THE JOURNAL OF FINANCE • VOL. XLVIII, NO. 2 • JUNE 1993

A Semiautoregression Approach to the Arbitrage Pricing Theory

JIANPING MEI*

ABSTRACT

This paper developes a semiautoregression (SAR) approach to estimate factors of the arbitrage pricing theory (APT) that has the advantage of providing a simple asymptotic variance-covariance matrix for the factor estimates, which makes it easy to adjust for measurement errors. Using the extracted factors, I confirm the finding that the APT describes asset returns slightly better than the CAPM, although there is still some mispricing in the APT model. I find that not only are the factors "priced" by the market, but the factor premiums move over time in relation to business cycle variables.

A MAJOR PROBLEM IN studying asset pricing is that pervasive factors affecting asset returns are unobservable. Although the arbitrage pricing theory (APT) has been studied empirically for a decade, estimation methods for factors and factor loadings remain a focus of research.[1] In this paper, I develop a new semiautoregression (SAR) approach to estimating factors of the APT model.

The intuition underlying the SAR approach is that for any given factors, there is an approximate linear relationship between the returns and factor loadings. So, if we have more returns than factor loadings, we can approximate the loadings by using returns from a subsample. Once we have proxies for the loadings, we can then perform large cross-sectional regressions to estimate factors associated with returns from other sample periods.[2]

* Department of Finance, Stern School, New York University. I am greatly indebted to John Campbell for his comments and encouragement, to Douglas Holtz-Eakin for providing me with part of the program for the autoregression, and to Wayne Ferson for letting me use his data. The paper has benefited greatly from valuable suggestions by René Stulz, the editor, and an anonymous referee. Helpful comments from Gregory Chow, Silverio Foresi, Larry Lang, Bruce Lehmann, Crocker Liu, Albert Margolis, Burton Malkiel, Whitney Newey, Richard Quandt, Bob Stambaugh, and seminar participants at Princeton University and at the NBER summer workshop are gratefully acknowledged. The paper was originally entitled "Extracted factors and time-varying conditional risk premiums: A new approach to the APT." This research is partly supported by the John M. Olin Foundation for Study of Economic Organization and Public Policy.

[1] Recent studies include Connor and Korajczyk (1988), Lehmann and Modest (1988), and Shukla and Trzcinka (1990).

[2] My technique is similar in spirit to the asymptotic principal components technique developed by Connor and Korajczyk (1988), who use large cross-sectional samples. It retains several important advantages of their approach, such as allowing for time-varying risk premiums, avoiding the estimation of factors loadings, and being applicable to an *approximate* factor model.

This approach differs from conventional factor extraction techniques, such as maximum likelihood factor analysis and the principal components approach, in several ways. First, my approach does not require the distributional assumptions made in the maximum likelihood estimation. Second, it places little restriction on the time and cross-sectional variation of idiosyncratic shocks.

A distinctive advantage of the SAR approach is that it provides a simple asymptotic variance-covariance matrix for the factor estimates, which can be used to control for measurement errors. This gives the approach an edge both in testing the APT and in using it to evaluate performance, where one needs to take into account the measurement errors in the benchmark portfolios when calculating abnormal performance. The approach combines the simplicity of regression analysis with the flexibility of the Newey-West adjustment procedure, which makes it both intuitive and easy to use.

Applying the SAR approach, I conduct an empirical study of the APT model. I also analyze the relationship between factor risk premiums and business conditions, integrating two approaches to asset pricing used in recent empirical studies. The first approach, represented by Chan, Chen, and Hsieh (1985), Chen, Roll, and Ross (1986), and Shanken and Weinstein (1990), focuses on whether some economic factors are "priced" by the market, assuming that the factor premiums are constant. The second approach, represented by Campbell (1987), Chen (1991), Fama and French (1988, 1989), Ferson and Harvey (1991), and Keim and Stambaugh (1986), examines the relationship between overall economic conditions and time-varying risk premiums for a small group of portfolios. Neither group of studies, however, estimates factors from a large panel data set or explicitly investigates the relationship between factor premiums and business conditions.

The paper is organized as follows. Section I demonstrates that historical returns can be used to approximate unobservable factor loadings and that factors can be estimated by performing a series of large cross-sectional semiautoregressions. Section II first estimates a five-factor APT model and then conducts some asset-pricing tests in comparison with the CAPM. Section III compares the advantages and disadvantages of my method with those of alternative approaches. Section IV studies the relationship between the extracted factors and some business cycle variables. Section V summarizes the results.

I. The Linear Factor Model and the Semiautoregressive (SAR) System

Assume that asset returns are generated by the following approximate K-factor model:

$$R_{it} = E_t(R_{it}) + f_{1t}\beta_{i1} + \cdots + f_{Kt}B_{iK} + \varepsilon_{it} \qquad i = 1,\ldots,N; \quad t = 1,\ldots,T. \tag{1}$$

where $f_t' = (f_{1t}, \dots, f_{Kt})$ is a $Kx1$ column vector of unobservable pervasive shocks, $\beta_i' = (\beta_{i1}, \dots, \beta_{iK})$ is a vector of factor loadings that are constant over the sample period, and $\varepsilon_t' = (\varepsilon_{1t}, \dots, \varepsilon_{Nt})$ represents a vector of idiosyncratic risk specific to each asset in time period t. I also assume that $E(f_t) = 0$, $E(\varepsilon_t/f_t) = 0$, and $\mathrm{Cov}(\varepsilon_t) = D_t$, where D_t is block diagonal, and that the size of the block is such that the error terms $\{\varepsilon_{it}, i = 1, 2, \dots\}$ satisfy the mixing conditions given by Connor and Korajczyk (1988).

Using the equilibrium APT of Connor and Korajczyk (1988), the above economy implies the following linear pricing relationship:

$$E_t(R_{it}) = \lambda_{0t} + \lambda_{1t}\beta_{i1} + \cdots + \lambda_{Kt}\beta_{iK}, \tag{2}$$

where $(\lambda_{1t}, \dots, \lambda_{Kt})$ is a vector of risk premiums corresponding to the pervasive shocks $(f_{1t}, \dots, f_{Kt})$, and λ_{0t} is the return on a zero beta portfolio. Combining (1) and (2), denoting $s_t' = (s_{1t}, \dots, s_{Kt}) = (f_{1t}, \dots, f_{Kt}) + (\lambda_{1t}, \dots, \lambda_{Kt})$, we obtain,

$$R_{it} = \lambda_{0t} + s_{1t}\beta_{i1} + \cdots + s_{Kt}\beta_{iK} + \varepsilon_{it} \qquad i = 1, \dots, N; \quad t = 1, \dots, T. \tag{3}$$

for simplicity, I will call s_{kt} factors and f_{kt} systematic shocks later in the paper.

If we were able to observe β_{ik}, it would be straightforward to obtain the factor estimates of (3) by running a cross-sectional regression of R_{it} on β_{ik}. Since β_{ik} is not observable, we need to construct a proxy for the factor loadings. Although there might be some ad hoc ways to do so, we want to derive the proxy directly from the return-generating process of (3). This approach has the advantage of using the return-generating process directly and allowing us to test some restrictions imposed by the asset-pricing model.

To derive a proxy for the unobservable β_{ik}, I take (3) for asset i from time period 1 to K, stack the equations move λ_{0t} and ε_{it} to the other side, and denote:

$$\psi = \begin{bmatrix} s_{11} & \cdots & s_{K1} \\ \vdots & \ddots & \vdots \\ s_{1K} & \cdots & s_{KK} \end{bmatrix}, \qquad \beta_i = \begin{bmatrix} \beta_{i1} \\ \vdots \\ \beta_{iK} \end{bmatrix} \qquad \text{and} \quad \gamma_i = \begin{bmatrix} R_{i1} - \lambda_{01} - \varepsilon_{i1} \\ \vdots \\ R_{iK} - \lambda_{0K} - \varepsilon_{iK} \end{bmatrix}.$$

It follows from (3) that

$$\Psi\beta_i = \gamma_i. \tag{4}$$

Assume that there are no redundant factors in the model so that Ψ is nonsingular. I normalize factors and factor loadings by setting

$$\tilde{s}_t = (\tilde{s}_{1t}, \dots, \tilde{s}_{Kt})' = (\Psi^{-1})' s_t \qquad \text{and} \quad \tilde{\beta}_i = \left(\tilde{\beta}_{i1}, \dots, \tilde{\beta}_{iK}\right)' = \Psi\beta_i,$$

where s_t is the original vector of factors. Then, I can rewrite (3) as

$$R_{it} = \lambda_{0t} + \tilde{s}_{1t}\tilde{\beta}_{i1} + \cdots + \tilde{s}_{Kt}\tilde{\beta}_{iK} + \varepsilon_{it}. \qquad i = 1, \dots, N; \quad t = K+1, \dots, T. \tag{5}$$

From (4), we have:

$$\tilde{\beta}_i = \Psi\beta_i = \gamma_i = \begin{bmatrix} R_{i1} - \lambda_{01} - \varepsilon_{i1} \\ \vdots \\ R_{iK} - \lambda_{0K} - \varepsilon_{iK} \end{bmatrix}. \quad (6)$$

Equation (6) has a simple intuitive explanation. It says that historical excess returns can approximate the factor loadings because idiosyncratic shocks are just random noise. Substituting (6) into (5) for time period $t(t > K)$ and collecting terms, we obtain:

$$R_{it} = \tilde{s}_{0t} + \tilde{s}_{1t}R_{i1} + \cdots + \tilde{s}_{Kt}R_{iK} + \eta_{it}, \qquad i = 1,\ldots,N; \quad t = K+1,\ldots,T \quad (7)$$

where:

$$\tilde{s}_{0t} = \lambda_{0t} - \sum_{v=1}^{K} \tilde{s}_{vt}\lambda_{0v} \quad \text{and} \quad \eta_{it} = \varepsilon_{it} - \sum_{v=1}^{K} \tilde{s}_{vt}\varepsilon_{iv} \qquad t = K+1,\ldots,T. \quad (8)$$

Therefore, we have transformed the K-factor model of (3) into a K-lag semiautoregressive model. For any time period t $(K + 1 \leq t \leq T)$, we can run a cross-sectional regression of R_{it} on a fixed set of regressors, $(1, R_{i1},\ldots,R_{iK})$, to obtain estimates of the normalized factors, $\tilde{s}_{kt}$. The autoregressors in (7), unlike those in a regular cross-sectional autoregressive model, $R_{it} = a_{0t} + a_{1t}R_{i,t-1} + \cdots + a_{Kt}R_{i,t-K} + \eta_{it}$, which vary both across i and over time t, vary only across i. Thus, I call (7) a semiautoregressive (SAR) model.

The intuition behind (7) is that historical excess returns are useful in explaining current cross-sectional returns because they span the same return space as $\tilde{\beta}_i$ and thus can be used as proxies for systematic risks. The substitution of excess returns for unobservable betas is similar in spirit to the technique of substituting "mimicking factor portfolio" returns for unobservable factors used by Jobson (1982), and Huberman, Kandel, and Stambaugh (1987).

To estimate the semiautoregression (7), I rewrite it as:

$$\begin{bmatrix} R_{1t} \\ \vdots \\ R_{Nt} \end{bmatrix} = \begin{bmatrix} 1 & R_{11} & \cdots & R_{1K} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & R_{N1} & \cdots & R_{NK} \end{bmatrix} \begin{bmatrix} \tilde{s}_{0t} \\ \tilde{s}_{1t} \\ \vdots \\ \tilde{s}_{Kt} \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \vdots \\ \eta_{Nt} \end{bmatrix}, \qquad t = K+1,\ldots,T. \quad (9)$$

I stack equation (9) by time to obtain a system of equations. This system looks very similar to the classic seemingly unrelated regressions (SUR) system, but here the system is a set of *cross-sectional* regressions for different time periods instead of the conventional *time series* regressions for different individuals. Through their correlation with $\varepsilon_{i,t-1}, \ldots, \varepsilon_{i,t-K}$, the autoregressors $R_{ik}(k = 1, \ldots, K)$ are correlated with the error term η_{it}. η_{it} is heteroskedastic and cross-sectionally correlated. It follows from (8) that as long as $\varepsilon_{i\tau}$ and $\tilde{s}_{kt}$ are independent, η_{it} will satisfy the mixing condition. This is because when $\tilde{s}_{kt}$ is given, η_{it} is a linear function of $\varepsilon_{i\tau}$, which satisfies the mixing condition. Thus an instrumental variables regression is needed to estimate (9), with appropriate adjustments to account for heteroskedasticity and cross-sectional correlations. Since the R_{is}'s $(K < s < t)$ are orthogonal to η_{it} and are correlated cross-sectionally with the autoregressors $R_{ik}(1 < k < K)$ on the right side of (9), we can use them as instruments to estimate the autoregression.[3] The orthogonal condition is satisfied because the R_{is} contains only idiosyncratic risk $\varepsilon_{is}(K < s < t)$ and the η_{it} contains only idiosyncratic risk $\varepsilon_{i\tau}(\tau = 1, \ldots, K, t)$ according to (8). There is no overlap of idiosyncratic risk in R_{is} and η_{it}, and ε_{it} is assumed to be independent over time.

I use a modified $3SLS$ (three-stage least squares) cross-sectional regression technique, developed by Holtz-Eakin, Newey, and Rosen (1988), to accomplish the estimation task.[4] To take into account the cross-sectional correlations of idiosyncratic risk in model (1), I introduce the Newey-West (1987) adjustment matrix into the estimation procedure. If the data satisfy the orthogonal condition and the other regularity conditions in Newey-West (1987), the estimates will be consistent despite the presence of heteroskedasticity and cross-sectional correlations. For convenience, I call the estimate, $\tilde{s}_{kt}$, the kth extracted factor.

Intuitively, the estimation procedure is a special case of the $3SLS$ instrumental estimation technique. In the first stage, a simple instrumental variables regression is used to obtain an estimate of (9) for each time period, ignoring heteroskedasticity and cross-sectional correlations. In the second and third stages, I use moving averages of the residuals from the first stage regression to calculate a Newey-West adjustment matrix. I then use a generalized least squares (GLS) method to obtain a more efficient estimate of the autoregression for all periods. This procedure has two distinctive advantages. First, unlike most generalized method of moments or maximum likelihood estimation procedures, it does not require nonlinear optimization, which makes it easy to use. Second, it is based on a simple regression framework, which makes it intuitive and easy to understand. A detailed description of the estimation procedure is given in the appendix.

[3] R_{is} $(K < s < t)$ are perfect instruments for R_{ik} $(1 < k < K)$ if the idiosyncratic shocks are zero. In this case, they are perfectly correlated because they span the same return space according to equation (7).

[4] See the Appendix or Holtz-Eakin, Newey, and Rosen (1988) for details of the estimation procedure.

II. Empirical Implementation of the Semiautoregressive Approach

A. Data Source and the Extraction of Factors

The data are obtained from the December 1989 version of the Center for Research in Security Prices (CRSP) monthly return file. I construct four data sets, covering 1969 to 1973, 1974 to 1978, 1979 to 1983, and 1984 to 1988. I choose these time periods to make this study comparable to Connor and Korajczyk (1988) and other previous studies. Each data set includes all New York Stock Exchange (NYSE) securities with no missing information on returns during the five-year period. The numbers of securities available are 1105, 1276, 1210, and 1089, respectively. The riskless return is assumed to be equal to the return on the thirty-day treasury bill.

Using the four panel data sets, I estimate a five-factor model for 1970 to 1973, 1975 to 1978, 1980 to 1983, and 1985 to 1988, using data from 1969, 1974, 1979, and 1984 for lags and instruments.[5] For each five-year period, I use the first five months of returns as the semiautoregressors. Then, starting from the thirteenth month, I choose returns from $t - 1$ to $t - 7$ as instruments to estimate factors for any given month t. I first sort the returns data by four-digit SIC industry code and then use a moving average window of thirty, the average number of firms in most industries, to calculate the Newey-West adjustment matrix. I implicitly assume that the idiosyncratic shocks to different stocks, which are far apart in the sense of belonging to different industries, are not correlated. After obtaining the factor estimates, I rescale them so that the equally weighted portfolio has unit factor loadings on each factor.

Figure 1 plots the first factor extracted from a five-factor model for the 1970 to 1973 period. I also provide 95% confidence intervals for the extracted factors. The upper bond is calculated by adding 1.96 times the standard deviation to the extracted factor and the lower bond by subtracting 1.96 times the standard deviation. The standard errors are taken from the variance-covariance matrix of extracted factors. The boundaries are generally quite tight, suggesting that the estimation errors are small.

The factors extracted here play a relatively equal role in explaining portfolio returns. This finding differs from that of Connor and Korajczyk (1988),

[5] I have also estimated the APT model with more than five factors. Like Connor and Korajczyk (1988), I find there are more than five factors at work in the economy, but factors in excess of five generally do not play an important role in explaining variations in asset returns, although they are statistically significant. The testing of the number of factors in the economy is straightforward, since a K-factor model translates to a K-lag semiautoregressive model. The procedure is to first estimate the *unrestricted* ($K + 1$ factor) model, then the *restricted* (K) model, and calculate the difference in their sum of squared residuals, Q_U and Q_R. If the restricted model is true, we expect the difference ($L = Q_R - Q_U$) to be small. Under the null hypothesis, L follows a χ^2 distribution with degrees of freedom equal to the difference between the number of coefficients to be estimated in the two systems. If the restriction does not hold, the difference will be large, indicating a rejection of the restricted model. See Holtz-Eakin, Nwey, and Rosen (1988) for details.

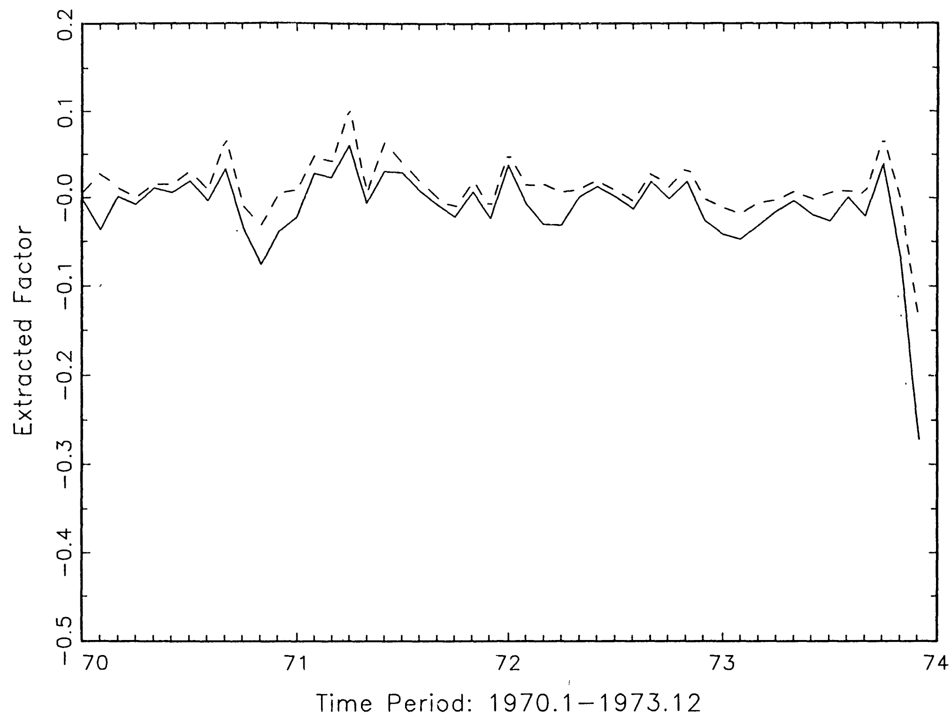

Figure 1. The first factor ($\tilde{s}_{1t}$) extracted from a five-factor APT model (1970 to 1973). The factor is extracted by the semiautoregressive approach. Ninety-five percent confidence bounds are provided for the extracted factor. The upper bound is calculated by adding 1.96 times the standard deviation to the extracted factor and the lower bound by subtractiing 1.96 times the standard deviation. The standard deviations are taken from the variance-covariance matrix of the extracted factors.

who extract factors using the asymptotic principal components technique. Their first factor, which corresponds to the largest eigenvalue, generally explains over 99% of the variation of the excess returns of the equally weighted portfolio, and other factors have little explanatory power over the variation. The maximum explanatory power of any one factor in my study is about 43% of the variance. Moreover, the factor that explains most could be the first factor in one sample period and the fourth factor in the next period. Thus, no factor estimates mimic the equally weighted portfolio in the SAR estimation.

B. Test of the APT Model

To test some restrictions imposed by the linear pricing relationship, I perform a time series regression of returns of the size-decile portfolios on the extracted factors:

$$R_{it} = a_{i0} + b_{i0}\tilde{s}_{0t} + b_{i1}\tilde{s}_{1t} + \cdots + b_{ik}\tilde{s}_{kt} + \varepsilon_{it}. \tag{10}$$

By comparing (10) with (5) and (8), one can see that the first two coefficients in (10) should satisfy the following constraints: $a_{i0} = 0$, $b_{i0} = 1$. The intuition behind the restriction is that there should be no mispricing by the asset-pricing model and the coefficient on the zero-beta rate (λ_{0t}) should be one.

The regression results for the five-factor model are shown in Table I. The model generally explains between 75% and 98% of the variance of the decile portfolio returns after an adjustment is made for degrees of freedom. The hypothesis H_0: $a_{i0} = 0$ is rejected in one out of two tests at the 5% significance level, indicating the existence of mispricing in the data. The hypothesis H_0: $b_{i0} = 1$ is rejected in one out of four tests, and the joint hypothesis H_0: $a_{i0} = 0$, $b_{i0} = 1$ is rejected in one out of two tests with the

Table I

Goodness of Fit and Tests of Mispricing on Decile Portfolios

Time series regression of monthly decile returns on factors estimated by the semiautoregressive (SAR) method. Factors are estimated using monthly stock returns on 1105, 1276, 1210, 1089, securities over the periods 1969 to 1973, 1974 to 1978, 1979 to 1983, and 1984 to 1988. P_1 is the significance level at which the hypothesis H_0: $a_{\iota 0} = 0$ is rejected, P_2 is the significance level at which the hypothesis H_0: $b_{i0} = 1$ is rejected, and P_3 is the significance level at which the joint hypothesis H_0: $a_{i0} = 0$, $b_{i0} = 1$ is rejected. $\bar{R}^2$ is the adjusted R^2 for the regression: $R_{it} = a_{i0} + b_{i0}\tilde{s}_{0t} + b_{i1}\tilde{s}_{1t} + \cdots + b_{ik}\tilde{s}_{kt} + \varepsilon_{it}$. The numbers in parentheses are the standard errors for the statistics, P_1, P_2, P_3, and $\bar{R}^2$, respectively, due to measurement errors. Decile 1 is composed of small stocks from the first decile of size-sorted portfolios and Decile 10 is composed of large stocks from the last decile of size-sorted portfolios.

	Time Period 1970–1973				Time Period 1975–1978			
Decile	P_1	P_2	P_3	$\bar{R}^2$	P_1	P_2	P_3	$\bar{R}^2$
1	0.169	0.641	0.364	0.922	0.765	0.297	0.493	0.954
	(0.072)	(0.298)	(0.149)	(0.005)	(0.231)	(0.207)	(0.266)	(0.006)
2	0.598	0.794	0.831	0.948	0.010	0.001	0.000	0.971
	(0.126)	(0.263)	(0.095)	(0.002)	(0.007)	(0.001)	(0.000)	(0.002)
3	0.128	0.482	0.223	0.976	0.001	0.151	0.000	0.969
	(0.051)	(0.229)	(0.078)	(0.001)	(0.000)	(0.113)	(0.000)	(0.002)
4	0.167	0.640	0.326	0.974	0.001	0.021	0.000	0.972
	(0.055)	(0.238)	(0.086)	(0.001)	(0.001)	(0.025)	(0.000)	(0.001)
5	0.065	0.422	0.116	0.972	0.000	0.522	0.000	0.975
	(0.031)	(0.207)	(0.050)	(0.001)	(0.000)	(0.224)	(0.000)	(0.001)
6	0.247	0.369	0.327	0.973	0.000	0.736	0.001	0.979
	(0.064)	(0.168)	(0.094)	(0.001)	(0.000)	(0.206)	(0.001)	(0.001)
7	0.148	0.179	0.120	0.970	0.000	0.146	0.000	0.974
	(0.064)	(0.119)	(0.061)	(0.002)	(0.000)	(0.082)	(0.000)	(0.001)
8	0.013	0.976	0.044	0.963	0.029	0.661	0.093	0.967
	(0.007)	(0.280)	(0.020)	(0.002)	(0.016)	(0.241)	(0.044)	(0.002)
9	0.169	0.422	0.305	0.954	0.511	0.265	0.085	0.938
	(0.070)	(0.266)	(0.162)	(0.003)	(0.127)	(0.024)	(0.064)	(0.005)
10	0.020	0.000	0.000	0.836	0.918	0.013	0.039	0.809
	(0.008)	(0.001)	(0.000)	(0.008)	(0.160)	(0.012)	(0.036)	(0.016)

Table I—*Continued*

	Time Period 1980–1983				Time Period 1985–1988			
Decile	P_1	P_2	P_3	$\bar{R}^2$	P_1	P_2	P_3	$\bar{R}^2$
1	0.001	0.991	0.002	0.811	0.897	0.173	0.383	0.896
	(0.001)	(0.548)	(0.002)	(0.007)	(0.167)	(0.098)	(0.156)	(0.007)
2	0.000	0.592	0.000	0.910	0.190	0.141	0.216	0.959
	(0.000)	(0.561)	(0.000)	(0.008)	(0.070)	(0.076)	(0.098)	(0.003)
3	0.000	0.920	0.000	0.945	0.440	0.725	0.734	0.954
	(0.000)	(0.962)	(0.000)	(0.010)	(0.114)	(0.212)	(0.119)	(0.002)
4	0.000	0.027	0.000	0.965	0.083	0.057	0.073	0.973
	(0.000)	(0.038)	(0.000)	(0.002)	(0.047)	(0.047)	(0.054)	(0.002)
5	0.000	0.648	0.000	0.972	0.530	0.070	0.192	0.977
	(0.000)	(0.466)	(0.000)	(0.002)	(0.166)	(0.059	(0.133)	(0.001)
6	0.000	0.511	0.000	0.967	0.438	0.032	0.038	0.981
	(0.000)	(0.443)	(0.000)	(0.002)	(0.157)	(0.042)	(0.049)	(0.001)
7	0.000	0.599	0.000	0.966	0.615	0.230	0.336	0.974
	(0.000)	(0.291)	(0.000)	(0.002)	(0.177)	(0.176)	(0.191)	(0.002)
8	0.000	0.430	0.001	0.933	0.946	0.037	0.091	0.979
	(0.000)	(0.147)	(0.001)	(0.003)	(0.261)	(0.060)	(0.131)	(0.002)
9	0.000	0.210	0.000	0.941	0.498	0.834	0.731	0.964
	(0.000)	(0.152)	(0.000)	(0.005)	(0.185)	(0.479)	(0.181)	(0.003)
10	0.022	0.019	0.013	0.747	0.494	0.056	0.157	0.908
	(0.013)	(0.022)	(0.001)	(0.016)	(0.155)	(0.052)	(0.125)	(0.005)

four time periods. Before interpreting these results as evidence against the model, one should remember that the measurement errors in the factor estimates have been ignored. The measurement errors tends to bias the slope coefficients downward and the intercept upward, and they could also lead to inconsistent estimates for the variance-covariance matrix of the coefficients.

To gauge the impact of measurement error on the test results, I calculated the standard deviations for the test statistics, which are written as a nonlinear function of the extracted factors. Defining the extracted factors as γ and their covariance matrix as V, the standard error of the statistics, which is a nonlinear function $f(\gamma)$, can be calculated as $\sqrt{f_\gamma(\gamma)' V f_\gamma(\gamma)}$. The standard error for the adjusted R^2 is extremely small, suggesting that the "true" factors probably explain as much variation in decile portfolio returns as the extracted factors.[6] Although the standard errors for the p-values are generally larger, they are quite small if the p-values are below 10%. Thus, my basic conclusion about the significance of most tests is unaffected by mea-

[6] Using $\bar{R}^2 \pm 1.96\hat{\sigma}$ to construct the confidence interval for R^2 could result in the intervals crossing the boundaries of zero and one. One common solution is to use the Fisher transformation $z = f(R^2) = \ln(R^2) - \ln(1 + R^2)$ to construct the confidence interval for z and then do a reverse transformation to obtain the confidence interval for R^2. The same technique applies to the construction of the confidence intervals for significance levels.

surement errors. This also confirms the result in Figure 1 that the measurement errors in the extracted factors are small.

To compare factors extracted by the SAR approach with those estimated by Connor and Korajczyk (1988), I regress the excess returns of several market indices on a constant and on factors estimated for the same periods and compare the results with Connor and Korajczyk's. Indices used are the value-weighted (VW) and equally weighted (EW) portfolios of NYSE and Amex stocks, a long-term low-grade corporate bond portfolio (JBRET), and a long-term government bond portfolio (UTS). Returns on these indices are all taken from Ibbotson Associates (1990).

The results are shown in Table II. The first column provides some evidence of mispricing by the five-factor model. The second column provides the adjusted R^2 for the regression. As pointed out earlier, if the linear factor model is correctly specified and the linear pricing relationship of (2) holds, the intercept should be zero in the excess return regression. When I rescale

Table II

Comparison between the SAR Approach and the Principal Components Approach

Time series regression of monthly market index returns, in excess of one-month Treasury bill returns, on a constant and either a) factors estimated by the semiautoregressive method, or b) factors estimated by the principal components approach of Connor and Korajczyk. Indices are the value-weighted (VW) and equally weighted (EW) portfolios of NYSE and Amex stocks, a long-term low-grade corporate bond portfolio (JBRET), and a long-term government bond portfolio (UTS), obtained from Ibbotson Associates (1990). Mispricing is defined as the intercept of the regression.

	The SAR Approach		The Connor & Korajczyk Approach	
Index	Mispricing (% per Annum)	$\bar{R}^2$	Mispricing (% per Annum)	$\bar{R}^2$
		1969–1973		
VW	1.89	0.915	0.85	0.938
EW	−3.72	0.936	−1.20	0.997
JBRET	3.97	0.092	0.67	0.340
UTS	2.41	−0.004	4.49	0.291
		1974–1978		
VW	−5.54	0.910	−5.52	0.962
EW	4.20	0.973	0.56	0.998
JBRET	−1.99	0.366	−2.52	0.553
UTS	−0.28	0.295	−0.63	0.194
		1979–1984		
VW	−2.72	0.887	−3.72	0.941
EW	1.20	0.966	0.06	0.997
JBRET	−1.33	0.138	−2.28	0.479
UTS	−0.90	0.185	−6.85	0.437

the intercept so that the mispricing is given in percent per annum, the factors extracted under the two approaches produce similar but not identical results. Connor and Korajczyk's factors generally explain a little more of the variation in asset returns. Both sets of factors misprice the market indices in similar directions but the magnitude of the mispricing is slightly different. My results point more strongly to positive abnormal returns for small stocks, which carry more weight in the EW portfolio, for the sample periods 1974 to 1978 and 1979 to 1984.[7]

Figure 2 plots the average January mispricing by the five-factor model and the CAPM. The mispricing for the APT model is obtained by adding a January dummy variable to the time series regression (10) and taking the slope coefficient on the January dummy as a measure of the January mispricing. The mispricing for the CAPM is estimated by the slope coefficient on the January dummy in the regression of the monthly excess return on a constant, a January dummy, and either (a) the monthly excess return on the value-weighted portfolio of NYSE stocks, or (b) the monthly excess return on the equally weighted NYSE portfolio. The mispricing is estimated for four subperiods (1970 to 1973, 1975 to 1978, 1980 to 1983, and 1985 to 1988). The mispricing presented in Figure 2 is the average across these subperiods. As can be seen, the APT model has the least mispricing for most of the portfolios.

To further compare the performance of factors and market portfolios as measures of systematic risk, I first run time series regressions of individual stock excess returns on the factors to obtain the factor loadings. I then run cross-sectional regressions of the five-year mean excess returns on the estimated factor loadings to obtain unconditional means of risk premiums. This two-pass regression approach makes my results comparable to those of Shukla and Trzcinka (1990) and other previous studies. I estimate similar regressions for the CAPM, using two market portfolios. The results are presented in Table III. The five-factor model explains about 17 to 42% of the cross-sectional variation of mean excess returns, whereas the market model, using the value-weighted or equally weighted portfolio as the benchmark, explains only about 0 to 18% or 2 to 28% of the same variation.[8] I also discover that most factors are "priced" in the APT model, with some risk premiums being as high as 10% per annum or as low as -17%. As the t-statistics in Table III show, most unconditional factor premiums are highly significant.

Following Connor and Korajczyk (1988), I perform Hotelling T^2-tests on the hypothesis that the unconditional mean factor risk premiums are equal to zero. The results are presented in the third line of each row in Table III. Like Connor and Korajczyk, I find little evidence of nonzero unconditional factor

[7] See Connor and Korajczyk (1988), Lehmann and Modest (1988), Chan, Chen, and Hsieh (1985), Gultekin and Gultekin (1987), and Jegadeesh (1990).

[8] This result is similar to those in recent studies by Connor and Korajczyk (1988), and Lehmann and Modest (1988), and Shukla and Trzcinka (1990), who find that the APT better describes expected returns on individual stocks than the CAPM.

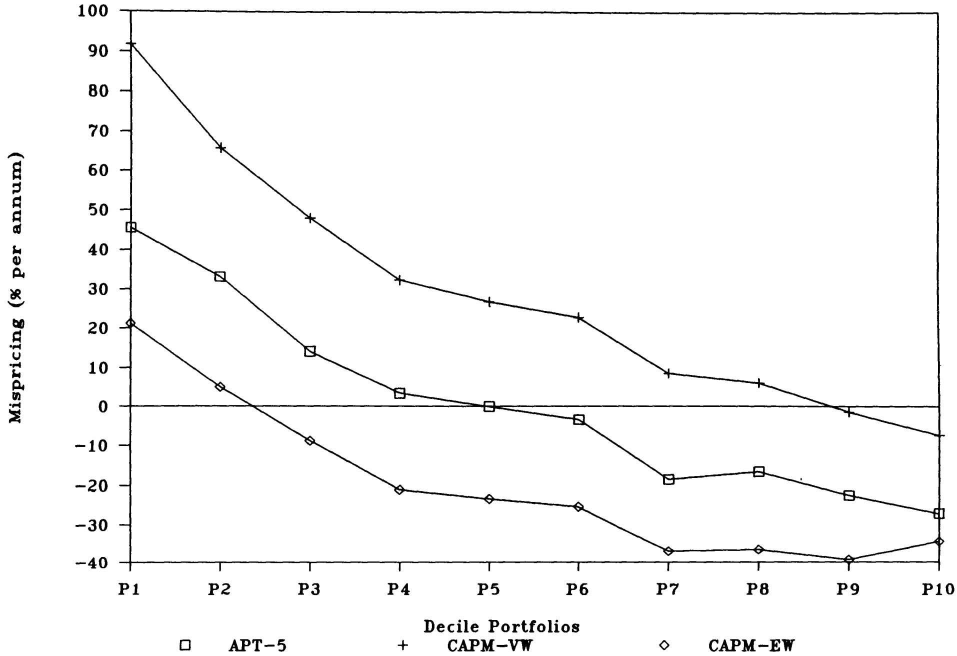

Figure 2. January mispricing, in percentage per annum, for decile portfolios formed by size. Portfolio 1 represents the smallest firms and portfolio 10 represents the largest firms. Mispricing is estimated by the slope coefficient on the January dummy in the regression of monthly excess returns on a constant, a January dummy, and either (a) the monthly excess return on the CRSP value-weighted portfolio of NYSE stocks (CAPM-VW), or (b) the monthly excess return on the CRSP equally weighted portfolio of NYSE stocks (CAPM-EW), or (c) the factor estimates from a five-factor APT model using the semiautoregression approach (APT-5). The mispricing is estimated for four subperiods (1970 to 1973, 1975 to 1978, 1980 to 1983, and 1985 to 1988). The mispricing presented here is the average mispricing across the four subperiods.

premiums. I believe the Hotelling T^2-test may not be applicable here because the assumption of i.i.d. sample distribution is violated by the nonrandom time variation in the factor premiums estimated over time. Although the t-statistics in Table III are highly significant, I do not control for the estimation errors on the factor loadings and thus the standard errors may be biased. On the other hand, since the estimates of the unconditional risk premiums are obtained from the large cross-sectional regression with over a thousand observations, the effect of measurement error will be small if the true betas are uncorrelated with the error terms.[9]

Figure 3 depicts the general mispricing for the competing models. The general mispricing for the APT and the CAPM is defined as the intercepts of the following cross-sectional regressions: $\bar{r}_i = \alpha_0 + \lambda_1 \beta_{i1} + \cdots + \lambda_t \beta_{i5} + \varepsilon_i$

[9] See Shukla and Trzcinka (1990) for details.

Table III

Test of Unconditional Factor Pricing

Unconditional factor risk premiums and their t-statistics, obtained from a cross-sectional regression of five-year mean excess security returns ($\bar{r}_\iota$) on factor loadings ($\beta_{\iota 1}, \ldots, \beta_{i5}$), are shown in the first two lines of each row. The p-values from Hotelling T^2-tests are given in the third line in parentheses. Factor loadings are estimated by a time series regression of excess returns on factors, which are estimated by a semiautoregressive method using monthly stock returns on 1105, 1276, 1210, 1089 securities over the periods 1969 to 1973, 1974 to 1978, 1979 to 1983, and 1984 to 1988.

	$\bar{r}_\iota = \alpha_0 + \lambda_1 \beta_{\iota 1} + \cdots + \lambda_5 \beta_{\iota 5} + \varepsilon_i$ [a]								
Period	$\alpha_0 \times 1200$	$\lambda_1 \times 1200$	$\lambda_2 \times 1200$	$\lambda_3 \times 1200$	$\lambda_4 \times 1200$	$\lambda_5 \times 1200$	$\bar{R}^2$	$\bar{R}^2$(VW)	$\bar{R}^2$(EW)
1969–1973	2.089	−17.601	−1.749	0.031	4.214	−0.629	0.420	0.179	0.281
	(2.853)[b]	(−17.855)	(−9.803)	(0.148)	(2.573)	(−0.525)			
	(0.846)[c]	(0.094)	(0.476)	(0.737)	(0.914)	(0.497)	(0.346)[d]		
1974–1978	4.417	3.557	1.098	−0.076	1.066	2.262	0.171	0.070	0.183
	(6.534)	(3.793)	(4.893)	(−1.218)	(5.229)	(2.983)			
	(0.314)	(0.562)	(0.191)	(0.979)	(0.622)	(0.460)	(0.166)		
1979–1983	5.844	0.579	1.755	−1.613	0.423	6.081	0.263	0.160	0.207
	(10.378)	(5.169)	(4.364)	(−1.919)	(2.223)	(6.830)			
	(0.659)	(0.090)	(0.924)	(0.386)	(0.320)	(0.526)	(0.018)		
1984–1988	7.963	−13.672	−2.383	7.006	10.206	−5.455	0.287	0.000	0.022
	(12.625)	(−8.089)	(−6.334)	(4.834)	(8.692)	(−5.620)			
	(0.876)	(0.530)	(0.246)	(0.729)	(0.328)	(0.592)	(0.089)		

[a] $\bar{r}_\iota$ is the five-year mean excess return on individual stocks.
[b] t-statistics.
[c] p-value for the Hotelling T^2-test (distributed $F_{1,47}$) that $\lambda_j = 0$.
[d] p-value for the Hotelling T^2-test (distributed $F_{5,43}$) that $\lambda_1 = \cdots = \lambda_5 = 0$.

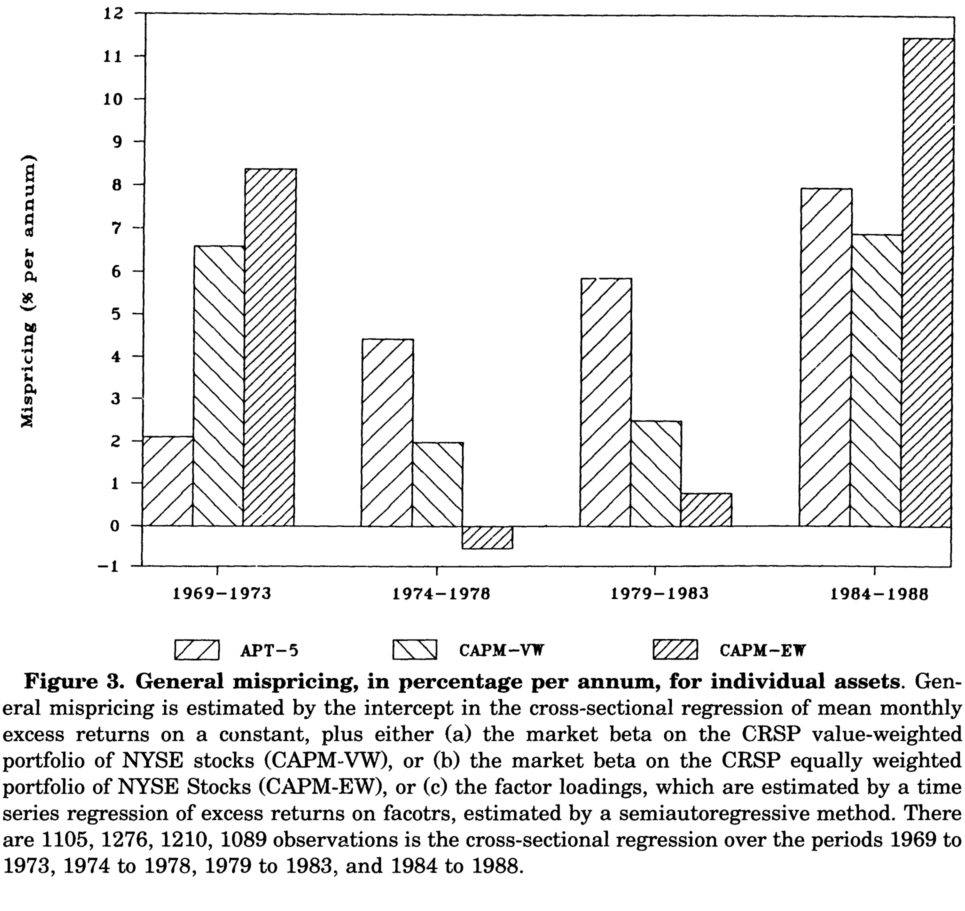

Figure 3. General mispricing, in percentage per annum, for individual assets. General mispricing is estimated by the intercept in the cross-sectional regression of mean monthly excess returns on a constant, plus either (a) the market beta on the CRSP value-weighted portfolio of NYSE stocks (CAPM-VW), or (b) the market beta on the CRSP equally weighted portfolio of NYSE Stocks (CAPM-EW), or (c) the factor loadings, which are estimated by a time series regression of excess returns on facotrs, estimated by a semiautoregressive method. There are 1105, 1276, 1210, 1089 observations is the cross-sectional regression over the periods 1969 to 1973, 1974 to 1978, 1979 to 1983, and 1984 to 1988.

and $\bar{r}_i = \alpha_0 + \lambda_1 \beta_{im}$, where $\bar{r}_i$ is the five-year mean excess return on individual stocks. The APT model has generally less mispricing, except for the 1979 to 1983 period.

III. Comparison with Alternative Approaches for Extracting Factors

A competing estimation method in the literature is the maximum likelihood factor analysis used by Brown and Weinstein (1982), Chen (1983), Ferson, Kandel, and Stambaugh (1987), Lehmann and Modest (1988), and Roll and Ross (1980). This approach places stringent assumptions on the returns data, such as normality and independent distribution over time. Although one can allow the factor premiums to vary over time, the estimation procedure becomes quite complicated. Moreover, it is difficult to obtain the asymptotic variance-covariance matrix for the factor estimates. Although one could use a Monte Carlo simulation, previous studies have not tried that approach, possibly because of difficulties involved in setting the simulation parameters. Thus with the exception of Shanken (1992), few attempts have been made to measure the estimation errors in the factor (or beta) estimates directly, even though the estimates are used later for hypothesis testing. Another limitation of the approach is that it generally has trouble analyzing a large cross-sectional data set, because the number of beta parameters tends to explode as one tries to estimate betas for a large number of assets.

The principal components approach is much easier to use and requires relatively fewer distributional assumptions about the return-generating process. The method also allows for fairly arbitrary time variation in factor risk premiums and an approximate factor structure. By taking advantage of a large cross-sectional data set, it is possible to obtain factor estimates with fairly small estimation errors.[10] This approach has the pitfall, however, of ignoring the firm-specific variation in returns. This could be a serious problem if firm-specific variation accounts for a large percentage of total variation in returns or if the variation changes substantially over time and across firms. As with the maximum likelihood estimation, there is also no simple solution to the asymptotic variance-covariance matrix for the principal components. Thus few attempts have been made to measure the estimation errors in the factor estimates directly.

The technique I develop in this paper compliments the maximum likelihood estimation and principal components technique in the following ways. First, my approach does not require the restrictive assumptions made in the maximum likelihood estimation. Second, I put little restriction on the time and cross-sectional variation of firm-specific shocks. Third, I provide a simple asymptotic variance-covariance matrix of factor estimates. This is important because it provides a simple way of adjusting for measurement errors. My technique, however, has one limitation that alternative methods do not share: of the same data set, my estimation covers a slightly shorter period because of the use of lagged returns as autoregressors and instruments.

IV. Extracted Factors and Economic Risks

Numerous studies have discovered that excess returns are predictable (for instance, Campbell (1987), Fama and French (1988, 1989), Ferson (1989), Ferson and Harvey (1991), and Keim and Stambaugh (1986)). This finding emerges from regressing excess returns on some predetermined state variables, such as the treasury bill rate, the term spread, and the default spread. As Fama and French (1989) point out, predictability per se does not necessarily imply market inefficiency if risk premiums vary over time because of changing business conditions and changing risk perceptions. Since all these

[10] See, for instance, Connor and Korajczyk (1988), Trzcinka (1986), and Shukla and Trzcinka (1990).

Table IV

Test of Time-Varying Factor Premiums

Time series regression of estimated factors *for time period t* on economic variables observed at *the end of time period t − 1.* Factors, $\tilde{s}_{kt}$, are estimated by a semiautoregressive (SAR) method using monthly stock returns on 1105, 1276, 1210, and 1989 securities over the periods 1969 to 1973, 1974 to 1978, 1979 to 1983, and 1984 to 1988. Economic variables used are a constant, a January dummy, the lagged return on the value-weighted portfolio (VW), the one-month treasury bill rate (TB), the lagged spread between the yields of a long-term AAA corporate bond and the one-month treasury bill (TERM), and the lagged dividend yield on the value-weighted portfolio (DivYld).

$$\tilde{s}_{kt} = d_0 + d_1 \text{ Jan. Dum.} + d_2(\text{VW})_{t-1} + d_3(\text{TB})_{t-1} + d_4(\text{TERM})_{t-1} + d_5(\text{DivYld})_{t-1} + \varepsilon_{jt}$$

Factor	d_0	d_1	d_2	d_3	d_4	d_5	F-test	$\overline{R}^2$
				1970–1973				
$\tilde{s}_{1t}$	−0.029	0.006	0.000	0.001	0.002	0.006*	0.002	0.543
$\tilde{s}_{2t}$	0.008	−0.002**	0.000	−0.001	−0.001	−0.001	0.000	0.115
$\tilde{s}_{3t}$	−0.001	0.002	0.000	0.000	0.000	0.000	0.149	−0.068
$\tilde{s}_{4t}$	0.048	0.033*	0.000	−0.011	−0.011	0.010	0.000	0.036
$\tilde{s}_{tt}$	0.034	−0.005	0.001*	−0.004	−0.004	−0.001	0.000	0.178
				1975–1978				
$\tilde{s}_{1t}$	0.054	0.000	0.001	−0.008	−0.009	0.002	0.007	−0.028
$\tilde{s}_{2t}$	0.005	−0.006*	0.000	−0.001	−0.001	0.002	0.001	0.069
$\tilde{s}_{3t}$	−0.002	0.001	0.000	0.000	0.000	0.000	0.002	0.005
$\tilde{s}_{4t}$	0.007	0.003	0.000	0.000	−0.001	−0.001	0.008	0.048
$\tilde{s}_{5t}$	−0.008	−0.012	0.001***	−0.005	−0.003	0.010	0.000	0.154

Table IV— *Continued*

$$\tilde{s}_{kt} = d_0 + d_1 \text{ Jan. Dum.} + d_2(\text{VW})_{t-1} + d_3(\text{TB})_{t-1} + d_4(\text{TERM})_{t-1} + d_5(\text{DivYld})_{t-1} + \varepsilon_{jt}$$

Factor	d_0	d_1	d_2	d_3	d_4	d_5	F-test	$\bar{R}^2$
				1980–1983				
$\tilde{s}_{1t}$	−0.002	0.000	0.000	0.000	0.000	0.000	0.011	0.328
$\tilde{s}_{2t}$	0.010	0.002	0.000	0.000	0.000	0.002*	0.000	0.107
$\tilde{s}_{3t}$	−0.104*	0.006	−0.002*	0.004*	0.005*	0.017*	0.000	0.173
$\tilde{s}_{4t}$	−0.016	0.000	0.000*	0.000	0.000	0.003*	0.000	0.143
$\tilde{s}_{5t}$	0.051*	0.015	0.001	−0.002	−0.002	−0.008*	0.000	0.092
				1985–1988				
$\tilde{s}_{1t}$	0.030**	0.006	0.000	0.000	0.001	−0.008**	0.000	0.517
$\tilde{s}_{2t}$	−0.008	0.006**	0.000	0.000	0.000	0.002	0.000	0.364
$\tilde{s}_{3t}$	0.018	0.002	0.000	0.001	−0.003**	−0.004	0.000	0.240
$\tilde{s}_{4t}$	−0.010	−0.001	0.000	0.000	0.000	0.002	0.344	0.566
$\tilde{s}_{5t}$	0.005	0.005	0.000	−0.001	0.002**	−0.002	0.000	0.201

* Significant at 10% level.
** Significant at 5% level.
*** Significant at 1% level.

studies concentrate on a small group of asset portfolios, it will be interesting to find out whether this predictability applies more universally.[11]

I investigate this question by studying the relationship between factors extracted from returns of a large number of assets and variables closely related to business conditions. The variables used here are a constant, a January dummy, the return on the value-weighted portfolio (VW), the one-month treasury bill rate (TB), the spread between the yields of a long term AAA corporate bond and the one-month treasury bill (TERM), and the dividend yield on the value weighted portfolio (DivYld). I obtain all series from CRSP, except for the yields on the AAA corporate bond, which are from the Federal Reserve Bullentin. These variables have been used extensively in previous studies and have been found to be closely related to economic conditions.

I use lagged values of these variables to forecast factors, which are sums of risk premiums and pervasive shocks. My objective is to see whether these variables help explain variation in factor risk premiums through time. If the premiums are constant, the slope coefficients should be zero because, by definition, the forecasting variables should not be able to predict the pervasive shocks.

Table IV presents time series regressions of the factors on lagged values of the forecasting variables. The regressions are performed for 1970 to 1973, 1975 to 1978, 1980 to 1983, and 1985 to 1988. The results clearly show that these forecasting variables have significant predictive power over the factors. In some cases, the variables explain over 10% of the variation in the extracted factors, after degrees of freedom are adjusted for. The joint test of all five variables being zero is strongly rejected by the data at the 5% significance level in nine of ten tests.

The predictability of extracted factors reported in Table IV is consistent with other studies that use similar variables to predict excess returns on stock and bond portfolios. For example, Campbell (1987) reports an unadjusted R^2 of 11.2% on the value-weighted index predicted by a set of term structure variables. Harvey (1989) reports an average unadjusted R^2 of 10% on the value-weighted index and size-decile portfolios. Also, Fama and French (1988), using a slightly different set of variables, report an unadjusted R^2 of 4% on the value-weighted index. The contribution of this paper is to show that this predictability exists not only among a set of market portfolios but also among the factors of the APT model, which implies that excess returns of almost all assets are predictable.

Figure 4 plots the first extracted factor and its conditional factor premium from a five-factor model for 1970 to 1973. One can see clearly that the conditional factor premium does vary over time.

So far, we have proceeded under the assumption that betas are constant during the sample period. There are many good reasons to question the

[11] In a recent paper, Ferson and Korajczyk (1991) also study the predictability of returns on a large number of stocks, using economic variables and principal components.

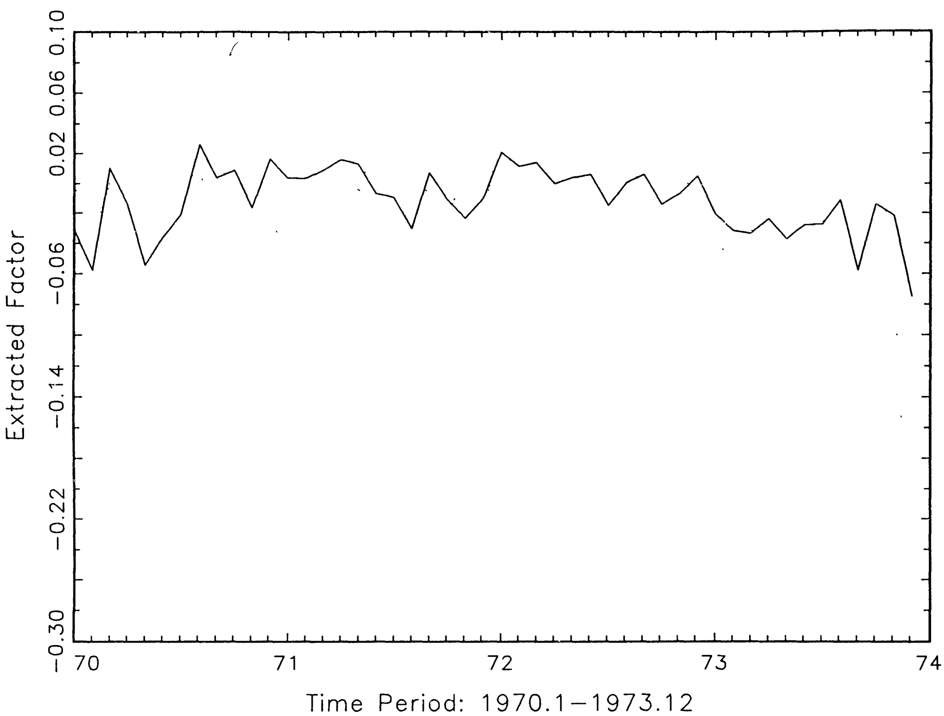

Figure 4. Plot of first extracted factor ($\tilde{s}_{1t}$) and its conditional factor premium (1970 to 1973). The dotted line is the extracted factor and the solid line is the conditional factor premium. The factor is extracted from a five-factor APT model by the semiautoregressive approach. The conditional factor premium is defined as the predictable part of the factor in the time series regression of the factor on a constant, a January dummy, the lagged return on the value-weighted portfolio (VW), the one-month treasury bill rate (TB), the lagged spread between the yields of a long term AAA corporate bond and the one-month treasury bill (TERM), and the lagged dividend yield on the value-weighted portfolio (DivYld).

validity of this assumption, however. Changes in a firm's debt-equity ratio, the introduction of new products, and mergers and acquisitions, to name a few, all affect the firm's sensitivity to pervasive shocks in the economy. Recent studies have also found evidence that betas vary over time (Ferson and Harvey (1991)). It is easy to show that as long as the betas vary linearly with some state variables, the technique developed here can be modified and still be applicable.

V. Conclusion

This paper develops a semiautoregression (SAR) approach to estimating and testing the APT model. It adds a new tool to the empirical researcher's arsenal, which usually consists of maximum likelihood factor analysis, the principal components technique, "mimicking portfolios" regression analysis, and the observable factors approach.[12] One major advantage of my approach is that it provides a simple asymptotic variance-covariance matrix for the factor estimates, which makes it easy to adjust for measurement errors in a finite sample.

I demonstrate that historical returns can be used to approximate the unobservable factor loadings and that factors can be estimated by running a series of semiautoregressions. By combining the autoregression procedure of Holtz-Eakin, Newey, and Rosen (1988) with the Newey-West adjustment, I estimate the APT model and find that the measurement errors in the extracted factors are small. Using the asymptotic variance-covariance matrix for the factor estimates, I show how to take measurement errors into account in the second-state hypothesis testing. My empirical work confirms the result of previous studies that the APT offers a slightly better description of asset returns than the CAPM. I also study the relationship between the extracted factors and business cycle variables, finding that these variables help predict movements in factor premiums over time.

Appendix

The semiautoregression (9) is estimated in the following framework. Using Holtz-Eakin, Newey, and Rosen's (1988) notation, I write:

$$y_t = (R_{1t}, R_{2t}, \ldots, R_{Nt})'$$

as $Nx1$ vectors of observations for a given time period t.

Denote the variables on the right side of equation (9) as $W = (e, y_1, \ldots, y_k)$, e as a $Nx1$ vector of ones, $u_t = (\eta_{1t}, \eta_{2t}, \ldots, \eta_{Nt})'$ as the transformed error terms, and $s_t = (s_{0t}, s_{1t}, \ldots, s_{Kt})'$. Then (9) can be written as

$$y_t = Ws_t + u_t. \qquad (t = M + 1, \ldots, T) \tag{A1}$$

To transform (A1) over time into a system of equations, we stack the equation by time and denote

$$\begin{aligned} Y &= (y'_{m+1}, \ldots, y'_T)', && ((T - M)Nx1) \\ B &= (s'_{m+1}, \ldots, s'_T)', && ((K + 1)(T + M)x1) \\ U &= (u'_{m+1}, \ldots, u'_T)', && ((T - M)Nx1) \\ W &= \text{diag}(W', \ldots, W'), && ((T - M)Nx(K + 1)(T - M)) \end{aligned}$$

where diag$(W', \ldots, W')$ denotes a block diagonal matrix with W placed on the diagonal. Thus (9) can be written as

$$Y = WB + U. \tag{A2}$$

[12] See Brown and Otsuki (1989), Burmeister and McElroy (1988), Jobson (1982), and Huberman, Kandel, and Stambaugh (1987), among others, for regression analysis. See Chan, Chen, Hsieh (1985), Chen, Roll, and Ross (1986), and Shanken and Weinstein (1990) for observable factor studies.

Since variables known from time $K + 1$ to $t - 1$ are orthogonal to the error terms in (9) for time t, the following qualify as instrumental variables

$$Z_t = (e, y_{t-1}, \ldots, y_{t-p}),$$

where $t - p > K$. Denoting $Z = \mathrm{diag}(Z'_{m+1}, \ldots, Z'_T)$ and using the $3SLS$ estimation method given by Holtz-Eakin, Newey, and Rosen (1988), we can get a consistent estimate of B:

$$\hat{B} = \left[W'Z(\hat{\Omega})^{-1}Z'W\right]^{-1} W'Z(\hat{\Omega})^{-1}Z'Y \tag{A3}$$

in which $\hat{\Omega}$ is the covariance matrix of $Z'U$. See Newey and West (1987) for the calculation $\hat{\Omega}$. The variance-covariance matrix of B is given by: $\Theta = Var(\hat{B}) = [W'Z(\hat{\Omega})^{-1}Z'W]^{-1}$. The estimate of B given by (A3) is consistent despite the correlation between the autoregressors and the error terms, and the presence of heteroskedasticity and cross-sectional correlation. As pointed out by Holtz-Eakin, Newey, and Rosen (1988), the consistency is based mainly on the orthogonal condition that the error terms are uncorrelated with the instrumental variables used in estimating (A2). It is special case of Hensen's (1982) GMM consistent estimator.

The weighted sum of the squares of residuals is calculated as follows:

$$Q = (Y - W\hat{B})'Z(\hat{\Omega})^{-1}Z'(Y - W\hat{B})/N \tag{A4}$$

REFERENCES

Brown, Stephen and Mark Weinstein, 1983, A new approach to testing asset pricing models: The bilinear paradigm, *Journal of Finance* 38, 711–743.

——— and Toshiyuki Otsuki, 1989, Macroeconomic factors and the Japanese equity markets: the CAPMD project, Working paper, New York University.

Burmeister, Edwin and Marjorie McElroy, 1988, Joint test of factor sensitivities and risk premia for the arbitrage pricing theory, *Journal of Finance* 43, 721–733.

Campbell, John, 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.

Chan, K. C., Nai-fu Chen, and David Hsieh, 1985, An exploratory investigation of the firm size effect, *Journal of Financial Economics* 14, 451–471.

Chen, Nai-Fu, 1983, Some empirical tests of the theory of arbitrage pricing, *Journal of Finance* 38, 1392–1414.

———, 1991, Financial investment opportunities and the macroeconomy, *Journal of Finance* 46, 529–554.

———, Richard Roll, and Stephen Ross, 1986, Economic forces and the stock market, *Journal of Business*, 59, 386–403.

Connor, Gregory and Robert Korajczyk, 1988, Risk and return in an equilibrium APT: Application of a new test methodology, *Journal of Financial Economics* 21, 255–289.

Fama, Eugene and Kenneth French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.

———, 1989, Business conditions and expected return on stocks and bonds, *Journal of Financial Economics* 25, 23–49.

Ferson, Wayne, 1989, Changes in expected security returns, risk, and level of interest rates, *Journal of Finance* 44, 1191–1217.

——— and Campbell Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

Ferson, Wayne and Robert Korajczyk, 1991, Do arbitarge pricing models explain the predictability of stock returns?, Working paper no. 115, Northwestern University.

Ferson, Wayne, Shmuel Kandel, and Robert Stambaugh, 1987, Test of asset pricing with time-varying expected risk premiums and market betas, *Journal of Finance* 42, 201–219.

Gultekin, Mustafa, and Bulent Gultekin, 1987, Stock return anomalies and tests of the APT, *Journal of Finance* 42, 1213–1224.

Harvey, Campbell R., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289–318.

Holtz-Eakin, Douglas, Whitney Newy, and Harvey Rosen, 1988, Estimating vector autoregessions with panel data, *Econometrica* 56, 1371–1395.

Huberman, Gur, Shmuel Kandel, and Robert Stambaugh, 1987, Mimicking portfolios and exact arbitrage pricing, *Journal of Finance* 42, 1–9.

Keim, Donald and Robert Stambaugh, 1986, Predicting returns in the stock and bond markets, *Journal of Financial Economics* 17, 357–390.

Lehmann, Bruce and David Modest, 1988, The empirical foundations of the arbitrage pricing theory, *Journal of Financial Economics* 21, 222–254.

Jegadeesh, Narasimhan, 1990, Evidence of predictable behavior of security returns, *Journal of Finance* 45, 881–898.

Jobson, J. D., 1982, A multivariate linear regression test for the arbitrage pricing theory, *Journal of Finance* 37, 1037–1042.

Newey, Whitney, and Kenneth West, 1987, A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica* 55, 703–708.

Roll, Richard, and Stephen Ross, 1980, An empirical investigation of the arbitrage pricing theory, *Journal of Finance* 35, 1073–1103.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *Review of Financial Studies* 5, 1–33.

——— and Mark Weinstein, 1990, Macroeconomic variables and asset pricing: Further results, Working paper, University of Rochester.

Shukla, Ravi, and Charles Trzcinka, 1990, Sequential tests of the arbitrage pricing theory: A comparison of principle components and maximum likelihood factors, *Journal of Finance* 45, 1541–1564.

Trzcinka, Charles, 1986, On the number of factors the arbitrage pricing model, *Journal of Finance* 41, 347–368.