Bicycle legislation and safety information

Let H be the event that a child wears a helmet, B be the event that a child is from Beachwood, and M be the event that a child is from Morland Hills. We are told that twice as many students are from Beachwood as from Morland Hills, so $P(B) = 2/3$ and $P(M) = 1/3$. We are also told that $P(H|B) = .68$ and $P(H|M) = .21$. We want $P(H)$. We can lay this out in terms of a "hypothetical 300,000" (using a total that makes the calculations easier) as follows:

	H	H'	
B	$(200,000)(.68) = 136{,}000$		200,000
M	$(100,000)(.21) = 21{,}000$		100,000
	157,000		300,000

After filling in the missing entries by subtraction, the table gives $P(B|H')$ directly as $P(B|H') = 64000/143000 = .448$.

Best-of-7 playoff series

Say there are two teams, A and B. We will define the event A as being team A winning a game, and the event B as being team B winning a game. Since the teams are evenly-matched, $P(A) = P(B) = .5$.

(a) The series ends after 4 games if one team wins all four games; that is $AAAA$ or $BBBB$. By independence,

$$P(AAAA) = P(A)P(A)P(A)P(A) = (.5)^4 = .0625;$$

since the same is true for $BBBB$, the probability of a 4-game series is $.0625 + .0625 = .125$.

(b) A 5-game series means that it was 4-1 for one of the teams. Say A wins the series; this can only happen via $AAABA$, $AABAA$, $ABAAA$, or $BAAAA$, each of which has probability $.5^5 = .03125$, giving a probability of $(4)(.03125) = .125$. The same is true if B wins the series, so the probability of a 5-game series is $.125 + .125 = .25$.

(c) Say one team is ahead 3-2 after 5 games. If that team wins game 6 the series is over at 6 games, while if that team loses game 6 the series goes 7 games (whoever ultimately wins). Thus, the determinant of whether the series ultimately goes 6 or 7 games is only whether the team that was ahead 3-2 wins game 6. Since the teams are evenly-matched, it is equally likely that the series will be 6 or 7 games. Note that since the probability of a series going 4 or 5 games is $.125 + .25 = .375$, the probability of a 6-game series and the probability of 7-game series each equal $(1 - .375)/2 = .3125$.

(d) The equal probability, constant probability, and independence assumptions are all questionable here. Even in the finals or World Series teams might not be evenly-matched, and in earlier rounds they usually are not, since the teams are seeded. Independence is also questionable given that teams might react differently if they are up 3-1 versus down 3-1 (being full of confidence or demoralized). The probability of a team winning a particular game probably depends on whether it is at home or away, who is pitching (in baseball), and so on. The following table illustrates this for the first 100 best-of-7 World Series:

Length	Theoretical percentage	Observed percentage
4	12.50%	21%
5	25.00%	22%
6	31.25%	21%
7	31.25%	36%

Note that there are more 4- and 7-game series than expected, and fewer 5- and 6-game series. Why would that be?