

Extra random variable problems

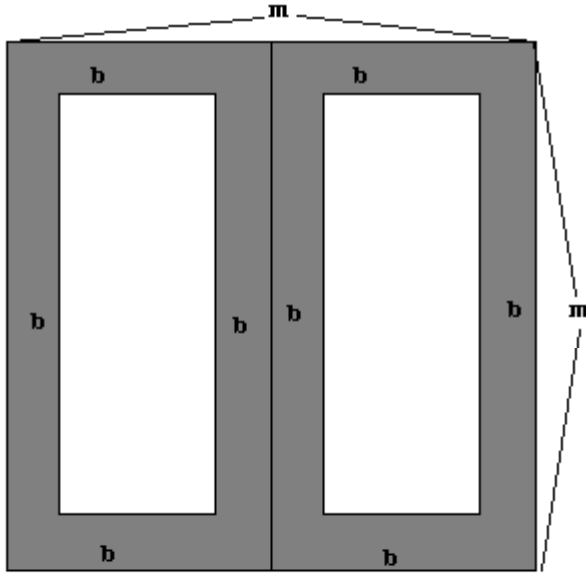
1. A study done by PaineWebber Group Inc. and the Gallup Organization in July 1997 reinforces what many investment advisors have noticed: the years of bull markets have resulted in investors expecting far higher returns on investments than was previously true. According to the survey, when asked what level of return would qualify as a “good” return, the responses were as follows:

Return expected	Proportion
0 – 5%	.19
5 – 10%	.29
10 – 15%	.34
15% and higher	.18

Treating these proportions as probabilities in the population of investors, what is the expected value of the “good” investment annual return? What is the standard deviation of this value? Use the midpoint of each interval as the value of “good” investment return for that interval, and 20% for the interval “15% and higher.” Do you think investors’ expectations are reasonable?

2. One of the most serious problems facing research libraries is the preservation of the materials that comprise their collections — materials that are deteriorating because of their chemical composition, the mechanics of their construction, and the effects of uncontrolled environmental conditions. In the late 1970s and early 1980s several large research libraries, including the Library of Congress, the New York Public Library, and the Yale University Library, undertook extensive surveys of their books to try to determine the extent of damage (for a description and discussion of the first and largest of these surveys, see “The Yale survey: a large-scale study of book deterioration in the Yale University Library” by G. Walker, J. Greenfield, J. Fox, and J.S. Simonoff, *College and Research Libraries*, **46**, 111–132 [1985]). The New York Public Library study resulted in the following estimated proportions of books requiring the following repair strategies: none needed, .815; restoration, .044; microfilming, .107; full repair needed, .034. These strategies are given in order from least difficult to most difficult.
 - (a) If a score is attached to each level of repair strategy from 1 (least difficult) through 4 (most difficult), what is the expected deterioration score of a randomly chosen book from the library?
 - (b) A better way to evaluate the repair problem is to attach repair costs to each of the types of repair. If the typical repair costs are \$0 for a book that requires no repair, \$25 for a book that requires restoration, \$75 for a book that requires microfilming, and \$500 for a book that requires full repair, what is the expected repair cost for a randomly chosen book from the New York Public Library?
3. According to the 1997 Annual Survey of the Pharmaceutical Research and Manufacturers of America, in 1995 28.7% of domestic U.S. scientific and professional research and development personnel in research-based pharmaceutical companies held a Ph.D. degree. Let D be the number of Ph.D. holders in a randomly chosen research group from a research-based pharmaceutical company. Assume that the total number of personnel is very large (it was actually close to 35,000).
 - (a) If the research group consists of 7 people, what is the expected number of Ph.D.’s in the group? What is the standard deviation of the number of Ph.D.’s in the group?
 - (b) What is the exact probability that exactly two of the people in the group are Ph.D.’s?
 - (c) What is the exact probability that at least three of the people in the group are Ph.D.’s?
 - (d) Consider four randomly chosen research groups of size 7 (from the set of all groups). What is the exact probability that exactly half of these groups have at least three of the people being Ph.D.’s?
4. Over the period 1962–1995, the daily change in the Standard and Poor’s 500 Index had mean .00032, with standard deviation .00859. Assume that the returns follow a normal distribution (actually, stock returns have a distribution that has longer tails than those of a normal distribution).
 - (a) What is the probability of a day’s return being greater than .01?
 - (b) What is the probability of a day’s return being less than -.01?
 - (c) What is the probability of a day’s return being greater than 0?
 - (d) What is the probability of a day’s return being between -.015 and .015?

- (e) What is the probability of a day's return being less than .025?
- (f) A mutual fund's prospectus states that their goal is to match the Standard and Poor's return, while giving less risk by reducing the standard deviation of the return, through clever investment strategies. To back this up, they claim that their fund will yield positive daily returns on 78.81% of all days. Assuming that their fund does indeed have the same average return as the S & P index does, what is the standard deviation of the daily return for this fund?
5. Have you ever noticed that it seems that when you are using a road map to find a specific location it seems to lie in an awkward place, such as along the edge of the map or along the central crease? We might call this "Murphy's Law of Maps" — "If a location can lie in an awkward part of the map, it will." The figure below illustrates the situation for a typical square map (folded in half) with width and length equal to m . The "Murphy zone" of awkward places is a strip of width b along the edges of the map and along the central crease, and is given in gray in the figure.



- (a) The area of a rectangle is equal to the product of its width and height (you remembered that from high school, didn't you?). Verify that the area of the entire map is m^2 , while the total area of the two unshaded non-Murphy zones is $m^2[1 - 6(b/m) + 8(b/m)^2]$.
- (b) If locations are equally likely to fall anywhere on the map, the probability that a randomly chosen location will fall in the Murphy zone is just the ratio of the area covered by the Murphy zone to the total area of the map. Verify that this equals $6(b/m) - 8(b/m)^2$.
- (c) In the figure the width of the strips comprising the Murphy zone (b) is one-tenth the width of the entire map (m). Given this, what is the probability that a randomly chosen location will fall in the Murphy zone? Does the answer surprise you? What does it say about this example of Murphy's Law ("Anything that can go wrong, will go wrong")?
- (d) A travel agent is planning trips to the same general location for 40 different customers. If we consider the travel destinations of each customer as independent of each other, what distribution does the number of destinations that fall in the Murphy zone have? Assume the same map structure as in part (c) for this, and all subsequent questions.
- (e) What is the expected number of destinations in the Murphy zone?
- (f) What is the standard deviation of the number of destinations in the Murphy zone?
- (g) What is the (approximate) probability that at least 15 destinations will end up in the Murphy zone?
- (h) What is the (approximate) probability that exactly 19 destinations end up in the Murphy zone?
6. Among the top 50 stock mutual funds, as measured by their ten-year annualized return, the distribution of loads was as follows on May 18, 1998:

Load	Proportion
0%	.34
3%	.30
4.75%	.08
5.5%	.10
5.75%	.10
6% and higher	.08

Treating these proportions as probabilities for the process of “top” mutual funds, what is the expected value of the load? What is the standard deviation of this value? Use the value 7.0 for the interval “6% and higher.” If you were asked to give a “typical” value for the load of top mutual funds, what value would you give? Why?

7. A poll conducted by *Newsday* and the *Los Angeles Times* during March 8 – April 1, 1998, asked Jews around the United States their feelings about Israel. One question that was asked was as follows: “How close do you feel to Israel?” The following distributions of answers were given by women and men:

	Very close	Fairly close	Fairly distant	Very distant
<i>Women</i>	.26	.33	.25	.16
<i>Men</i>	.23	.38	.28	.11

Treating these proportions as probabilities for the responses of all Jewish adult women and men in the U.S., respectively, do the probability distributions seem very different to you? Support your answer using numerical calculations and (possibly) graphical representations. For example, you could assign what you consider to be reasonable numerical values to the four levels Very close, Fairly close, Fairly distant, and Very distant, and then calculate and compare appropriate numerical summaries of the probability distributions for the two random variables, Women’s response and Men’s response.

8. According to a 1997 survey of companies in the Fortune 500 by the research group Catalyst, 2.5% of the top earners in such companies are women (the survey was based on the top five earners among the corporate officers of the companies). Let W be the number of women among the top five earners in a randomly chosen Fortune 500 company.
- What is the expected number of women in the top five earners? What is the standard deviation of the number of women?
 - What is the exact probability that exactly two of the people in the group are women?
 - What is the exact probability that none of the people in the group are women?
 - Consider ten randomly chosen Fortune 500 companies. What is the exact probability that all ten of these companies have no women among the top five earners of the company?
9. Over the period 1990–1995, the daily change in the Standard and Poor’s 500 Index had mean .00039, with standard deviation .00721. Assume that the returns follow a normal distribution (actually, stock returns have a distribution that has longer tails than those of a normal distribution).
- What is the probability of a day’s return being less than .01?
 - What is the probability of a day’s return being greater than 0?
 - What is the probability of a day’s return being between -.01 and .01?
 - What is the probability of a day’s return being less than .005?
 - A mutual fund’s prospectus states that their goal is to beat the Standard and Poor’s return, while giving the same risk by matching the standard deviation of the return, through clever investment strategies. To back this up, they claim that their fund will yield positive daily returns on 78.81% of all days. Assuming that their fund does indeed have the same standard deviation of return as the S & P index does, what is the average daily return for this fund?
10. The Moody’s corporate bond default study mentioned earlier also tracks the changes in rating for bonds that have not been retired. Between January 1998 and January 1999, corporate bonds rated as A moved the following rating levels:

Movement	Percentage
2	.09%
1	2.78%
0	84.42%
-1	9.37%
-2	.46%

That is, for example, .09% of A bonds moved up two rating classes to Aaa during that year; 9.37% of A bonds moved down one rating class to Baa. Treating these values as probabilities for the process of movement of A bonds, what is the expected value of the one-year movement? What is the standard deviation of the movement? If you were asked to give a “typical” value for the movement of A bonds, what value would you give? Why?

11. According to a 1996 survey of New York college students conducted by the Office of Alcoholism and Substance Abuse Services (the results of which were released on March 10, 1999), 41% of students were binge drinkers (defined as having had five or more drinks in one sitting within the previous two weeks). Let B be the number of binge drinkers among six randomly chosen students.
 - (a) What is the expected number of binge drinkers in the group of students? What is the standard deviation of the number of binge drinkers?
 - (b) What is the exact probability that exactly two of the people in the group are binge drinkers?
 - (c) What is the exact probability that at least one of the people in the group is a binge drinker?

12. Over the period February 1996 through October 1998, the monthly change in the Standard and Poor’s 500 Index had mean .01784, with standard deviation .04793. Assume that the returns follow a normal distribution (actually, stock returns generally have a distribution that has longer tails than those of a normal distribution).
 - (a) What is the probability of a day’s return being greater than .01?
 - (b) What is the probability of a day’s return being less than 0?
 - (c) What is the probability of a day’s return being between -.05 and .05?
 - (d) A mutual fund’s prospectus states that their goal is to match the Standard and Poor’s return, while giving the smaller risk by having smaller standard deviation of the return, through clever investment strategies. To back this up, they claim that their fund will yield positive monthly returns on 94.52% of all months. Assuming that their fund does indeed have the same mean return as the S & P index does, what is the standard deviation of the monthly return for this fund?