CDS for Big Bank Corp. was quoted around 160 bps on January 1, 2009. This means that if you want to buy the 5-year protection for a $100 million exposure to Big Bank Corp. credit, you would pay 40 bps, or $400,000, every quarter as an insurance premium in the event of default. Figure 6 plots the one-month LIBOR-REPO spread, three-month LIBOR-REPO spread, and five-year CDS median over the sample period. The first two measures follow each other closely, while the CDS median has maintained an upward trend throughout the sample period.

3.1.2 The Liquidity Factor

For the liquidity factor, I use the premium between on-the-run and off-the-run ten-year U.S. Treasury securities. The on/off-the-run premium results from the fact that off-the-run securities are sold at a discount to comparable on-the-run securities. On-the-run securities are more liquid relative to off-the-run securities due to search technologies in these markets, their large volume at issuance, sophistication of the participating investors, relative supply of each in the secondary market, etc., as stressed in papers such as Duffie et.al. (2005), Vayanos and Weill (2008), and Pasquariello and Vega (2009). The on-the-run data series is the yield of the current ten-year on-the-run security as provided by the Federal Reserve Bank of New York, while the off-the-run data series is a synthetic series of the off-the-run equivalent yield of the current ten-year on-the-run security using a zero-coupon yield curve. Longstaff (2004) discusses a similar measure using the difference between Treasury bond prices and the prices of bonds issued by the Resolution Funding Corporation (Refcorp), which is fully-collateralized by the U.S. government. Schwarz (2009) uses a measure of liquidity similar to Longstaff (2004), particularly the difference between German government bonds and bonds issued by the KfW development bank, both of which are backed by the German government.

Empirical investigations of liquidity in the U.S. Treasury market have been examined by various authors. Brandt and Kavajecz (2003) examine price discovery throughout the relationship between liquidity, order flow, and the yield curve. Fleming (2003) documents the time series behavior of a set of liquidity measures, including the on/off-the-run premium, using high-frequency data and points out the advantages and disadvantages of certain measures over others. Finally, Chordia et.al.(2005) examines liquidity across equity and fixed-income markets and shows using vector autoregressions that liquidity within and across these two markets is highly correlated, implying that a common factor drives liquidity in

these markets.

The measure of liquidity I use is meant to capture market liquidity, the ease at which an asset can be traded. Brunnermeier and Pederson (2009) provide a model that stresses the difference between an asset's market liquidity and a trader's funding liquidity, or the ease at which traders can obtain funding. Market liquidity is asset-specific, and is modeled as any deviation of an asset's price from its fundamental value. The on- and off-the-run securities have identical future cash flows, and thus they should have the same fundamental value. The on/off-the-run premium exists because one (or both) of the prices differs from the fundamental value. Funding liquidity is agent specific, but the two concepts are mutually reinforcing. However, since market liquidity is asset-specific, issues can arise using a U.S. Treasury measure of liquidity for an analysis studying the LIBOR market. While a LIBOR measure of liquidity would be most desirable, calculating such a measure using traditional metrics such as bid-ask spreads, market frequency and volume, or price impact coefficients leaves a liquidity measure that is contaminated by the default characteristics of LIBOR. The on/off-the-run premium avoids this issue, since the underlying bonds are backed by the (essentially) default-free U.S. government.

3.2   Estimation Procedure

In order to estimate the model, define $\eta_{t,n}$ as the vector of residuals:

$$\eta_{t,n} \;=\; z_t^{(n)} - \hat{z}_t^{(n)} \tag{30}$$

$$=\; z_t^{(n)} - \hat{a}_n - \hat{b}_n^{\mathrm{T}} X_t \tag{31}$$

$$=\; z_t^{(n)} - (\hat{a}_{L,n} - \hat{a}_{O,n}) - (\hat{b}_{L,n}^{\mathrm{T}} - \hat{b}_{O,n}^{\mathrm{T}}) X_t, \tag{32}$$

where $z_t^{(n)}$ denotes the actual LOIS spreads and $\hat{z}_t^{(n)} = \hat{a}_n - \hat{b}_n^T X_t$ are predicted LOIS spreads from the theoretical model. Using these errors η_t, the estimation procedure involves two

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stages:

(i) Fix μ to match the unconditional mean of X_t. Using seemingly unrelated regressions (i.e. OLS equation-by-equation), estimate $\Theta_1 \equiv (\Phi, \Sigma, \gamma_0, \gamma_1)$.

(ii) Holding Θ_1 fixed, estimate $\Theta_2 \equiv (l_0, l_1)$ using non-linear least squares, where the minimization problem is given by:

$$\min_{l_0, l_1} \sum_{t=1}^{T} \sum_{n=1}^{N} \eta_t^{(2)}. \tag{33}$$

Since the objective function is highly non-linear, I estimate the model 10,000 times in order to get reliable starting values for the minimization procedure. I then estimate the model around these starting values to find the minimum. I compute first-stage robust GMM standard errors. To correct for any first-stage estimation error, second stage standard errors are computed by bootstrapping the data sample 10,000 times and taking the posterior standard deviation of the parameter estimates.

3.3 Parameter Estimates

Table 2 reports estimation results for Θ_1, which includes the VAR dynamics and the short rate equation for $i_t^{L,(1)}$. The first row of the matrix Φ corresponds to the equation for the Federal Funds rate r_t, the second row corresponds to the equation for the credit factor C_t, and the third row corresponds to the equation for the liquidity factor L_t. The first column of Φ corresponds to the parameter estimate for the impact of r_{t-1} on each of the variables in X_t, while the second and third columns correspond to the effects of C_{t-1} and L_{t-1} on the variables in X_t, respectively. C_{t-1} is estimated to be statistically significant in each of the equations of the VAR, while r_{t-1} and L_{t-1} are significant only in the equations corresponding to themselves. This implies that the credit factor Granger-causes the other

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factors, thus having significant predictive power for movements in both the Federal Funds rate and the liquidity factor. Innovations to each of the factors exhibit low correlation as reported by the estimated off-diagonal elements of $\Sigma\Sigma^T$. This implies that the Cholesky identification of shocks is satisfied and does not restrict the parameter estimates.[5]

The short rate dynamics for the estimation $\hat{i}_t^{L,(1)}$ can be found on the right side of Table 2. The first element of γ_1 corresponds to the Federal Funds rate (which is identical to the short-term OIS rate $i_t^{O,(1)}$), and is constrained to be one as given by equations (9) and (10). From the estimates, a one percent increase in the credit factor increases the short-term LIBOR rate by 0.8%, while a one percent increase in the liquidity factor actually decreases the short-term LIBOR rate by 0.95%. Conditional on movements in credit, liquidity imposes a negative premium on the short-term rate. This could be due to a larger amount of longer-term investors in the market, or due to expectations of falling short-term interest rates in the future.[6]

Figure 7 plots the two short-term rates and the estimated $\hat{\gamma}_t$ premium. The solid black line is the short-term OIS rate $i_t^{O,(1)} = r_t$, the green line is the estimated short-term LIBOR rate $\hat{i}_t^{L,(1)}$, and the dashed line is the estimate of $\hat{\gamma}_t = \hat{i}_t^{L,(1)} - i_t^{O,(1)}$. In the early part of the sample before the crisis, the $\hat{\gamma}_t$ premium was zero, and began increasing in the summer of 2007. During the panic in late 2008, the premium increased to over 2%, and has settled back down to near zero since. Even though the liquidity factor was estimated with a negative coefficient in the equation for $\hat{\gamma}_t$, the estimated effect of the credit factor is far larger than that for the liquidity factor and results in a positive $\hat{\gamma}_t$.

Turning to the dynamic responses of each of the factors to shocks to the VAR, Figure 8

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[5]Estimating an AR(1) process for the Federal Funds rate r_t shows that there is a unit root in the process for r_t. However, the VAR(1) specification allows me to pin down the stationary dynamics of the Federal Funds rate, since it not only depends on its own lags but lags of both the credit factor C_t and the liquidity factor L_t. The eigenvalues of the system are all within the unit circle, with the modulus of the eigenvalue vector given by (0.99, 0.91, 0.91).

[6]In the robustness exercises reported in the appendix, both the one-month LIBOR-REPO spread and CDS median credit factors do not exhibit this negative coefficient on the liquidity factor in the equation for γ_t.

plots the impulse response functions (IRF) from the VAR. Each panel depicts the Cholesky-decomposed response of a one-standard deviation shock to the relevant factor. The first row of Figure 8 shows how each of the factors responds to shocks to the Federal Funds rate r_t. Interestingly, a positive shock to r_t has a negative impact on the credit factor C_t. Common intuition would associate effective loosening of policy (i.e. a decrease in r_t) with a similar decrease in the credit factor, yet the standard policy of lowering the Federal Funds rate during this crisis did not lower the credit factor. Pressures due to credit remained in markets even though the Federal Reserve plummeted the Federal Funds rate down to zero. However, the liquidity factor L_t showed no significant reaction to a shock to r_t. Turning to the second row of Figure 8, we can examine the impact of shocks to C_t. Neither r_t nor L_t exhibit significant reactions to shocks to C_t. Lastly, row three shows that shocks to L_t have a positive impact on C_t. This means that as there is less liquidity in the markets (i.e. an increase in L_t), there is a higher amount of credit pressures, as well.

Table 3 reports the variance decomposition of each of the factors from the VAR. Each panel represents how much of the forecast error variance of a particular factor is due to each of the factors in X_t as the forecast horizon increases. Panel A decomposes how much of the forecast error variance of r_t is due to each of the factors. At the four-week horizon, most of the variation is due to r_t itself. However, as the horizon increases, C_t begins to explain more of the variation. 41% of the unconditional forecast error variance (i.e. $h = \infty$) of r_t is due to C_t, while only 14% is due to L_t.

Turning to Panel B, the majority of the variation in C_t is due to itself, with only a small decrease in the proportion of the variance explained by itself as the horizon increases. Lastly, Panel C shows how the forecast error variation of L_t is decomposed. At the shortest horizon, L_t explains the largest proportion. However, by the twelve-week horizon, C_t begin to explain the majority proportion, with a maximum of 68% of the variation at the twenty-four week horizon. The results of the variance decomposition again highlight the predictive

power of the credit factor for not only itself, but also both the Federal Funds rate and liquidity factor.

Finally, Table 4 reports estimates of the market price of risk parameters Θ_2 from the second stage estimation. The first row of the table corresponds to r_t, the second row corresponds to C_t, and the third row corresponds to L_t. The first column reports the constant risk premia parameters l_0. Each factor has significantly-priced constant risk premia. Turning to l_1, we can identify if there is any time-varying risk premia associated with the factors. Indeed, all but three of the coefficients are significantly different from zero, implying there is also significant time-variation in risk premia associated with all of the factors. Thus, the financial crisis proves to be a time period useful for estimating risk premia, and there is strong evidence for risk aversion (i.e. $\lambda_t \neq 0$) during the crisis.

3.4   Matching Moments

Table 5 reports the means, standard deviations, and first-order autocorrelations of the LOIS spreads from both the data and predicted by the model. The model captures the unconditional moments of LOIS spreads very well, including the sharp spike in volatility witnessed during the crisis, and only slightly underestimates the autocorrelation exhibited by LOIS spreads. All three moments are (weakly) increasing functions of maturity of the LOIS spread, and the model is able to capture this general trend well.

4   The Dynamics of LOIS Spreads

This section will discuss the behavior of LOIS spreads generated by the model by analyzing the response coefficients from the linear equation for spreads (29) and the dynamic response of spreads to each of factors in X_t.

4.1 The Response Coefficients

Figure 9 plots the estimated response coefficients from equation (29). For each maturity, the response coefficient corresponds to how much the LOIS spread responds to a one percent increase in each of the factors. The dark solid line plots the estimated coefficients for r_t, the light solid line corresponds to the coefficients for C_t, and the dashed line corresponds to the coefficients for L_t.

r_t has a negligible, negative impact on the term structure of LOIS spreads, reflecting the fact that the LOIS spread has removed general movements in short-term interbank interest rates captured by r_t. C_t has a positive impact on LOIS spreads that declines slightly with maturity. At the one-month maturity, a one percent increase in C_t implies an approximately 0.8% increase in the LOIS spread. This effect decreases to a 0.48% increase in the LOIS spread at the twelve-month maturity.

L_t has an initially negative impact on LOIS spreads that quickly increases to a positive impact as the maturity increases. The negative response coefficient at the shorter-term LOIS maturities is related to the negative coefficient estimated on L_t in the equation for γ_t; conditional on credit, there is a negative correlation between the short end of the LOIS yield curve and liquidity. However, as the maturity of the LOIS spread increases, L_t has a positive impact on LOIS spreads. At the twelve-month maturity, a one percent increase in the liquidity factor has an almost 1% increase in the LOIS spread. This reflects the importance of liquidity as maturity increases; longer-term assets generate a larger liquidity premium.

Figure 9 also reports OLS coefficients and standard error bands from the following regression:

$$z_t^{(n)} = \alpha_n + \beta_{1,n} r_t + \beta_{2,n} C_t + \beta_{3,n} L_t + v_t. \tag{34}$$

The point estimates from the OLS estimation are reported as symbols in the figure. Small

dots correspond to the estimates of $\beta_{1,n}$, small stars correspond to estimates of $\beta_{2,n}$, and small crosses correspond to estimates of $\beta_{3,n}$. The small dots are on top of the dark solid line, reflecting the fact that the response coefficients for r_t estimated by the model are almost identical the OLS point estimates from equation (34). The standard error bands around these estimates are also tight, and the response coefficient curve from the model lies within the standard error band. The model also does well of matching the response coefficients on C_t to those predicted from the OLS regressions.

In contrast, the model does not match the response coefficients on L_t estimated by OLS. The model predicts that the response coefficients on L_t are lower than the OLS point estimates for all maturities, even more so for the shorter maturities. In addition, the standard error bands are much wider for the liquidity factor. At the one-month maturity, the OLS coefficient is insignificantly different from zero, implying that liquidity has almost no effect on the shortest end of the LOIS yield curve. However, the response coefficient curve from the model lies almost entirely within the OLS standard error bands, so the model-predicted response coefficients are not statistically significantly different from the OLS point estimates.

The coefficients from this OLS estimation are related to those reported in Taylor and Williams (2008), McAndrews et.al. (2008), and Schwarz (2009), which all use OLS regressions to ascertain the impact of credit and liquidity on LOIS spreads. The linear relationship between LOIS spreads and the factors X_t predicted by the model implies that the response coefficients derived from the model should be close to OLS coefficient estimates, since OLS is the most efficient estimation procedure for linear models. However, the small differences between the estimates points out that policymakers can use this model as another means of estimating the impact of policies aimed at credit and liquidity. In addition, as I will show later, the model estimated here identifies risk premia separately, while the OLS coefficients do not identify the role of risk premia on the movements in LOIS spreads.

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4.2 LOIS Spread Dynamics

I examine IRFs of LOIS spreads with one-, three-, and twelve-month maturities to shocks to the state vector X_t. The affine model provides IRFs for all maturities as closed form solutions that are functions of the parameters. A derivation is provided in the appendix. Estimating the IRFs of spreads using a vector autoregressions that stacks the state vector X_t on top of all of the spreads used in estimation does not preclude arbitrage, and would not allow me to examine the IRFs of maturities of LOIS spreads not included in the VAR.

Figure 10 shows the IRFs of the one-, three-, and twelve-month LOIS spreads from the model to shocks to each of the factors. These IRFs are derived from the Cholesky-factorized, one-standard-deviation shocks to the original VAR in (8) and the linear equations for spreads (29). The solid line draws out the dynamics response of spreads to shocks to the Federal Funds rate r_t, the dashed line corresponds to shocks to the credit factor C_t, and the dotted line corresponds to shocks to the liquidity factor L_t.

Shocks to r_t cause a decrease in spreads, with the largest effect on the one-month spread. The maximum impact on spreads happens between ten and twenty weeks after the initial shock, highlighting how changes in the Federal Funds rate has large, persistent effects on LOIS spreads. However, this also demonstrates how ineffective standard policy was during the financial crisis. By lowering the Federal Funds rate, the Fed hoped to decrease both LIBOR and OIS rates and, in turn, decrease the spread between them. However, the spread between LIBOR and OIS continued to increase as the Federal Funds rate declined.

Shocks to C_t and L_t cause increases in spreads for all maturities. The credit factor has the largest impact at the one-month maturity, while the liquidity factor has the largest impact at the twelve-month maturity. Although the magnitude of the effect is larger for a one standard deviation shock to the credit factor, this is partly driven by the fact that the credit factor was approximately three times more volatile than the liquidity factor during

this time period. The standard deviations of C_t and L_t were 0.62 and 0.18, respectively. However, scaling the impulse responses by the size of the standard deviation still results in a larger response of LOIS spreads at the one- and three-month maturities.

Table 6 reports the variance decomposition of LOIS spreads from the model. Each panel reports how much of the forecast error variance of each of the LOIS spreads used in estimation is due to each of the factors. At the shorter horizons, the credit factor explains most of the forecast error variance of LOIS spreads, and continues to explain a significant proportion of the variance as the horizon increases. Put together, the credit and liquidity factors combined explain between 20% (for the one-month LOIS) and 50% (for the twelve-month LOIS) of the long-term forecast error variance, with most of the weight being on the credit factor. Even though general interest rate movements captured by the Federal Funds rate play the largest role in the long-run, there is explanatory power of the credit and liquidity factors for LOIS spreads.

5   The Role of Risk Premia

The critical contribution of the model derived in Section 3 is that the no-arbitrage restrictions imply LOIS spreads can be decomposed in the following way:

$$z_t^{(n)} = \underbrace{\mathbb{E}_t\left[\sum_{i=0}^{n-1} z_{t+i}^{(1)}\right]}_{\text{Expectations Hypothesis}} + \underbrace{RP_t}_{\text{Time-Varying Risk Premia.}} \tag{35}$$

Equation (35) states the LOIS spreads for each maturity can be partitioned into two components. The first is related to time-varying expectations of the future spread between LIBOR and the OIS (or Federal Funds) rate. Under the theory of the Expectations Hypothesis (EH), longer-term interest rates are expectations of future short-term interest rates plus a constant term premium. Here, the EH implies that longer-term LOIS spreads would be

expectations of future short-term LOIS spreads plus a constant term premium. The second term of equation (35) is time-variation in risk premia (RP) associated with changes in risk attitudes of investors. Any movements in risk aversion during this period would be captured by this term.

5.1 Decomposing the Response Coefficients

Recall the equation for LOIS spreads: $z_t^{(n)} = a_n + b_n^{\mathrm{T}} X_t$. Movements in LOIS spreads over time are driven by the term $b_n^{\mathrm{T}} X_t$. Therefore, any time-variation in risk premia that exists in LOIS spreads has to be contained within the term $b_n^{\mathrm{T}} X_t$. In order to isolate the effects of risk premia on LOIS spreads, I decompose the response coefficients b_n into an EH term and a time-varying risk premia (RP) term:

$$b_n = b_n^{EH} + b_n^{RP}, \tag{36}$$

where the EH term is computed under the assumption that $l_1 = 0$, i.e. the market price of risk is constant and given by $\lambda_t = l_0$. If it were the case that time-varying risk premia were not fundamental for pricing in this model, then the coefficients in b_n^{RP} would be small. It should also be noted that while λ_t is constrained to be a constant under the EH, the adjustment term γ_t given by equation (11) that drives the wedge between the short-term LIBOR and OIS rates is not constant under the EH. This term is deterministically known and enters the expectations of future spreads.

Table 7 reports the response coefficients for the LOIS spreads from the estimation, along with the EH and RP components. Each row corresponds to a particular maturity for the LOIS spread. The first three columns report the response coefficients b_n as estimated in the model and shown in Figure 9. Columns four through six report the response coefficients under the EH, where $l_1 = 0$. Lastly, columns seven through nine report the time-varying risk

premia portion of the response coefficients b_n^{RP}. The columns labeled 'r' are the coefficients associated with the Federal Funds rate, the columns labeled 'C' are associated with the credit factor, and the columns labeled 'L' are associated with the liquidity factor.

At the one-month maturity, the EH coefficients associated with the Federal Funds rate and credit factor are close in magnitude to the total response coefficients b_n associated with each factor. However, the EH implies that the response coefficient associated with the liquidity factor would be twice as negative as estimated by the model (-1.35 under the EH vs. -0.67 using the model), which causes the RP coefficient to be positive. At the three-month maturity, the EH response coefficients begin to diverge more from the total response coefficients. The EH coefficient on the Federal Funds rate is still reasonably close to the total response coefficient. However, the coefficient for the credit factor under the EH only picks up 54% of the total response coefficient, compared to 85% at the one-month maturity. The liquidity factor coefficient under the EH is even more negative at the three-month maturity than it was at the one-month maturity, predicting that the response of the three-month LOIS spread to the liquidity factor would be -1.63 rather than the -0.06 predicted by the model.

Moving to the twelve-month maturity, it becomes evident how much of a role risk premia play in pinning down the response coefficients. The Federal Funds rate response coefficient is -0.05 under the EH vs. -0.14 predicted by the model. The credit factor response coefficient under the EH is 0.1 vs. 0.44 predicted by the model, and thus the EH only captures 25% of the variation due to the credit factor at this maturity. Lastly, the liquidity factor continues to have a negative impact on LOIS spreads under the EH, with a response coefficient of -0.65 vs. the positive response coefficient of 0.93 predicted by the model. Across the entire term structure, the EH predicts that the liquidity factor has a negative impact on LOIS spreads, and thus time-varying risk premia are what drive the liquidity factor coefficient to be positive in the estimation of the model.

With this decomposition in hand, Figure 11 plots the actual three-month LOIS from the data (solid dark line), the three-month LOIS predicted by the model (solid light line), and the three-month LOIS predicted under the EH (dashed line). The results from Table 7 hint that the EH will not provide the best fit of the data, since the response coefficients (particularly on the liquidity factor) under the EH begin to diverge from the response coefficients predicted by the model. The EH is a constrained version of the model presented in Section 3 that imposes constant risk premia across the term structure, and measures how much expectations of the future short-term LOIS can predict the three-month LOIS.

According to Figure 11, the model with time-varying risk premia does a good job of matching the dynamics of the three-month LOIS during the crisis period. However, constraining risk premia to be constant over time strongly deteriorates the predictive power of the model, as shown by the dashed line relative to the light solid line. Although the credit and liquidity factors are used when computing the three-month LOIS predicted under the EH, there are risk premia associated with each of these factors that have strong predictive power for the term structure of LOIS.[7]

Finally, I can use the decomposition from equation (36) to see how much of the variability in LOIS spreads comes as a result of movements in risk premia. To do this, I compute the proportion of forecast error variance of the LOIS spreads that is due to time-varying risk premia, which I report in the first column Table 8. This is computed in a similar way that the variance decompositions for LOIS spreads reported in Table 6 were computed, but using the coefficients b_n^{RP} in place of the total response coefficients b_n. In addition, I report how much of this RP proportion of the total forecast error variance can be explained by each of the factors, shown in columns two through four. Only 4.06% of the forecast error

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[7]Indeed, as seen in Table 7, the lack of time-varying risk premia implies that longer-term spreads will diverge even further from the data series since the response coefficients are even further away from the total model-predicted response coefficients. Indeed, the EH at the three-month maturity was one the best-performing spreads of the bunch under the EH in terms of how closely it fit the actual data.

variance of the one-month LOIS is a result of time-varying RP. Of this 4.06%, 85.17% is due to movements in the Federal Funds rate, and a total of 14.83% is due to the credit and liquidity factors. However, as the maturity increases, so does the proportion of variance explained by time-varying RP. At the twelve-month maturity, 53.62% of the variation is attributable to time-varying RP, with a 21.58% share due to movements in the credit and liquidity factors. At all horizons, the Federal Funds rate maintains a high proportion of the variance due to RP. This is due to the fact that movements in the determinants of LOIS spreads, LIBOR and OIS, are highly correlated with movements in the Federal Funds rate.

5.2   What Predictive Power Does Time-Varying Risk Premia Have?

The results in the previous section provide evidence for the advantages of a model that includes time-variation in risk premia as a determinant for LOIS spreads. However, do time-varying risk premia actually have predictive power? What is required to answer this question is a measure of LOIS spreads that is consistent with time-varying risk premia; that is, a measure of these spreads in which time-varying risk premia drives all of the time-variation in the measure itself. The null hypothesis of the EH would be rejected if time-variation in this measure is significantly different from zero.

Zero-coupon bonds (such as LIBOR and OIS in the context of the model) allow for a convenient metric for studying time-varying risk premia. Consider the following trading strategy: Purchase a zero-coupon bond at time t at price $P_t^{(n)}$ that matures at time $t + n$. At time $t + 1$, sell the zero-coupon bond that now matures in $n - 1$ periods for the price $P_{t+1}^{(n-1)}$.

The gross return from this strategy is given by:

$$r_{t+1}^{(n)} \equiv \ln\left(\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\right) \tag{37}$$

$$= \ln(P_{t+1}^{(n-1)}) - \ln(P_t^{(n)}) \tag{38}$$

$$= ni_t^{(n)} - (n-1)i_{t+1}^{(n-1)}, \tag{39}$$

where $i_t^{(n)}$ is the yield on the n-period zero-coupon bond. The excess return of this strategy over holding the one-period zero-coupon bond with yield $i_t^{(n)}$ is thus given by:

$$rx_{t+1}^{(n)} = ni_t^{(n)} - (n-1)i_{t+1}^{(n-1)} - i_t^{(1)}. \tag{40}$$

Suppose we are using the derivation for the LIBOR zero-coupon bond price and yield. Recall that the prices and yields of LIBOR can be written as (using equations (19), (21), (22), and (25):

$$P_t^{L,(n)} = \exp(A_n + B_n^\mathrm{T} X_t) \tag{41}$$

$$A_{L,n+1} = A_{L,n} + B_{L,n}^\mathrm{T}(\mu - \Sigma l_0) + 0.5 B_{L,n}^\mathrm{T} \Sigma \Sigma^\mathrm{T} B_{L,n} - \gamma_0, \quad A_{L,1} = -\gamma_0 \tag{42}$$

$$B_{L,n+1}^\mathrm{T} = B_{L,n}^\mathrm{T}(\Phi - \Sigma l_1) - (1, \gamma_{1,1}, \gamma_{1,2}), \quad B_{L,1}^\mathrm{T} = -(1, \gamma_{1,1}, \gamma_{1,2}) \tag{43}$$

$$i_t^{L,(n)} = a_{L,n} + b_{L,n}^\mathrm{T} X_t, \tag{44}$$

where $a_{L,n} = -A_{L,n}/n$ and $b_{L,n}^\mathrm{T} = -B_{L,n}^\mathrm{T}/n$. The excess return for LIBOR can then be written as:

$$rx_{t+1}^{L,(n)} = ni_t^{L,(n)} - (n-1)i_{t+1}^{L,(n-1)} - i_t^{L,(1)}. \tag{45}$$

Taking expectations of equation (45) and recalling that the conditional mean of the state

vector X_t is linear provides an expression for the conditional expected excess return:

$$\mathbb{E}_t[rx_{t+1}^{L,(n)}] = -0.5B_{L,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{L,n-1} - B_{L,n-1}^{\mathrm{T}}\Sigma l_0 + B_{L,n-1}^{\mathrm{T}}\Sigma l_1 X_t \tag{46}$$

$$= a_{L,n}^x + b_{L,n}^{x\mathrm{T}}X_t, \tag{47}$$

where $a_{L,n}^x = -0.5B_{L,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{L,n-1} - B_{L,n-1}^{\mathrm{T}}\Sigma l_0$ and $b_{L,n}^{x\mathrm{T}} = B_{L,n-1}^{\mathrm{T}}\Sigma l_1$. From equation (47), I can decompose expected excess returns for LIBOR into a Jensen's term $-0.5B_{L,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{L,n-1}$, a constant risk premia term $B_{L,n-1}^{\mathrm{T}}\Sigma l_0$, and a time-varying risk premia term $B_{L,n-1}^{\mathrm{T}}\Sigma l_1$. It is important to note that *any* time variation in expected excess returns must be a result of time-varying risk premia.

A similar derivation can be performed to derived the expected excess return for the OIS:

$$rx_{t+1}^{O,(n)} = a_{O,n}^x + b_{O,n}^{x\mathrm{T}}X_t, \tag{48}$$

where $a_{O,n}^x = -0.5B_{O,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{O,n-1} - B_{O,n-1}^{\mathrm{T}}\Sigma l_0$, $b_{O,n}^{x\mathrm{T}} = B_{L,n-1}^{\mathrm{T}}\Sigma l_1$, and the coefficients $B_{O,n}$ are derived in equations (23) and (24). Combining equations (47) and (48) provides an expression for the relative excess returns between LIBOR and OIS:

$$rx_{t+1}^{(n)} = a_n^x + b_n^{x\mathrm{T}}X_t, \tag{49}$$

which again takes a linear form with $a_n^x = -0.5(B_{L,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{L,n-1} - B_{O,n-1}^{\mathrm{T}}\Sigma\Sigma^{\mathrm{T}}B_{O,n-1}) + B_{L,n-1}^{\mathrm{T}}\Sigma l_0 - B_{O,n-1}^{\mathrm{T}}\Sigma l_0$ and $b_n^{x\mathrm{T}} = B_{L,n-1}^{\mathrm{T}}\Sigma l_1 - B_{O,n-1}^{\mathrm{T}}\Sigma l_1$.

In order to test for the presence of time-varying risk premia in the data for the relative excess return between LIBOR and OIS, which is captured by the LOIS spread, I run the following regression:

$$rx_{t+1}^{(n)} = \varphi_0 + \varphi_1 r_t + \varphi_2 C_t + \varphi_3 L_t + \zeta_{t+1}. \tag{50}$$

The EH implies that $\varphi_1 = \varphi_2 = \varphi_3 = 0$, and thus any significant coefficients reject the null of a constant risk premia over the term structure of LOIS spreads.

Panel A of Table 9 reports the OLS results from equation (50) for the one-, three-, six-, nine-, and twelve-month LOIS spreads from the data. Small-sample standard errors are reported below the coefficient estimates, which were computed using a Monte Carlo simulation over 10,000 repetitions. In addition, I report the R^2 of the regression. Recall that in order to compute the one-week relative excess return at the three-month maturity, the exact formula would require data on the LIBOR and OIS rates at the 11-month maturity, which is not available. Therefore, in order to compute excess returns I use the following approximation from Campbell and Shiller (1991):

$$ arx_{t+1}^{(n)} = nz_t^{(n)} - nz_{t+1}^{(n)} - z_t^{(1)}. \tag{51} $$

At the one-month maturity, 25% of the variation in relative excess returns is due to time-varying risk premia, and this explanatory power decreases in maturity but is still in the range of 10%-15%. While the magnitude of the R^2's are not as high as those in Cochrane and Piazzesi (2005), there is still non-negligible predictive power of the factors that were chosen in this analysis, and it must be remembered that we are looking at a weekly return horizon.[8] All three of the factors in X_t have significant coefficients even out to the twelve-month maturity. The Federal Funds rate has a positive impact on relative excess returns of 0.33 at the one-month maturity that increases to 1.27 at the twelve-month maturity. For a one-percent increase in the Federal Funds rate, relative excess returns increase by between 0.33% and 1.27%. The credit factor has a negative impact on relative excess returns that varies from -0.73% at the one-month maturity and nearly doubles to -1.40% at the twelve-

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[8]Smith (2012) examines longer holding period returns on repo spreads as a method of understanding risk premia in rollover returns. Here, we focus on returns over the weekly horizon to accommodate the data frequency.

month maturity. Finally, the liquidity factor has a positive impact on returns ranging from 5.05% at the one-month maturity up to 20.06% at the twelve-month maturity. The results imply that increases in the Federal Funds rate and liquidity factor cause positive relative excess returns from week to week, while increases in the credit factor cause negative relative excess returns.

Panel B of Table 9 reports a similar regression of the mean of relative excess returns across maturities on each of the factors:

$$\overline{rx}_{t+1} = \overline{\varphi}_0 + \overline{\varphi}_1 r_t + \overline{\varphi}_2 C_t + \overline{\varphi}_3 L_t + \overline{\zeta}_{t+1}, \tag{52}$$

where $\overline{rx}_{t+1}$ is the mean of relative excess returns at each time period computed over the one-, three-, six-, nine-, and twelve-month maturities. The coefficients in Panel B closely resemble the coefficients for the six-month maturity reported in Panel A. All three factors remain predictive even for the mean excess returns, with the same signs as reported in Panel A.

Figure 12 plots the relative excess returns between LIBOR and OIS for the one- (dark solid), three- (light solid), and twelve-month (dashed) maturities. The returns are reported in percentage points. In early 2007 before the crisis began, the one- and three-month maturities show almost zero relative excess returns, a results of the fact that LOIS spreads were low and not volatile during this time period. In contrast, the panic in the fall of 2008 saw an unprecedented increase in relative excess returns. At the twelve-month maturity, relative returns during the panic ranged from almost -30% to 15%. Absent the predictability results presented in Table 9, the high volatility of excess returns are evidence of time-variation in risk premia during the crisis.

6 Conclusion

This paper estimated a no-arbitrage model of the term structure of money market spreads to identify how much of the sharp movements in spreads during the financial crisis was attributable to measures of interest rates, credit, and liquidity. The entire term structure of spreads is a series of longer-term, risk-adjusted expected values of future short-term spreads. The model is able to closely match movements of LOIS spreads, with the credit factor playing the dominant explanatory role. Risk premia explain up to 50% of the variation in spreads, and relative excess returns are predicted by the credit and liquidity factors. Comparison with OLS regression results shows that entertaining estimates from these no-arbitrage models could improve policy going forward, since risk premia can be addressed directly.

Continuing research in Smith (**?**) includes estimation of a similar model for the cross-section of LIBOR-participating banks. Given that the results of this paper conclude that credit and liquidity risks are crucial for understanding the run-up in LOIS spreads, it is important to analyze how LOIS spreads related to individual banks reacted to these particular factors. In Hafstead and Smith (2012), we examine the Bernanke, Gertler, and Gilchrist (1999) model and incorporate interbank lending to ascertain the general equilibrium effects of monetary policy rules that focus on interest rates spreads. Today, as LOIS spreads begin to fall back to more normal levels, the overall economy still remains in a deep recession, and thus it is crucial that policymakers going forward ascertain how policy must be adjusted in this new age of financial innovation.

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A Derivation of the Response Coefficients

In order to derive the resursive coefficients defined by equations (21) through (24), first recall that for the one-week maturity, equations (9) and (10) are satisfied, and thus the following must be the case:

$$P_t^{L,(n)} = \exp(-i_t^{L,(1)}) \tag{A.1}$$

$$= \exp(-\gamma_0 - \gamma_1^T X_t) \tag{A.2}$$

$$P_t^{O,(n)} = \exp(-i_t^{O,(1)}) \tag{A.3}$$

$$= \exp(-r_t) \tag{A.4}$$

Matching coefficients implies that $A_{1,L} = -\gamma_0, B_{1,L} = -\gamma_1, A_{O,1} = 0$, and $B_{O,1}^T = (1,0,0)$.

Conjecture that the n-period LIBOR and OIS prices are exponential affine and given by $P_t^{L,(n)} = \exp(A_{L,n} + B_{L,n} X_t)$ and $P_t^{O,(n)} = \exp(A_{O,n} + B_{O,n} X_t)$, respectively. I will show how the exponential affine form conjectured for the n-period LIBOR price will imply an exponential affine form for the $n+1$-period LIBOR price:

$$P_t^{L,(n+1)} = \mathbb{E}_t[m_{t+1} P_{t+1}^{L,(n)}] \tag{A.5}$$

$$= \mathbb{E}_t[\exp(-i_t^{L,(1)} - 0.5\lambda_t^T \lambda_t - \lambda_t^T \varepsilon_{t+1} + A_{L,n} + B_{L,n} X_{t+1})] \tag{A.6}$$

$$= \exp(-\gamma_0 - \gamma_1^T X_t + A_{L,n} + B_{L,n}(\mu + \Phi X_t)) \tag{A.7}$$

$$\times \ \mathbb{E}_t[\exp(-\lambda_t^T \varepsilon_{t+1} + B_{L,n} \Sigma \varepsilon_{t+1})] \tag{A.8}$$

$$= \exp\big(A_n + B_{L,n}(\mu - \Sigma l_0) + 0.5 B_{L,n}^T \Sigma \Sigma^T B_{L,n} - \gamma_0 \tag{A.9}$$

$$+ \ (B_{L,n}(\Phi - \Sigma l_1) - \gamma_1^T) X_t\big), \tag{A.10}$$

where I use the distributional assumption that $\varepsilon_{t+1} \sim$ iid $N(0,I)$. Matching coefficients

implies the recursive coefficients in (21) and (22). A similar derivation can be completed for the OIS pricing coefficients, which leads to the recursions in (23) and (24).

B   Derivation of Impulse Response Functions and Variance Decompositions

B.1   Impulse Response Functions

Recall that VAR(1) specified in equation (8):

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t. \tag{B.1}$$

Using a Cholesky decomposition of the matrix Σ, we can write the VAR(1) in VMA(∞) form:

$$X_t = \sum_{j=0}^{\infty} \theta_h \varepsilon_{t-h}. \tag{B.2}$$

The model implies that the n-period spread can be written as a linear function of the vector X_t:

$$z_t^{(n)} = a_n + b_n^{\mathrm{T}} X_t \tag{B.3}$$

$$= a_n + \sum_{j=0}^{\infty} \psi_j^n \varepsilon_{t-j}, \tag{B.4}$$

where the vector $\psi_j^n = b_n^{\mathrm{T}} \theta_j$ is the impulse reponse function for the n-period spread at future period j for shocks to the state vector X_t at time 0. Stacking all $n = [1, \ldots, N]$ yields together provides a convenient VMA(∞) representation of spreads:

$$z_t = a + \sum_{j=0}^{\infty} \Psi_j \varepsilon_{t-j} \tag{B.5}$$

where $z_t \equiv (z_t^{(1)}, \ldots, z_t^{(N)})$ and the nth row of Ψ_j is ψ_j^n.

B.2 Variance Decompositions of Spreads

Denote by $\hat{z}_{t+h|t}$ the time t optimal h-horizon forecast. Using the VMA(∞) representation from (B.5), the error of this forecast can be written as:

$$\hat{z}_{t+h|t} - z_{t+h} = \sum_{j=0}^{h-1} \Psi_j \varepsilon_{t+h-j}. \tag{B.6}$$

Let $\Psi_{kl,j}$ denote the kth row and lth column of Psi_j. Then the mean squared error (MSE) of the kth component of the forecast error can be written as:

$$MSE(\hat{z}_{t+h|t}^j) = \sum_{l=1}^{L} (\Psi_{kl,0}^2 + \cdots + \Psi_{kl,h-1}^2). \tag{B.7}$$

Then the percent of the forecast error variance at horizon h of spread j attributable to factor l is:

$$\Omega_{kl,h} = \frac{\sum_{j=0}^{h-1} \Psi_{kl,j}}{MSE(\hat{z}_{t+h|t}^j)} \tag{B.8}$$

Equation (B.8) decomposes the forecast error variance at horizon h to each of the factors of X.

C   Robustness

Tables 11 and 12 reports robustness results for the estimation in Section 3. The mean and standard deviation of predicted spreads is reported, along with the response coefficient estimates. Table 11 reports the results using the LIBOR data for different measures of the credit factor. Panel A reports the original estimates using the three-month LIBOR-REPO series. Panel B uses the one-month LIBOR-REPO series, Panel C uses the median of the five-year credit default swap rates for LIBOR-participating institutions, Panel D uses the mean of the one-month LIBOR-REPO, three-month LIBOR REPO, and CDS median rates, and Panel E uses the first principal component of the one-month LIBOR-REPO, three-month LIBOR REPO, and CDS median rates. Table 12 reports similar results using the Term Federal Funds data rather than LIBOR data. For comparison, Table 10 reports the mean and standard deviation of spreads from the data.

Time	Floating Rate Payer	Fixed Rate Payer	Net Payments
t	0	0	0
T	$-N \times i_{float} \times \frac{w}{52}$	$-N \times i_{fixed} \times \frac{w}{52}$	$-N \times \lvert i_{float} - i_{fixed} \rvert \times \frac{w}{52}$

Table 1: This table shows the payments made by each participant in an OIS swap. N is the notional amount of the swap, i_{float} is the floating payment of the swap, i_{fixed} is the fixed payment of the swap, w is the number of weeks in the swap, and T is the maturity date.

X_t	VAR Dynamics						Short Rate Dynamics	
	Φ			$\Sigma\Sigma^{\mathrm{T}}$			γ_0	γ_1
r_t	0.98	-0.12	-0.02	0.17	0.00	0.00	-0.12	1.00
	(0.02)	(0.03)	(0.15)				(0.05)	
C_t	-0.04	0.92	-0.52	0.01	0.25	0.00		0.80
	(0.04)	(0.07)	(0.48)					(0.14)
L_t	-0.00	0.02	0.90	-0.00	0.01	0.03		-0.95
	(0.00)	(0.01)	(0.04)					(0.29)

Table 2: This table reports the estimated coefficients of the VAR and short rate equations in Θ_1. The full sample of data is 01/01/2007 - 06/19/2009. The estimated VAR is $\tilde{X}_t = \Phi\tilde{X}_{t-1} + \Sigma\varepsilon_t$. X_t consists of the Federal Funds rate r_t, the credit factor C_t, and the liquidity factor L_t. $\tilde{X}$ is $X_t - \mathbb{E}[X_t]$ in an effort to pin down the unconditional mean of the vector X_t. Σ is Cholesky-factorized, and $\varepsilon_t \sim N(0,I)$. The short rate coefficients are estimated according to $i_t^{(1)} = \gamma_0 + \gamma_1 X_t$, where $i_t^{(1)}$ is the one-week LIBOR rate, and γ_1 is constrained as $\gamma_1 = (1, \gamma_{1,1}, \gamma_{1,2})$. Estimation is performed using ordinary least squares. Robust GMM standard errors are reported below coefficient estimates.

Horizon h	Percent attributed to		
(weeks)	r	C	L
Panel A: Federal Funds Rate			
4	90.91	9.08	0.01
12	59.68	38.79	1.53
24	46.47	46.90	6.63
48	44.88	43.31	11.81
∞	45.07	41.21	13.72
Panel B: Credit Factor			
4	0.15	98.64	1.21
12	0.98	90.67	8.35
24	2.92	83.96	13.12
48	4.46	81.99	13.55
∞	5.70	80.65	13.65
Panel C: Liquidity Factor			
4	1.62	25.52	72.86
12	3.31	63.04	33.65
24	8.27	68.20	23.53
48	16.05	60.62	23.33
∞	21.36	56.30	22.34

Table 3: The table reports unconditional forecast error variance decompositions (in percentages) for each of the variables in X_t. The forecast horizon h is in weeks. The full sample of data is 01/01/2007 - 06/19/2009. The estimated VAR is $X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t$. X_t consists of the Federal Funds rate r_t, the credit factor C_t, and the liquidity factor L_t. Σ is Cholesky-factorized, and $\varepsilon_t \sim N(0,I)$. Estimation is performed using ordinary least squares.

l_0		l_1	
-7.47	1.41	0.27	-0.74
(0.02)	(0.14)	(0.11)	(0.10)
1.03	-0.27	-0.14	-1.29
(0.09)	(0.17)	(0.34)	(0.19)
-0.24	-0.06	0.31	0.38
(0.04)	(0.16)	(0.14)	(0.08)

Table 4: This table reports estimates of the market price of risk parameters. The full sample of data is 01/01/2007 - 06/19/2009. The market prices of risk are given by $\lambda_t = l_0 + l_1 X_t$, where X_t consists of the interest rate factor r_t, the credit factor C_t, and the liquidity factor L_t. Estimation is performed using non-linear least squares. Bootstrapped standard errors are reported below coefficient estimates.

	Mean	Standard Deviation	Autocorrelation
One-Month LOIS	0.46	0.54	0.94
Three-Month LOIS	0.73	0.62	0.96
Six-Month LOIS	0.88	0.67	0.98
Nine-Month LOIS	0.88	0.67	0.98
Twelve-Month LOIS	0.87	0.67	0.98

PANEL B: MOMENTS OF SPREADS FROM MODEL

	Mean	Standard Deviation	Autocorrelation
One-Month LOIS	0.47	0.46	0.90
Three-Month LOIS	0.72	0.56	0.93
Six-Month LOIS	0.90	0.66	0.96
Nine-Month LOIS	0.87	0.68	0.97
Twelve-Month LOIS	0.97	0.64	0.97

Table 5: This tables reports the unconditional moments of the LOIS spreads $z_t^{(n)}$ from the data and predicted from the model. The full sample of data is 01/01/2007 - 06/19/2009. The moments include the mean, standard deviation, and first-order autocorrelation. Panel A reported the moments of spreads from the data, and Panel B reports the model-predicted moments of spreads.

Horizon h	Percent attributed to		
(weeks)	r	C	L
Panel A: One-Month LOIS			
4	5.41	94.41	0.18
12	38.44	61.04	0.52
24	67.14	32.08	0.78
48	79.88	19.43	0.69
Panel B: Three-Month LOIS			
4	8.03	91.63	0.34
12	43.01	55.88	1.11
24	67.50	31.35	1.15
48	77.08	21.18	0.94
Panel C: Six-Month LOIS			
4	12.20	85.32	2.48
12	48.11	48.66	3.23
24	65.22	32.13	2.65
48	71.33	26.44	2.24
Panel D: Nine-Month LOIS			
4	16.07	78.08	5.83
12	50.34	43.37	6.29
24	58.96	35.85	5.19
48	60.12	34.92	4.97
Panel E: Twelve-Month LOIS			
4	19.38	71.10	9.51
12	49.98	40.54	9.48
24	50.67	41.41	7.92
48	50.54	41.70	7.76

Table 6: The table reports unconditional forecast error variance decompositions (in percentages) for the one-, three-, six-, nine-, and twelve-month LOIS spreads. The forecast horizon h is in weeks. The full sample of data is 01/01/2007 - 06/19/2009. First-stage estimation is performed using ordinary least squares, and second-stage estimation is performed using non-linear least squares.

Spread	Total b_n			b_n^{EH}			b_n^{RP}		
	r	C	L	r	C	L	r	C	L
One-Month LOIS	-0.02	0.80	-0.67	-0.04	0.68	-1.35	0.02	0.12	0.68
Three-Month LOIS	-0.07	0.76	-0.06	-0.09	0.41	-1.63	0.02	0.35	1.57
Six-Month LOIS	-0.12	0.66	0.54	-0.09	0.20	-1.25	-0.03	0.46	1.79
Nine-Month LOIS	-0.14	0.55	0.84	-0.07	0.13	-0.87	-0.07	0.42	1.71
Twelve-Month LOIS	-0.14	0.44	0.93	-0.05	0.10	-0.65	-0.09	0.34	1.58

Table 7: This table reports the response coefficients derived from the model, and decomposes each response coefficient into an expectations hypothesis (EH) component and a time-varying risk premia (RP) component. The EH componenent is computed by setting the market price of risk parameters $l_1 = 0$. Each column reports the response coefficient associated with the given factor.

Spread	Risk Premia Proportion	RP attributed to		
		r	C	L
One-Month LOIS	4.06	85.17	12.74	2.09
Three-Month LOIS	16.36	82.03	15.26	2.71
Six-Month LOIS	28.67	77.55	18.88	3.56
Nine-Month LOIS	41.98	76.71	19.53	3.76
Twelve-Month LOIS	53.62	78.42	18.15	3.43

Table 8: This table reports the proportion (in percentages) of the unconditional forecast error variance that is attributable to time-varying risk premia (RP) in the model, as well as how much of the RP proportion is attributable to each of the factors. The unconditional forecast error variance is computed using a horizon of 100 weeks.

PANEL A: RELATIVE EXCESS RETURNS REGRESSIONS

Maturity	φ_0	φ_1	φ_2	φ_3	R^2
One-Month	-1.91	0.33	-0.73	5.05	0.25
	(0.47)	(0.08)	(0.14)	(0.94)	
Three-Month	-3.87	0.62	-0.87	9.89	0.14
	(1.15)	(0.21)	(0.34)	(2.30)	
Six-Month	-5.08	0.79	-1.12	13.18	0.12
	(1.77)	(0.31)	(0.51)	(3.51)	
Nine-Month	-6.92	1.08	-1.32	17.47	0.11
	(2.37)	(0.42)	(0.68)	(4.79)	
Twelve-Month	-8.17	1.27	-1.40	20.06	0.09
	(2.91)	(0.52)	(0.89)	(5.62)	

PANEL B: MEAN RELATIVE EXCESS RETURN REGRESSION

$\overline{\varphi}_0$	$\overline{\varphi}_1$	$\overline{\varphi}_2$	$\overline{\varphi}_3$	R^2
-5.19	0.82	-1.09	13.13	0.12
(1.63)	(0.29)	(0.50)	(3.32)	

Table 9: This table reports OLS regressions results for approximate expected relative excess returns. The full sample of data is 01/01/2007-06/19/2009. Approximate expected relative excess returns for maturity n are computed as $arx_{t+1}^{(n)} = nz_t^{(n)} - nz_{t+1}^{(n)} - z_t^{(1)}$, where $z_t^{(n)}$ is the n-period LOIS spread. Panel A reports the OLS regression coefficients that are computed from the regression $rx_{t+1}^{(n)} = \varphi_0 + \varphi_1 r_t + \varphi_2 C_t + \varphi_3 L_t + \zeta_{t+1}$ that uses the Federal Funds rate r_t, the credit factor C_t, and the liquidity factor L_t for the one-, three-, six-, nine-, and twelve-month maturities. Panel B reports the OLS regressions that are computed from the regression $\overline{rx}_{t+1} = \overline{\varphi}_0 + \overline{\varphi}_1 r_t + \overline{\varphi}_2 C_t + \overline{\varphi}_3 L_t + \overline{\zeta}_{t+1}$, where $\overline{rx}_{t+1}$ is the mean expected relative excess return over all maturities. Each column reports the coefficient attached to each of the corresponding factors. Small-sample, Monte Carlo standard errors are below the OLS coefficient estimates, and R^2's for each of the regressions are also reported.

| | LOIS SPREADS | | TERM FF-OIS SPREADS | |
Spread	Mean	Standard Deviation	Mean	Standard Deviation
One-Month	0.46	0.54	0.53	0.69
Three-Month	0.73	0.62	0.83	0.81
Six-Month	0.88	0.67	1.04	0.95
Nine-Month	0.88	0.67	1.06	0.98
Twelve-Month	0.87	0.67	1.11	0.99

Table 10: This tables reports the mean and standard deviation of money market spreads from the data. The columns labeled 'LOIS Spreads' report the moments for spreads between LIBOR and OIS rates. The columns labeled 'Term FF-OIS Spreads' report the moments for spreads between Term Federal Funds and OIS rates. The full sample of data is 01/01/2007 - 06/19/2009.

PANEL A: THREE-MONTH LIBOR-REPO SPREAD

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month LOIS	0.46	0.54	-0.02	0.80	-0.76
Three-Month LOIS	0.73	0.62	-0.07	0.76	-0.06
Six-Month LOIS	0.88	0.67	-0.12	0.66	0.54
Nine-Month LOIS	0.88	0.67	-0.14	0.55	0.84
Twelve-Month LOIS	0.87	0.67	-0.14	0.44	0.93

PANEL B: ONE-MONTH LIBOR-REPO SPREAD

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month LOIS	0.46	0.44	-0.06	0.68	-0.00
Three-Month LOIS	0.75	0.55	-0.15	0.61	0.24
Six-Month LOIS	0.86	0.64	-0.21	0.54	0.41
Nine-Month LOIS	0.90	0.66	-0.22	0.50	0.47
Twelve-Month LOIS	0.87	0.65	-0.22	0.46	0.48

PANEL C: MEDIAN OF 5-YEAR CDS RATES

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month LOIS	0.46	0.24	-0.02	0.35	-0.11
Three-Month LOIS	0.73	0.39	-0.09	0.35	-0.01
Six-Month LOIS	0.88	0.55	-0.17	0.31	0.10
Nine-Month LOIS	0.89	0.61	-0.22	0.24	0.18
Twelve-Month LOIS	0.87	0.58	-0.22	0.17	0.21

PANEL D: MEAN OF THREE FACTORS

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month LOIS	0.46	0.43	0.05	1.06	-0.60
Three-Month LOIS	0.73	0.56	0.01	0.97	0.64
Six-Month LOIS	0.88	0.68	-0.03	0.84	1.35
Nine-Month LOIS	0.88	0.67	-0.05	0.72	1.51
Twelve-Month LOIS	0.87	0.63	-0.05	0.63	1.47

PANEL E: FIRST PRINCIPAL COMPONENT OF THREE FACTORS

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month LOIS	0.42	0.45	0.02	0.59	-0.81
Three-Month LOIS	0.74	0.58	-0.06	0.57	-0.14
Six-Month LOIS	0.90	0.69	-0.12	0.49	0.44
Nine-Month LOIS	0.86	0.68	-0.14	0.40	0.66
Twelve-Month LOIS	0.88	0.61	-0.13	0.32	0.69

Table 11: This table reports the model-predicted means and standard deivations of LOIS spreads, along with the estimated response coefficients, from estimating the model using different measures of the credit factor. Panel A uses the three-month LIBOR-REPO spread, Panel B uses the one-month LIBOR-REPO spread, Panel C uses the median of the five-year credit default swap (CDS) rate for LIBOR-participating institutions, Panel D uses the mean of these three factors, and Panel E uses the first three principal components of these three factors. The response coefficients are derived from the equation $z_t^{(n)} = a_n + b_n^{\mathrm{T}} X_t$, where $z_t^{(n)}$ is the LOIS spread of maturity n, and X_t is the vector of factors including the Federal Funds rate r_t, the credit factor C_T, and the liquidity factor L_t. The full sample of data is 01/01/2007 - 06/19/2009.

PANEL A: THREE-MONTH TERM FF-REPO SPREAD

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month Term FF-OIS	0.52	0.59	-0.02	0.78	-0.54
Three-Month Term FF-OIS	0.84	0.84	-0.09	0.86	0.27
Six-Month Term FF-OIS	1.03	0.97	-0.14	0.84	0.71
Nine-Month Term FF-OIS	1.09	0.97	-0.15	0.79	0.82
Twelve-Month Term FF-OIS	1.10	0.92	-0.15	0.72	0.83

PANEL B: ONE-MONTH TERM FF-REPO SPREAD

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month Term FF-OIS	0.48	0.58	0.02	0.73	0.10
Three-Month Term FF-OIS	0.83	0.78	-0.07	0.79	0.45
Six-Month Term FF-OIS	1.04	0.94	-0.14	0.79	0.69
Nine-Month Term FF-OIS	1.08	0.98	-0.17	0.76	0.77
Twelve-Month Term FF-OIS	1.11	0.97	-0.18	0.72	0.78

PANEL C: MEDIAN OF 5-YEAR CDS RATES

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month Term FF-OIS	0.52	0.27	-0.07	0.21	0.01
Three-Month Term FF-OIS	0.83	0.58	-0.19	0.28	0.19
Six-Month Term FF-OIS	1.03	0.76	-0.25	0.34	0.34
Nine-Month Term FF-OIS	1.08	0.81	-0.27	0.36	0.39
Twelve-Month Term FF-OIS	1.10	0.82	-0.27	0.36	0.41

PANEL D: MEAN OF THREE FACTORS

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month Term FF-OIS	0.51	0.54	-0.05	0.94	-1.00
Three-Month Term FF-OIS	0.85	0.70	-0.11	0.96	-0.31
Six-Month Term FF-OIS	0.97	0.87	-0.15	0.93	0.36
Nine-Month Term FF-OIS	1.09	0.95	-0.17	0.89	0.75
Twelve-Month Term FF-OIS	1.10	0.97	-0.18	0.85	0.97

PANEL E: FIRST PRINCIPAL COMPONENT OF THREE FACTORS

Spread	Mean	Standard Deviation	$b_{1,n}$	$b_{2,n}$	$b_{3,n}$
One-Month Term FF-OIS	0.50	0.60	0.05	0.61	-0.34
Three-Month Term FF-OIS	0.86	0.82	0.01	0.68	0.59
Six-Month Term FF-OIS	1.00	0.96	-0.11	0.61	0.98
Nine-Month Term FF-OIS	1.11	0.97	-0.16	0.52	1.00
Twelve-Month Term FF-OIS	1.10	0.90	-0.18	0.44	0.93

Table 12: This table reports the model-predicted means and standard deivations of Term Federal Funds (Term FF)-OIS spreads, along with the estimated response coefficients, from estimating the model using different measures of the credit factor. Panel A uses the three-month Term FF-REPO spread, Panel B uses the one-month Term FF-REPO spread, Panel C uses the median of the five-year credit default swap (CDS) rate, Panel D uses the mean of these three factors, and Panel E uses the first three principal components of these three factors. The response coefficients are derived from the equation $z_t^{(n)} = a_n + b_n^{\mathrm{T}} X_t$, where $z_t^{(n)}$ is the Term FF-OIS spread of maturity n, and X_t is the vector of factors including the Federal Funds rate r_t, the credit factor C_T, and the liquidity factor L_t. The full sample of data is 01/01/2007 - 06/19/2009.
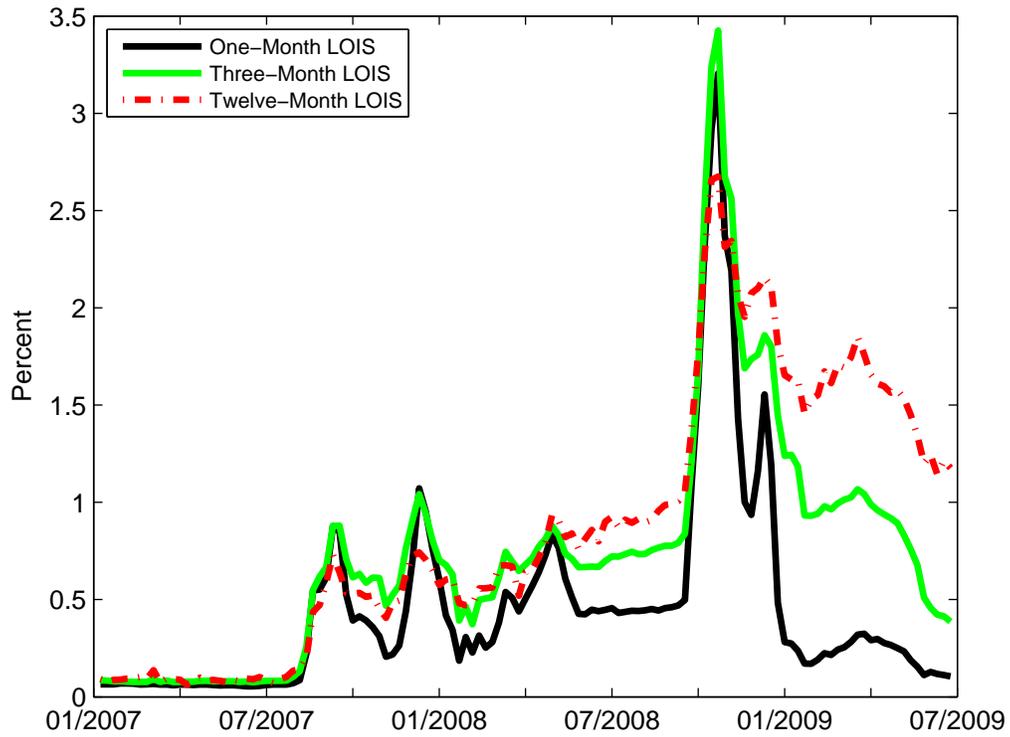
Figure 1: Weekly averages of daily data of spreads between LIBOR and OIS rates (LOIS spreads) at the one-, three-, and twelve-month maturities.
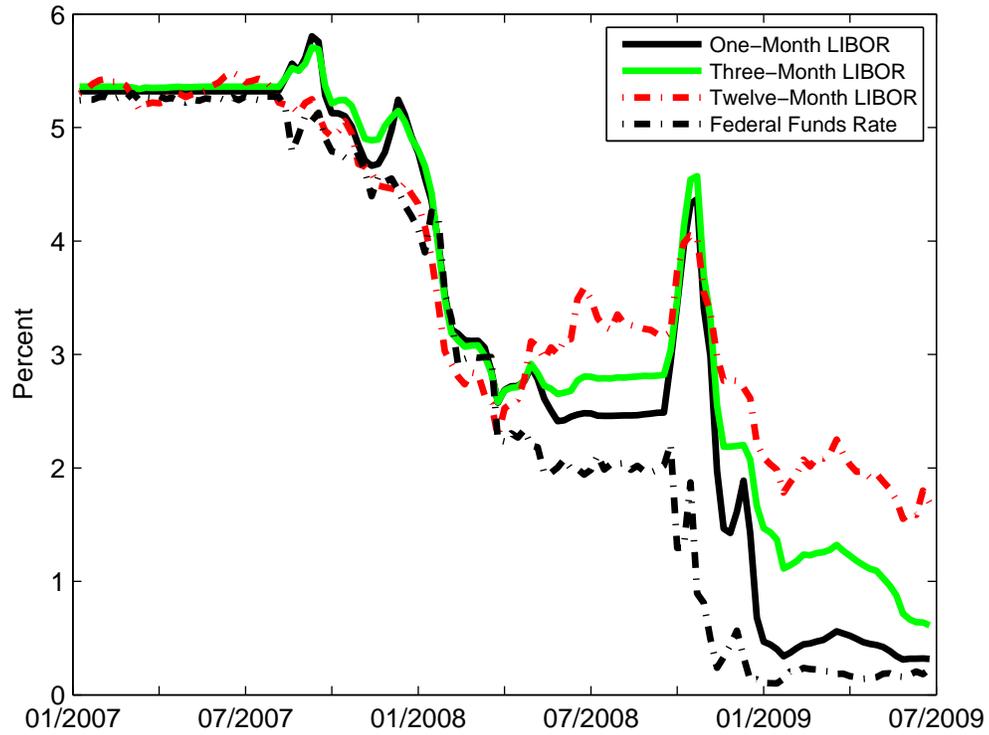
Figure 2: Weekly averages of daily data of the Federal Funds rate and the London Interbank Offering Rates (LIBOR) at the one-, three-, and twelve-month maturities for the US Dollar. LIBOR is computed using a trimmed mean of the offering rates of fifteen participants banks.
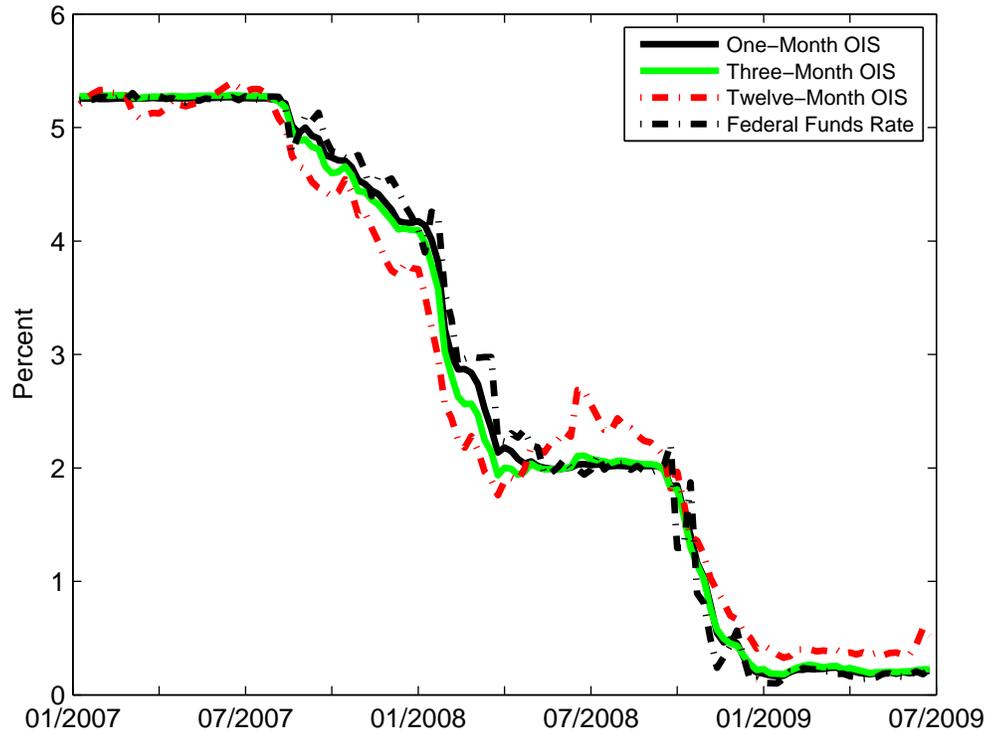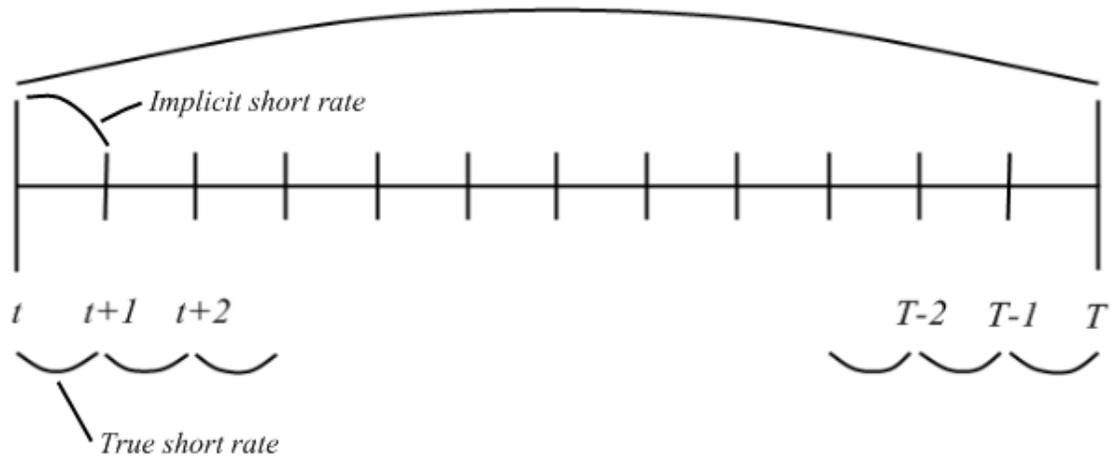
Figure 3: Weekly averages of daily data of OIS fixed rates at the one-, three-, and twelve-month maturities for the US Dollar. The index used for computed the floating leg is the Federal Funds rate.

Figure 4: Comparison of LIBOR vs. OIS.
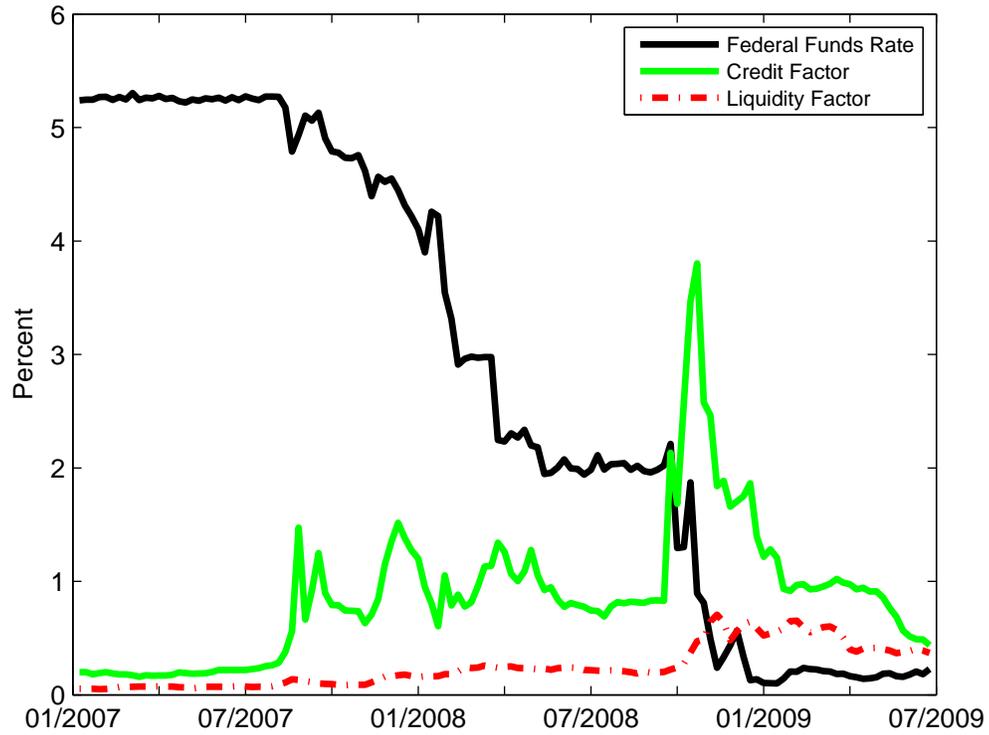
Figure 5: Weekly averages of daily data on the interest rate, credit, and liquidity factors. The interest rate factor is the Federal Funds rate, the credit factor is the three-month LIBOR-REPO spread, and the liquidity factor is the on/off-the-run ten-year U.S. Treasury premium.
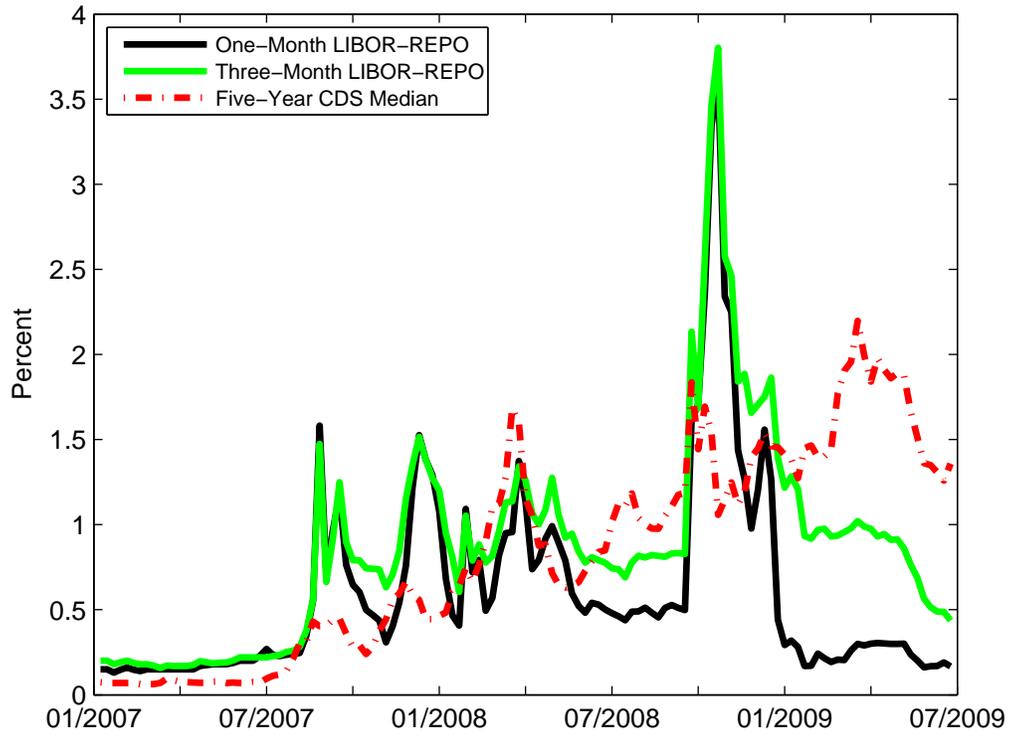
Figure 6: Weekly averages of daily data on the one-month LIBOR-REPO spread, three-month LIBOR-REPO spread, and the median five-year CDS rate for LIBOR participants institutions.
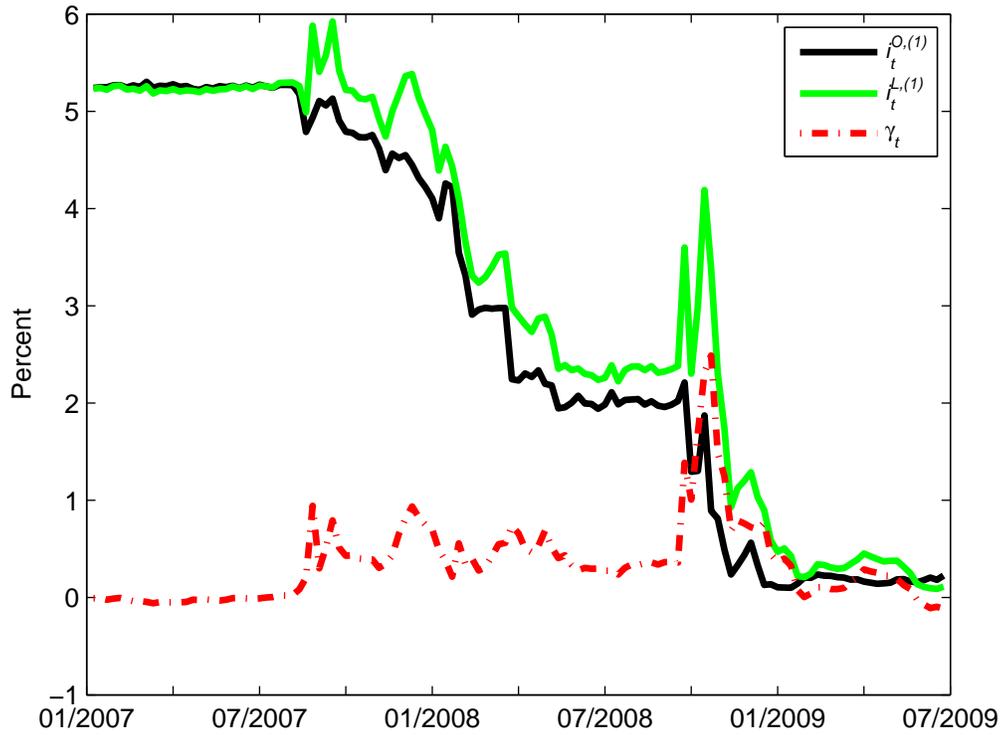
Figure 7: Comparison of LIBOR and OIS short rates from (9) and (10) and the γ_t premium. The full sample of data is 01/01/2007 - 06/19/2009. γ_t is estimated by regressing the one-week LIBOR rate minus the Federal Funds rate on a constant, the credit factor C_t, and the liquidity factor L_t. The parameters are reported in Table 2. The solid black line is the short-term OIS rate $i_t^{O,(1)} = r_t$, the green line is the estimated short-term LIBOR rate $\hat{i}_t^{L,(1)}$, and the dashed line is the estimate of $\hat{\gamma}_t = \hat{i}_t^{L,(1)} - i_t^{O,(1)}$.
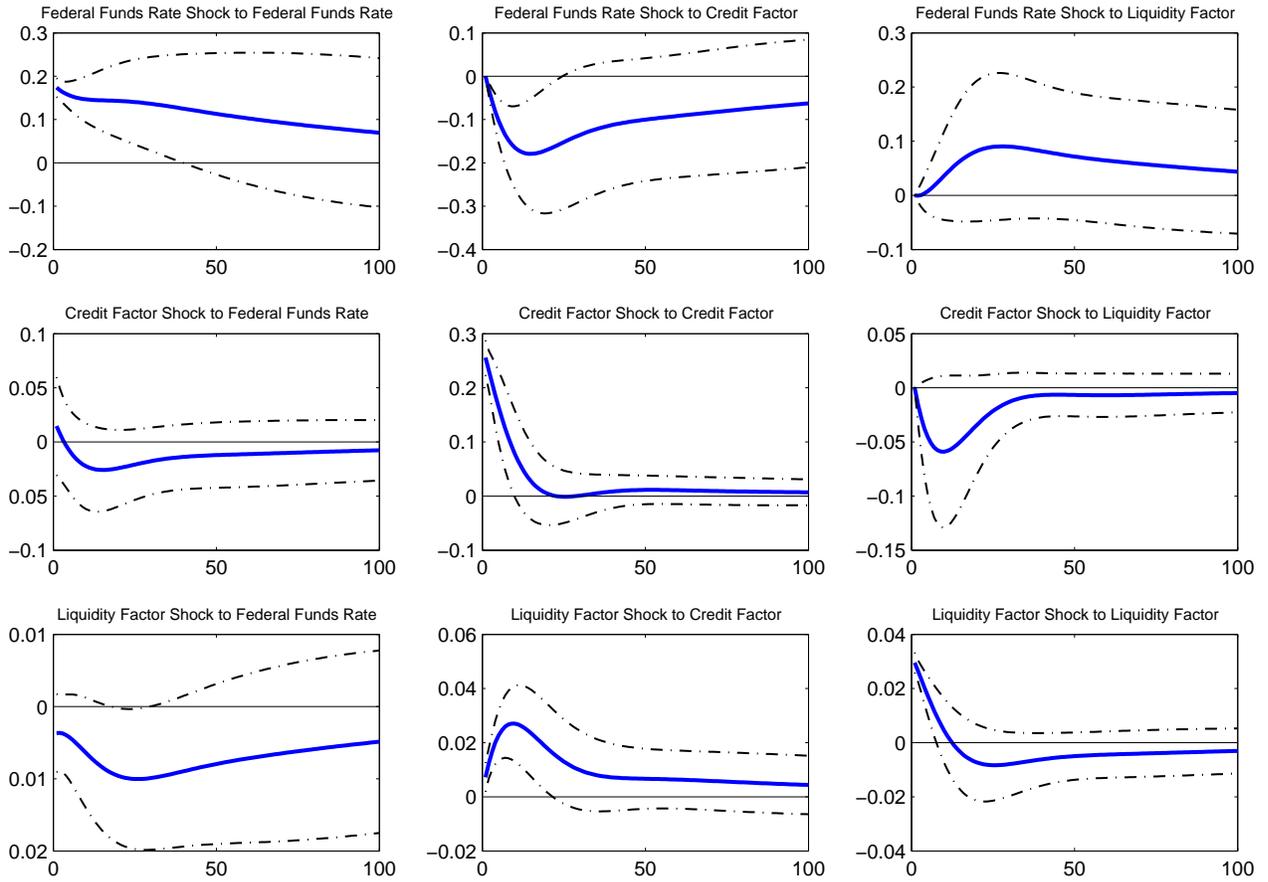
Figure 8: Impulse response functions of the VAR $X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t$. X_t consists of the interest rate factor r_t, the credit factor C_t, and the liquidity factor L_t. Σ is Cholesky-factorized, and $\varepsilon_t \sim N(0, I)$. Each figure plots the IRF of each of the factors to one standard deviation responses in a particular factor. Standard error bands are reported around estimated IRFs.
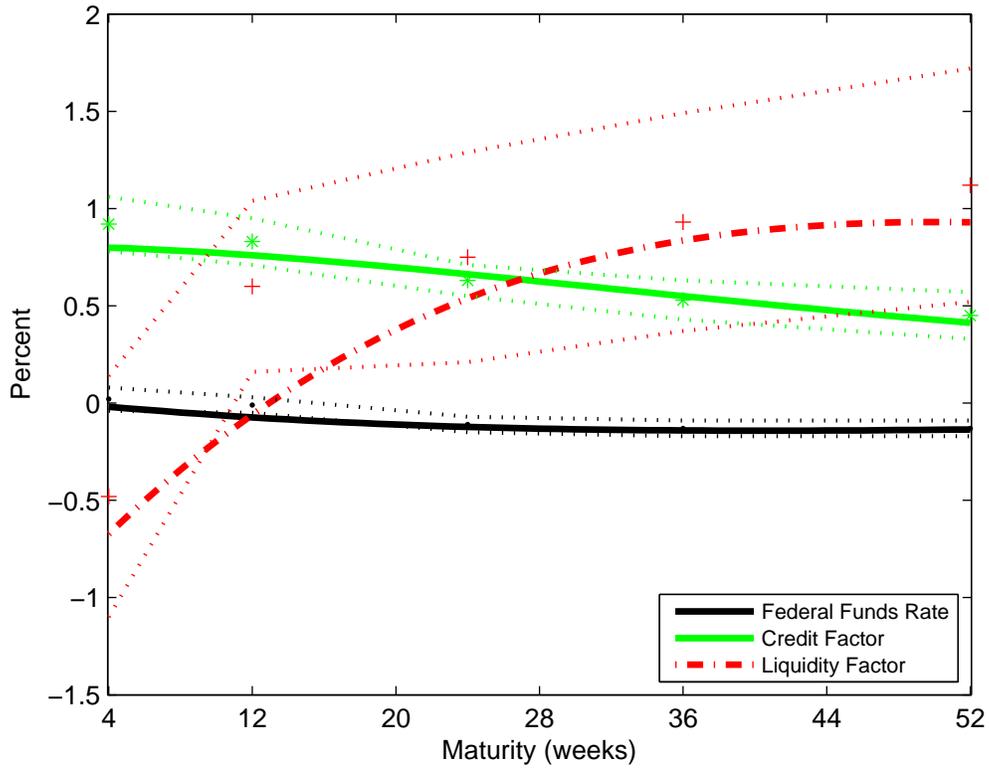
Figure 9: Estimated response coefficients b_n from the linear equations for LOIS spreads given by $z_t^{(n)} = a_n + b_n^\mathrm{T} X_t$ as a function of the maturity of the LOIS spread. The solid line shows how spreads react to the Federal Funds rate, the dashed line shows the reaction to the credit factor, and the dotted line shows the reaction to the liquidity factor.
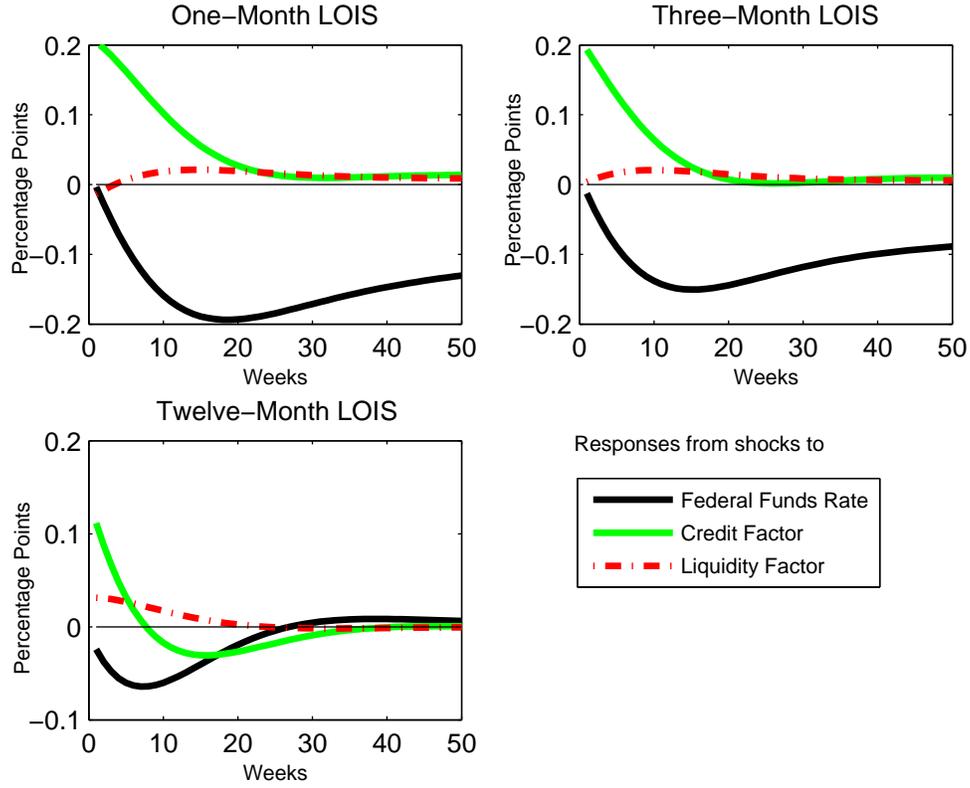
Figure 10: Impulse response functions of the one-, three-, and twelve-month LOIS spreads in the model to a one standard deviation shocks to a particular factor from X_t. The VAR is given by $X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t$. X_t consists of the interest rate factor r_t, the credit factor C_t, and the liquidity factor L_t. Σ is Cholesky-factorized, and $\varepsilon_t \sim N(0, I)$. IRFs are reported in percentage points.
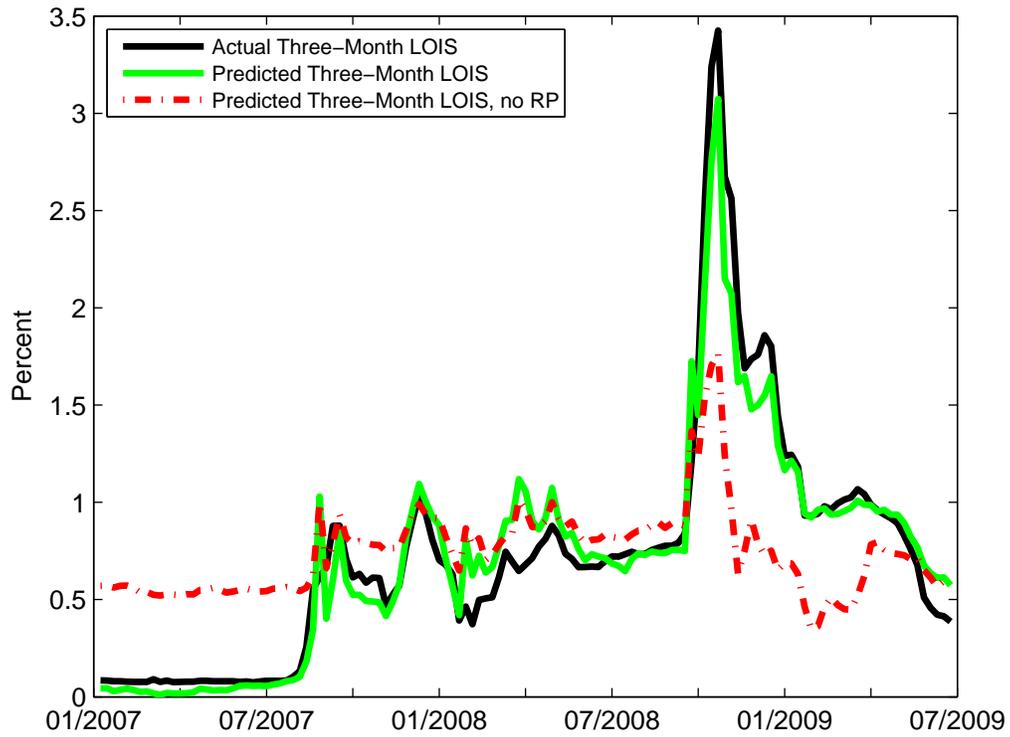
Figure 11: Three-month LOIS from the data, and predicted by the model presented here and a model under the EH. The solid dark line is the data, the solid light line is my model, and the dashed line is the model under the EH.
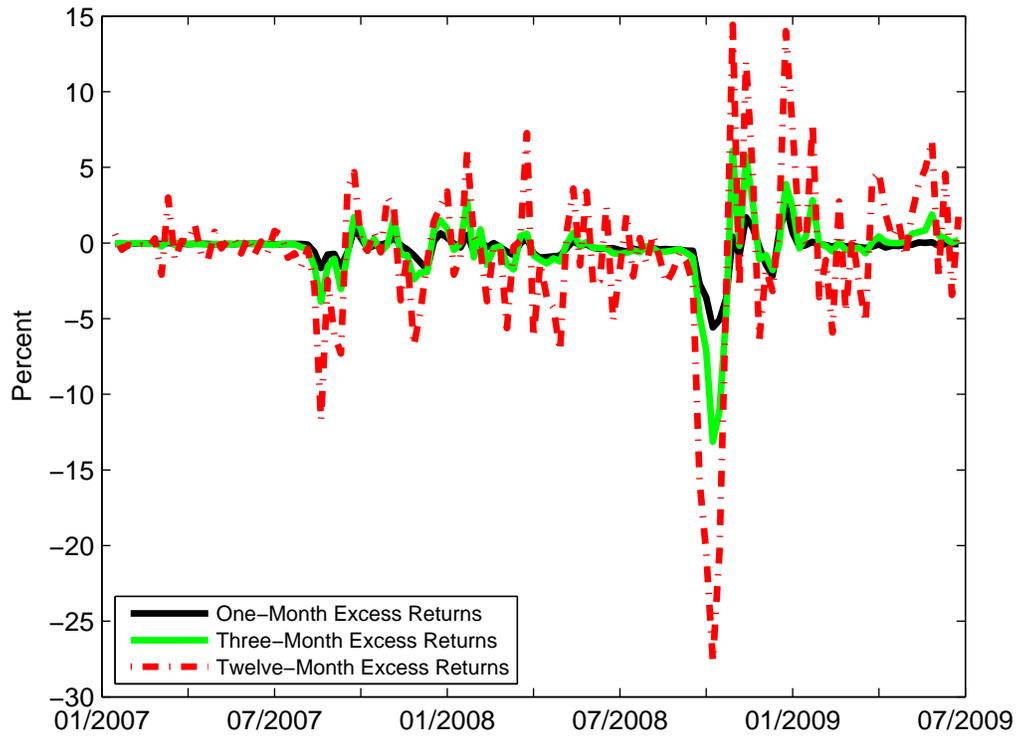
Figure 12: Relative excess returns between LIBOR and OIS for the one-, three-, and twelve-month maturities, reported in percentage points. The full sample of data is 01/01/2007-06/19/2009.