Securities Trading: Principles and Procedures

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Disclosures: I regularly teach (for compensation) in the training program of a firm that engages in high frequency trading. I have recently served (without compensation) on a CFTC advisory committee. I give presentations at financial institutions for which I sometimes receive honoraria.

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Preface

This manuscript is a set of draft teaching notes for a one-semester course entitled *Principles of Securities Trading*. The target audience is finance students planning careers in trading, investment management, or law, and information technology students who seeking to build trading and investment systems. The exposition draws on general economic principles, with an institutional focus on US equity markets.

The high level of institutional content underscores the realism and currency of the material. Given the speed with which markets evolve, however, it is likely (maybe even certain) that some of the details are out of date.

By way of full disclosure, I regularly teach (for compensation) in the training program of a firm that engages in high frequency trading. I have recently served (without compensation) on a CFTC advisory committee. I give presentations at financial institutions for which I sometimes receive honoraria.

Although these notes draw from the subject generally known as market microstructure, they certainly don't fully cover the field. There are many important areas of academic research that are barely touched upon: the econometrics of high-frequency data; measurement of liquidity; liquidity risk and commonality; liquidity and asset pricing; empirical analysis of price discovery; and so on. These omissions reflect the priority placed on simplifying the foundations of the subject, rather than discussing all the extensions.

The text is organized in parts (broad themes), chapters and sections.

- Part I starts with the basics. It introduces key terms and describes the important players. It explores the floor markets (pre-21st century) and their modern descendants, the continuous electronic limit order markets.
- Part II considers extensions and alternatives to the limit order markets: auctions, dealers and dark trading mechanisms.
- Part III examines informational efficiency. Many readers will have encountered the subject in an earlier finance class. They will have absorbed the idea that the market price of the stock incorporates and fully reflects the split, the takeover announcement, or whatever. The present approach discusses the trading processes that makes this incorporation possible. The role of trading procedure is particularly important with respect to private information, which can give rise to bid-ask spread effects, price impacts, market failures and so forth. Part III also discusses some issues of practical and legal importance: securities class action lawsuits and insider trading regulation.
- Part IV introduces algorithmic trading. The approach is incremental, moving from complex order types to statistical models and discussion of the order splitting problem.
- Part V covers current topics in regulation and high frequency trading.
- Part VI contains selected problems and exercises for some of chapters.
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Part I. Modern securities markets: the basics

Securities markets rely on highly-structured trading procedures and well-defined institutional roles. Part I introduces these institutions and procedures. This part discusses, by way of background, the floor markets. It then goes on to explore the descendants of these floors, our modern limit order markets.
Chapter 1. Introduction

We demand a lot of our securities markets. When we plan our investments or hedge risks, we rely on market prices to tell us the value of what we currently have and the cost of what we might attempt to do. We then enter the markets to trade and implement our decisions. As events unfold over time, we return to the markets to monitor our progress and revise our decisions. Finally, when we want to consume the gains from our investments or the hedge is no longer needed, we sell or settle the securities.

The traditional view of a perfect frictionless market is summarized in the conventional supply and demand framework. Each buyer and seller is assumed to be atomistic, that is, small relative to the market. Because each trader is small relative to the market, and knows that they're small, they believe that nothing they do will affect the market price. They willingly express their true preferences: when they are asked “How much would you buy if the price were $x$?,” for example, they answer honestly. (It does not occur to them to bluff or feign a weaker demand to obtain a lower price.) The buyers collectively define the demand curve (seeking to buy much at low prices, and little at high prices). The sellers define the supply curve. The price at which the total quantity demanded is equal to the quantity supplied defines the equilibrium price and quantity.

The process of arriving at the equilibrium point is (in principle at least) accomplished by an auctioneer. The auctioneer calls out a price, and asks, “Who wants to buy at this price? Who wants to sell?” The auctioneer than adjusts the price until total supply and demand are in balance, and the market clears.

Stock markets are often mentioned as settings that closely approximate this ideal. From one perspective, it's a reasonable conjecture. Stocks are held by thousands of investors, and thousands more might be standing by as potential buyers or sellers.

On closer examination, though, reality breaks from the model. While millions of people might hold a security, only a few might be actively participating in the market when we want to trade.
Ultimately the number of market participants might be as low as two: ourselves and our counter-party. All of a sudden, the large-number perfect-competition abstraction seems less relevant. We’re behaving strategically, taking into account the fact that our actions can change the price. Most of the time there is no one acting as an "auctioneer". In these interactions, the market procedure and rules matter very much.

These notes are about these rules, procedures, and the economic principles that shape them. Although we can’t avoid talking about the securities (the stocks, bonds, options, and so forth) these notes are not primarily about them, their characteristics, or their uses. The notes attempt to explain instead how they are traded.

Although the material applies to most security markets, the presentation is heavily focused on US equity markets. It is useful to have one actual functioning market as a central example, and in this respect, the US equity market is a good choice. The US equity market is large and active, and also exhibits an especially wide range of features. More broadly, the economic forces that have converged on it and shaped it are suggestive, for better or worse, of changes that are likely to play out elsewhere.

This is not to say that US markets are invariably at the forefront of technology. For example, near the close of the last century, when the rest of the world had long since adopted decimal prices, US markets were still trading in “eighths”. Moreover, if the present era can be called the age of electronic markets, the US was in most respects late to the party. Other countries (notably France and Canada) were well ahead of the US in broad adoption of market-unifying technology.

Nonetheless, when the US stock market finally did make the transition to electronic trading, it did so in a flexible and open fashion. The lead regulator, the Securities and Exchange Commission, mostly took the stance that a stock exchange was not a “natural monopoly”, and that there was much to be gained from competition to build better exchanges. This gave rise to rich experimentation with a variety of trading mechanisms and protocols, algorithmic trading, high-frequency trading and other practices that have spread to other markets.

The study of financial markets cuts across many disciplines, spanning almost everything from sociology to physics. The present perspective, though, draws mostly from financial economics. Within financial economics, the area that deals with the study, design, and regulation of trading mechanisms is known as market microstructure.

Market microstructure encompasses diverse lines of thought. Readers looking to supplement these notes might consider the following sources. Harris (2003) is a comprehensive review of trading mechanisms, styles and strategies. O’Hara (1995) covers the core economic principles. Hasbrouck (2006) discusses the empirical implications of these principles, and approaches to working with market data. Foucault, Pagano and Roell (2013) provides more depth on economic models and principles. For additional material on algorithmic trading, see Aldridge (2013); Cartea, Jaimungal and Penalva (2015); Johnson (2010); Kissell and Glantz (2003).

Like many other technology-driven sectors of the economy, securities markets have been subjected to many recent changes and disruptions, to the point where it is difficult to keep track of where they are and where they’ve been. Discussions that summarize the key change points include: Angel, Harris and Spatt (2011); (2015); O’Hara (2015).

The citations in these notes will point the reader to other background sources. Finally, although these notes are primarily focused on the “how” of trading, it is useful to have some sense of the “what” (is being traded), that is, the structure and characteristics of specific securities. In this regard, Bodie, Kane and Marcus (2011) is a useful source to have at hand.
Chapter 2. The Elements of a Securities Market: US Equities

The S&P Global Broad Market Index covers about 12,000 publicly-traded companies around the world. The 2015 end-of-year total market capitalization of these equities is approximately $55.6 Trillion.¹ The US accounts for about 51% of this.

Market capitalization is a “point in time” measure. Trading volume, though, is generally measured as a flow: shares (or dollars) traded per unit time. Usually, trading volume simply denotes the number of shares bought or, equivalently, the number of shares sold by all participants in the market. The most useful estimates of these figures are provided by the stock exchanges, where much of the trade occurs. Table 2.1 summarizes market capitalization and trading volume for a sample of the world’s largest exchanges.

Market capitalization and trading volume are positively related. This is to be expected because shares in large firms would be widely held and actively traded. An intuitive measure of trading activity that looks beyond this effect is turnover. Turnover is the ratio of trading volume to market capitalization. For a single firm, this is equivalent to the ratio of share volume to the number of shares outstanding. Because volume is a rate, turnover has a time dimension. The NYSE's annual turnover of 1.7 (170%) suggests that share of stock was traded, on average, 1.7 times over the year. This interpretation is only suggestive because it is an average estimate, and it is unlikely in practice that each share was actually bought and sold twice. It is more probable that most of the shares did not change hands during the year, while some shares were traded ten or more times.

¹ When we multiply the number of shares in a firm by the price per share, we arrive at the firm's equity market, the market value of all the firm’s shares. We can total this number for all the firms in a country to get a country’s equity capitalization, and total all the countries in the world to get a global figure.
A high turnover corresponds to short holding periods. With an average turnover of 4.5 times per year, the average holding period for the Shanghai Stock Exchange is under three months. On the Euronext and Nasdaq OMX exchanges, the average holding period is about 1.6 years.

<table>
<thead>
<tr>
<th>Exchange</th>
<th>Domestic Market Capitalization</th>
<th>Value of EOB Trading</th>
<th>Turnover</th>
<th>Currency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nasdaq - US</td>
<td>7,280,752</td>
<td>12,515,349</td>
<td>1.7</td>
<td>USD</td>
</tr>
<tr>
<td>NYSE</td>
<td>17,786,787</td>
<td>17,477,291</td>
<td>1.0</td>
<td>USD</td>
</tr>
<tr>
<td>Japan Exchange Group</td>
<td>589,788,805</td>
<td>671,342,413</td>
<td>1.1</td>
<td>JPY</td>
</tr>
<tr>
<td>Shanghai Stock Exchange</td>
<td>29,519,420</td>
<td>132,741,776</td>
<td>4.5</td>
<td>CNY</td>
</tr>
<tr>
<td>Euronext</td>
<td>3,025,561</td>
<td>1,882,173</td>
<td>0.6</td>
<td>EUR</td>
</tr>
<tr>
<td>NASDAQ OMX Nordic Exchange</td>
<td>1,160,512</td>
<td>684,237</td>
<td>0.6</td>
<td>EUR</td>
</tr>
</tbody>
</table>

“EOB” refers trading via the electronic order book. This is usually the principal trading mechanism, but not the only one. Capitalization and value of trading are measured in local currency (Millions).

Source: World Federation of Exchanges (http://world-exchanges.org)

2.1. Exchanges

An exchange usually consists of facilities for trading, such as a trading floor, software that defines the market or connects traders, and so on. An exchange establishes a regularization of the trading process. When we say that a security is exchange-traded, we mean that the trading process is structured, monitored and standardized.

Most exchange activity revolves around three areas: listing, trading and data. Briefly:

- When a firm lists on an exchange, the exchange is providing a kind of sponsorship. The firm pays a listing fee. In return the exchange provides trading services, and monitors and certifies financial statements and governance procedures.

- The trading services and facilities comprise computer systems, standardized trading procedures, and a certain amount of oversight.

- The trading generates market data: reports of trades, quote changes, and so on. These data are valuable for market participants, and their sale generates large revenues.

A firm usually lists with one exchange, or at least designates one as the primary listing exchange. The most important US listing venues are the NYSE, NYSE Arca, NYSE MKT, and NASDAQ. They are differentiated by listing fees and listing requirements, but also by public image, investors’ perceptions of the “kind” of firms that list there, and other intangibles.

The NYSE (we might call it “NYSE classic” to differentiate the former New York Stock Exchange from other exchanges that carry the NYSE brand) has the highest fees and tightest listing standards. It was historically the dominant US exchange, home to the “blue chip” companies, the largest and oldest industrial and financial companies. An NYSE listing carries associations of seniority and stability. NASDAQ-listed companies tend to be younger, smaller and more concentrated in technology. A NASDAQ listing carries associations of “entrepreneurial” and “growth”.

The American Stock Exchange (now NYSE MKT) historically occupied the space between the NYSE and NASDAQ. In the hypothetical corporate life cycle, a firm would first list on NASDAQ, move to the Amex when it grew a little larger, and ultimately step up to the NYSE. From a listing perspective, NYSE Arca represents a NYSE initiative to list companies whose profile comes closer to NASDAQ. In its materials, the NYSE commented, “NYSE Arca is a fully electronic exchange for growth-oriented enterprises. Listed companies can grow on NYSE Arca and transfer seamlessly to the NYSE once they meet the requirements.”

For trading purposes, a security is identified by its ticker symbol. Most NYSE-classic and Amex stock listings have ticker symbols of three letters or less, like IBM, GE, or C (Citigroup); most NASDAQ- and ARCA-listed stocks have four-letter symbols, like MSFT (Microsoft), INTC (Intel), and QCOM (Qualcom). Options and futures have more complicated symbols that encode references to maturity or exercise price.

Exchange Ownership

Until the end of the twentieth century, exchanges were historically member-owned cooperatives. The members were mostly brokers and traders; the cooperatives were organized as not-for-profit corporations. Memberships (sometimes also called “seats”) could be transferred, inherited, bought and sold. A membership comprised partial ownership of the exchange plus trading rights and privileges.

Beginning around 1990, exchanges began to reorganize themselves as for-profit-corporations, with publicly-traded shares. In this form, ownership and trading rights are separated: owning a share of the exchange does not confer trading privileges, and you can trade without owning any shares. The term “member” now generally refers to the second possibility, someone who has established a relationship with the exchange for purposes of trading.

Nowadays, although the exchanges continue to operate mostly independently, many are owned by large holding companies, and there have been many changes of name and ownership. NYSE Euronext formerly owned the New York Stock Exchange, MKT (the former American Stock Exchange), Arca, and the Paris, Amsterdam, Brussels and Lisbon Exchanges, among others. In 2013, NYSE Euronext was acquired by ICE (International Commodities Exchange, a large market for derivatives), and in 2014 Euronext was spun off as an independent company.

NASDAQ OMX owns the NASDAQ stock market, BX (the former Boston Stock Exchange), PHLX (the former Philadelphia Exchange), most of the Scandinavian and Baltic stock exchanges (except for the Norwegian Stock Exchange), and eSpeed (a trading platform for US Treasuries). Some of this consolidation has encountered resistance from regulators concerned about competitiveness. In 2011, a contemplated merger of the NASDAQ OMX and NYSE Euronext was effectively blocked by the US Department of Justice. A planned combination of the NYSE Euronext and DeutscheBourse (the principal German exchange) was similarly derailed.

2.2. Brokers

We can’t trade simply by visiting an exchange’s web site and giving a credit-card number. For a number of legal and practical reasons, the exchange requires a more substantial relationship, one that verifies our identity, capability, and authority to trade. Most customers establish this relationship indirectly, by setting up an account with a broker.

A broker conveys or represents customer orders to the market. In this capacity, the brokerage usually provides services directly related to trading: custody of securities purchased, cash
loans (for margin purposes), loans of securities (for short-sale purposes), record-keeping and tax reporting.

The process of representing customer orders might be as simple as directly conveying the customer's instructions, for example, “Buy 100 shares of Microsoft.” Typically, though, the conveyance requires the broker to make certain determinations and decisions. At an even more involved level, brokers may place at their clients’ disposal automated tools known as trading algorithms. Whatever the level of complexity, though, the broker is still representing the customer, acting as the agent.

A broker is an agent working on behalf of a customer (sometimes called the principal). In this capacity, the broker works under a legal obligation to act in accordance with the customer’s instructions and in the customer’s interest. In broker-customer relations, as in many other principal-agent arrangements, the customer may find it difficult to monitor the broker’s effort and actions. Did the broker really make a strong effort to complete our trade at the best possible price? It’s often tough to judge. The broker’s presumed superior expertise, that is, the very thing that makes the broker’s services valuable to us, also makes it more difficult to evaluate his performance. We will encounter in these notes many situations in which the divergence between the customers’ and brokers’ goals strongly influence market outcomes.

Brokers are sometimes differentiated by clientele and approach. Prime brokers provide transaction-related services for large and institutional customers. Individuals go to retail brokers. Retail brokers in turn are sometimes divided into “discount” or “online” brokers, who focus narrowly on trading services, and “full-service” brokers, who provide more comprehensive investment management and advice.

2.3. Traders and their motives

Trade arises from differences in investment goals, risk exposures, and beliefs about security values. People who are identical in all these respects would want, at any proposed price, to trade in the same direction (buy or sell), and a trade requires both a buyer and a seller. A potential buyer and seller might differ in many ways, large and small. But to get a big picture of the market, it is useful to think about broad groups or clienteles.

Investors are sometimes categorized on the basis of investment horizon. Long-term investors include institutions like endowment funds and individuals saving for retirement or a child’s education. Medium-term investors have holding periods on the order of a business cycle (3-5 years). These investors often seek to profit from changes in relative valuations of securities. Short-term traders have holding periods ranging from minutes to a few months.

Traders may also be classified by motive. There are many possible motives, of course, but the most important is information concerning the intrinsic value of the security. If our counterparty has superior information (most obviously of the illegal “insider” sort), then we are much more likely to lose. Informational traders usually need to trade quickly (before their information is made fully public) and stealthily (to avoid detection).

Non-informational motives for trade include hedging, liquidity, and gambling. Hedging trades aim at risk reduction. For example, high-level executives, as a result of stock and option grants, often have personal wealth that is very concentrated in the stock of a single firm (their own). By selling some of this stock, and reallocating to a mix of other stocks, they can reduce their overall risk. Liquidity motives stem from unexpected cash outflows and inflows. A mutual fund’s assets under management, for example, changes as customers invest or divest shares in the fund. On any given day, these are unlikely to be exactly offsetting, so the fund must trade.
2.4. The price

We often refer to the price of a security as if it were one well-defined number. In fact, the market usually provides us with several alternatives:

- The last sale price (the price of the most recent trade)
- The bid quote (the highest price that someone is publicly willing to pay)
- The ask or offer quote (the lowest price at which someone is publicly willing to sell).

“Ask” or “offer” are used interchangeably.

When a price is reported in public media, it is usually a last-sale price. The usefulness and validity of this price stems from the fact that the trade actually occurred. The buyer and seller didn’t just talk about a trade; they really bought and sold. On the other hand, the since the trade occurred before it was reported, the price is not completely current (and in fact might be quite old). The price that we would pay or receive in a trade that we’re currently contemplating might be quite different. The bid and ask are hypothetical prices. They are proposals that might or might not lead to a transaction.

The difference between the ask and bid quotes is the spread. Assuming that the bid and ask prices don’t change, the spread is the cost incurred by someone who buys and immediately sells the security, reversing the initial trade. Often, the bid and offer in a market are posted or set by different traders. Sometimes, however, they are set by one trader who is said to be making a market. If a buyer and seller were to arrive at the same time, the market-maker would buy at her bid price, and sell at her ask price. From the viewpoint of the market maker, then, the spread would represent her trading profit.

2.5. Make or take?

The first major decision – to buy, sell, or hold the security – lies in the realm of asset allocation or risk management, beyond the borders of the present discussion. But once this determination is made, we turn to the question of trading tactics. Here, we face a decision that is often simply stated as “make or take”. Specifically, when we go into the market to trade, should we take the best available price at the moment, or should we try to make our own price and await the arrival of a counterparty who finds our price agreeable?

We’ll start by assuming that we have a stock and a direction. Direction is shorthand for “buy” or “sell”. Let’s assume that we’re buying. A buyer entering the market has can trade immediately by taking the posted ask price. Or she can put in her own bid, hoping that a seller arrives, who is willing to accept her bid. Suppose market in the stock is $100 (per share) bid, offered at $101. She can buy immediately by paying $101, that is, taking someone else’s price. Or she might make price of her own, for example, by bidding $100.25. If an agreeable seller arrives, she’ll buy at 100.25.

The make or take decision is the choice of whether to take someone else’s offer and get an immediate execution, or to make a (lower) bid and hopefully buy at the better price. Making a bid entails some risk, because a seller might never arrive. The market might move higher, and the buyer might find herself chasing the stock, buying at a price higher than the original $101 offer, and therefore regretting her earlier decision to make.

The specifics of her decision are represented in her order. An order is a request, usually conveyed to the market through a broker. All orders indicate direction (buy or sell) and quantity. Most of the time, an order has a price limit, e.g., “buy 100 shares, limit $102.” That is, don’t pay above $102 per share. An order with a price limit is usually called a limit order. If the market ask
price is $101 when the buy order arrives, the buy order is considered marketable. There is an immediate execution, at $101.

A market order is communicated without a price limit. In the case of a buy order, it says “I will pay the market offer, however high that offer might be.” If the market offer price is $101, then someone sending in a market buy order expects to pay 101. But prices can change rapidly, and if the market offer price is $110 when the order actually arrives, the buyer will pay $110.

Someone putting in a limit order priced at 102 in this situation, also expects to buy at 101. But if the price goes above 102, the order will not be executed. Because market orders can sometimes lead to nasty surprises, some markets do not accept unpriced orders. Similar remarks apply, but in the opposite direction to sell limit and market orders.

A participant in a trade is sometimes called a side. A trade has at least one buying side (buyer) and at least one selling side (seller). There may be many sides if there are multiple buyers and/or sellers. Sides may also be classified as active or passive. The passive side refers to the trader who is posting the bid or ask/offer and stands willing and available for trade. The passive side is also called the resting side.

In any given trade, the active side might be the buyer or the seller. We refer to these situations differently. An active seller hits the bid. An active buyer lifts the offer (or lifts the ask). This distinction might seem unnecessary. In the construction “hit the ask,” for example, it seems clear that the seller is passive and the buyer is active. Indeed there are many instances of the expression online and in print. To the traditionalist, though, “hit the ask” sounds wrong, and may even be taken as a mark of the speaker’s ignorance.

The make/take choice often involves a trade-off between risk and reward. A trader who wishes to buy the stock can execute immediately by paying the offer price. The relative reward to using a limit buy order (a bid priced below the offer) is that the stock might be purchased more cheaply. The risk is that bid won’t be hit, and the security won’t be purchased. The consequences of this execution failure might be minor (if the trader is only marginally inclined to own the security), but can be major if the desire to own the security (for investment or hedging purposes) is strong. Finally, a limit order usually entails waiting (for the arrival of an order that executes it). Delay causes risk because security prices are constantly in motion, and may impose also impose psychological cost from postponed cognitive closure (resolution, removal of uncertainty).

2.6. Liquidity (and other terms of the art)

Some terms that we’ll encounter are everyday words, but nevertheless possess, in the context of trading and markets, particular meanings or connotations.

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2 Why did the hit/lift convention develop? I’m not aware of any authoritative pronouncements, but I suspect that it arose from the need for clarity and consistency. The trading process requires fast and accurate communication. All errors have consequences. Many of the worst errors involve direction: buying when you intended to sell or selling when you wanted to buy. As you read this and contemplate things at leisure, an error of direction might seem unlikely or even preposterous. If you’ve ever participated in an open-outcry floor market (real or simulated), though, you’ve probably seem more than a few. The hit/lift construction adds a little more information that helps clarify intent.


*Liquidity* is a broad term that summarizes the level of cost and difficulty that we encounter when we try to trade. In a liquid market, trading is cheap and easy. Moving beyond this generalization, liquidity is sometimes partially characterized by the attributes of immediacy, tightness, depth and resiliency:

- **Immediacy** is the ability to trade quickly.
  - Modern electronic securities exchanges that can be accessed instantaneously over the internet or some similar network have high immediacy. So-called over-the-counter markets that might require a customer to verbally contact many or more dealers have low immediacy.
- **Tightness** (of the bid-ask spread) implies that a round-trip purchase and sale can be accomplished cheaply.
- **Depth** refers to the existence of substantial buy and sell quantities at prices close to the best bid and offer.
  - Suppose the market in stock A is "$10.00 bid for 5,000 shares, and 10,000 shares offered at $10.05", and for stock B, "$10.00 bid for 100 shares, and 100 shares offered at $10.05." The tightness for A and B are the same, but A has greater depth.
- **Resiliency**, in the sense of "bounce back," suggests that any price changes that might accompany large trades are short-lived and quickly dissipate.

Liquidity varies across securities: larger, more widely-held securities generally enjoy better liquidity than smaller issues. Liquidity also varies across time. Some of this variation is predictable. The market for a US stock is more liquid during regular trading hours (9:30-16:00, Eastern Time) than after-hours. But some of the time variation is random and unpredictable.

Liquidity is sometimes characterized as a network effect or network externality. Just as one person’s benefit from a telephone depends on how many other people can be reached over the telephone system, liquidity depends on how many other people hold and (by implication) trade the security. If many people are active in a market, it is easier to find a counterparty.³

*Transparency* refers to the amount of information available about the market and trading process. In US equity markets, we generally know the full history of trades (volumes and prices) as well as past and current bids and asks. In currency (FX) markets, trades are not reported and bids and asks are not as freely available. As a relative statement, US equity markets are transparent, and currency markets are opaque. It should be noted that good market transparency doesn’t imply that there is full or adequate information about the fundamentals of the security.

Transparency is an attribute of the market, not the security being traded. The term *pre-trade transparency* is sometimes used to refer to information available before the trade, such as the bid, the offer, and recent price history. *Post-trade* transparency refers to information available after the trade, such as the trade price, executed volume, and (sometimes) identity of the counterparty.

³ “Liquidity” can take on a different meaning in other contexts. In corporate finance and monetary economics, liquidity can refer to how easily something can be converted into cash (either by selling it or borrowing against it). On a corporation’s balance sheet, for example, holdings of Treasury bills are considered liquid assets because they can easily be sold if the firm needs cash. Inventories might also be considered liquid under the assumption that the firm could borrow money from a bank using the inventories as collateral. When it is necessary to make the distinction, liquidity in the sense just described is called funding liquidity, and liquidity in reference to trading purposes is called market liquidity (Brunnermeier and Pedersen (2009)).
Latency refers to delays encountered in submitting orders and having them acted upon. Immediacy and latency both refer to speed, but while immediacy is a general attribute that encompasses the whole trading process, latency is more narrowly defined. It is usually measured (in milliseconds or microseconds) as the time that elapses from the receipt of an order at the trading center’s computer to the dispatch of a responding message from the computer. It is an attribute of the market’s technology.

2.7. Regulation

Most countries recognize the crucial role that a well-functioning security market plays in raising capital, allocating capital, and hedging. Due to the broad extent of these markets, the most visible regulation usually exists at the national level, supplemented by efforts at consistency, cooperation and coordination to manage trans-national concerns.

The pre-eminence of national regulation does not imply, however, that all markets and aspects of trading are closely overseen by federal governments. Rules and procedures are instituted and monitored by participants, industry associations, exchanges, even, in some cases, state governments.

“Securities,” in US law, comprise corporate stocks and bonds, state and local bonds, and stock options; they are overseen by the Securities and Exchange Commission (SEC, www.sec.gov). “Commodities,” including commodity futures and many financial futures are regulated by the Commodities Futures Trading Commission (CFTC, www.cftc.gov). Other financial derivatives (such as swaps) are regulated jointly by the SEC and CFTC.4 The markets for US Treasury securities are regulated by the Department of Treasury and the Federal Reserve Bank. Currency (foreign exchange, “FX”) markets are regulated indirectly in that the largest participants are banks, which are regulated by multitude of agencies. In addition, since currency forwards and futures have FX as their underlying, the CFTC also possesses derived authority.

The stock, stock option, and (to a lesser extent) bond markets are the most prominent markets. The SEC regulates them under the authorization of several Congressional acts. The 1933 Securities Act mostly applies to the primary market for corporate securities, that is, the initial sale of the securities by a corporate issuer. The 1934 Securities Act regulates secondary trading, that is, transactions where the seller is not the issuer. (Most of these notes are devoted to secondary markets.) The 1975 Securities Act updated certain aspects of the 1934 Act, most importantly giving the SEC the power to oversee and facilitate the transition to electronic markets.

The Acts leave most details of rule-making to the SEC. The SEC in turn delegates some its authority to the exchanges or the Financial Regulatory Authority (FINRA, www.finra.org). FINRA is a non-government, not-for-profit corporation that oversees trading and many aspects of broker-customer relations.5 The power sharing arrangements are sometimes awkward. If the SEC wishes all exchanges to adopt a rule, it must “request” that each exchange make a rule “proposal,” which the SEC then approves.

In the course of describing market operations, we’ll cover some SEC rules that apply directly to the trading process. The SEC also oversees, however, many aspects of the corporate disclosure

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4 In the US Code of Federal Regulations, the most relevant material is found under Title 17, Commodity and Securities Exchange (www.ecfr.gov).
5 FINRA administers the examinations that securities professionals must pass in order to practice. Many employees of securities firms (such as retail stock brokers) take the “Series 7” exam.
process and insider trading. These rules affect the information environment in which trading occurs. Information is the primary input to the trading decision, so it’s not surprising that almost anything that affects its production, communication and use exerts a powerful indirect influence on the market. These information-relevant rules, too, are important if we hope to understand trading.

The CFTC was created by the Commodities Futures Trading Commission Act of 1974. Some of the things that it regulates seem very similar to things regulated by the SEC. A trader seeking broad exposure to the market, for example, might buy an S&P Index ETF (an exchange-traded fund, regulated by the SEC) or go long a stock index futures contract (regulated by the CFTC). The similarities are strong enough that we might expect agreement about how the market should be organized and regulated. In practice, though, the ETF and the futures contract are traded under substantially different rules, and regulatory philosophies differ significantly.

In the European Union, securities overseen by the European Commission’s Internal Market and Services Directorate General, Directorate G – Financial Markets. The overarching regulation is the Markets in Financial Services Directives 2 (“MiFID 2”). Much authority still resides with the exchanges and their home countries.

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Summary of terms and concepts

Exchanges; listing; brokers (retail, prime, discount, full-service); “make or take”; hit the bid/lift the offer; active vs. passive/resting/standing; liquidity (immediacy, breadth, depth, resiliency); transparency (pre- and post-trade), latency; SEC; 1933 Act; primary market; 1934 Act; secondary market; CFTC; FINRA.
Many of today’s securities markets started as floor markets. A floor market is simply some central place where people go to trade. The facilities can be modest. The New York Stock Exchange operated for a while in the Tontine Coffee House (a sort of precursor to Starbucks). The American Stock Exchange started as the New York Curb Market, operating on the sidewalk outside of the NYSE’s building.

On the floor, the traders meet face to face. They negotiate, bargain, and attempt to reach agreement on terms of trade. A trade is not inevitable: the attempt at agreement might break down, and then someone walks away. Although most trades are bilateral (one buyer, one seller), the negotiation takes place in a crowd. Everyone can see and hear the proposed terms of trade. Anyone can jump in, perhaps displacing a buyer or seller who has dominated the negotiation up to that point.

As a financial institution, “the floor” reached the zenith of its scope and power in the last half of the twentieth century, when it dominated stocks, futures and options. At the end of the century, however, most markets transitioned to screen-based electronic trading, and floors closed. The London Stock Exchange closed its floor in 1992; The Chicago Mercantile Exchange closed its trading pits in 2015. At this point, the transition is nearly complete. The era of floor markets is over.

So why study them? There are several reasons. Most importantly, many trading practices, rules, and regulations arose in floor markets, and are best understood in the context of a floor market. An electronic market will sometimes exhibit behavior that at first glance looks like something completely new because it embodies advanced technology (particularly when that technology features speed). But then on closer examination, it becomes more familiar, an adaptation of something we’ve seen in the trading floors of earlier eras. Simply put, floor markets still provide
a useful touchstone in understanding current markets. Throughout these notes we’ll see many examples.6

A simulated floor market can be organized as a game in which participants are given incentives to trade, and are left to bargain with each other using whatever procedures they wish. When such a game is used in an introductory economics class, an unstructured trading procedure is a virtue because it demonstrates that even under minimal rules, players responding to market forces will tend to arrive at outcomes that closely approximate the economist’s ideal of a competitive equilibrium.

Real floor markets, however, are highly structured. Day to day, their trading activities involve the same people, and over time these people have evolved standardized practices and rules. This chapter focuses on these rules and practices. It describes the organization and procedures of a typical floor market, based on the rules of Chicago Mercantile Exchange (CME). This approach reflects a deliberate emphasis here on operational efficiency. Viewing the Exchange as a factory, operational efficiency means that trades are produced quickly, and with minimal effort on the part of the traders. (In later chapters we’ll examine allocational and informational efficiencies.)

The Chicago Mercantile Exchange (CME, “the Merc”) started in the 19th century trading agricultural futures. A wheat futures contract, for example, calls for the delivery of a given amount of wheat on a given maturity date. The price that will be paid for the wheat is determined in the market when the contract is traded, but the actual exchange of wheat and money won’t occur until maturity. This “deferred settlement” feature means that prior to maturity futures contracts can be traded without actually transferring the underlying commodity. For purposes of speculation and hedging, a position in the futures contract can be similar to, but much more convenient than, direct ownership of the underlying. The CME originally listed contracts in grains (such as wheat, corn, and soybeans) and livestock (such as cattle and hogs).

3.1. Floor procedures

To explore how the floor works, we’ll dip into an official CME Rulebook, from a vintage around the turn of the millennium (2004, to be precise). The rules cover almost all aspects of CME governance and procedures. We’ll be focusing on Chapter 5, which deals with trading practices. Featured prominently near the beginning is Rule 520: TRADING CONFINED TO EXCHANGE FACILITIES:

All trading ... must be confined to transactions made on the Exchange; and ... must be confined to the designated trading area during Regular Trading Hours ... Any member violating this rule shall be guilty of a major offense.

The wording reflects the CME’s organizational structure: it has members, like a club or other association. To participate directly in the trading process, one must be a member. Memberships can be bought sold or leased, but the total number of memberships is limited. Membership is therefore somewhat exclusionary. I do not have to be a member to buy or sell a wheat contract, but if I am not a member, I must pay someone who is a member to act as my broker. This gives rise to a division between floor traders, who have direct access to the market and its information, and

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6 It’s also worth noting that at least a few trading firms – including some of the most advanced – run simulated floor markets for training purposes. In a floor market, one is constantly reminded that other traders are human, and therefore (at least in a trading context) scheming, deceptive, inconsistent, and capable of spontaneous improvisation.
off-floor traders, who necessarily see less of the trading process and must wait a bit longer to see their orders executed.

With Rule 520, the Exchange membership is essentially asserting that there is one market, and that the market is under their control. Furthermore, although the floor is often viewed as an arena of pure competition, anyone who tries to compete by accommodating customers’ desire to trade in a place or at a time not approved by the Exchange is “guilty of a major offense.” The members are agreeing that while they might compete strenuously against each other within the club and according to its rules, they will not attempt to set up a separate club.

Viewed from this perspective, the rule might be interpreted as an anticompetitive attempt to amass economic power against the interests of those who are not members of the club. But there is another aspect to the rule. The centralization and consolidation of trading, the bringing together of all buyers and sellers at a particular place and during particular times, makes it easier for us to find counterparties and negotiate with them, in full confidence that there are no other secret or hidden markets where we might find better terms of trade. In this sense the rule advances the operational efficiency of the market, facilitating rapid negotiation and high trading volume, ultimately benefiting members and non-members alike.

The tension between these two views of Rule 520 arises in many of the rules that markets devise for their members. Is a market a “natural monopoly”? Should all markets be consolidated, or should we encourage competition and accept the resulting fragmentation? We’ll return to the debate later in these notes.

A member on the floor might be trading on his own account, that is, relying on personal funds or those of his employer. Alternatively, he might be acting as a broker, an agent for a customer who is not a member and can’t directly participate in the floor trading. An order conveyed by an off-floor customer would specify direction (buy or sell) and quantity (number of contracts). The order might be “at the market” (a market order), which instructs the broker to try to execute the trade as quickly as possible, at the best price currently available in the market. In terms of “make or take,” a market order directs the broker to “take”. The broker should not delay in hopes of getting a better price.

Alternatively, the customer might submit a limit order. In addition to direction and quantity, a limit order has a limit price. “Buy three December wheat, limit $4.00,” instructs the broker to buy (“go long”) three wheat futures contracts that mature in (the nearest upcoming) December. The broker can pay as much as $4.00 [per bushel], but the buyer would obviously like to buy the contracts at a lower price, if possible.

So given this order, “buy three, limit $4.00,” what would our floor trader do next? We turn to Rule 521: Pit Trading

All transactions ... shall be by open outcry in the established pit for that transaction ...

The CME trading floor is bigger than a soccer field, and it is very crowded. The pit is the smaller portion of the floor that is designated as the place to trade, say, wheat. The pit has a distinctive shape. It is constructed as a set of nested octagons (eight-sided shapes) that slope downwards toward the center, like a sports arena. This ensures maximum visibility for the traders. “Open outcry” means simply that bids to buy and offers to sell must be made orally.

Most of the people in the pit will be traders, but there may also be exchange employees such as reports, who record the prices of trades as they happen, and exchange officials who oversee the activity. Traders, reporters and officials are distinguished by the color of their jackets. A trader will also be wearing a badge that displays a short code that identifies him for trading purposes. Our broker is “ALN”. Rule 521 continues:
A bid shall be made only when it is the best bid available in the pit.

Why is this necessary? Can’t a potential seller hear all bids and simply ignore all bids except for the very highest? This might work in principle, but the pit is a crowded and noisy place. If thirty buyers were simultaneously announcing their bids, it would be difficult for a potential seller to hear and keep track of which bid was best.

A bid is made by stating the price first and quantity next (such as "38.50 on 2," etc.) and by holding a hand outstretched with the palm towards the bidder indicating the quantity by the number of fingers shown.

The price-first-quantity-next convention for bids is followed in most floor markets. This makes sense when you fill in the missing words: [I’m] bidding $4 for 3 [contracts]. The hand gesture is also significant. With the palm faced inwards, I’m miming the act of pulling the contracts towards myself. Similarly:

An offer shall be made only when it is the best available offer in the pit. An offer is made by stating quantity first and price next (such as "2 at 38.50") and by holding a hand outstretched with palm away from offeror indicating quantity by the number of fingers shown.

Filling in the missing words, “[I’m offering] two [contracts] at $5.” With the palm facing away, I’m pushing the contracts away, toward the buyer.

So we follow our broker ALN attempting to buy three contracts limit $4. Suppose that he enters the pit, hears BEV bidding $3.50 for seven, and CAM asking $4.10 for five. He could, within the rules, bid the customer’s limit price, “Bidding $4 for three.” But it’s not a good idea to start any negotiation by stating your worst acceptable deal. Perhaps ALN can fill the order at a better (lower) price: “Bidding $3.80 for three.”

At this point CAM might lower her price: “Asking $3.90 for five.” ALN might counterbid closer to CAM’s offer, and we might see convergence to the point where agreement (and a trade) seem very likely.

Now how does a trade actually happen? You might think that a trade would occur automatically whenever there happened to be a match between the bid and offer. So suppose that ALN bids $3.85 and CAM offers at $3.85. While this is indeed the case, this agreement is not sufficient to cause a trade to occur. Rule 521 continues:

When a trader desires to buy the going offer in the pit, he shall by outcry state "buy it" or "buy them" or "buy" followed by the quantity desired, as the case may be. When selling, the trader shall similarly, by outcry, state "sell it" or "sell them" or "sell" followed by the quantity desired.

Bids and offers are passive. Statements like, “Bidding $3.80 for two” or, “Offering $3.90 for four,” simply indicate availability. For a trade to occur, someone has to take action, shouting “Sell it!” to hit another trader’s bid, or “Buy it!” to lift another’s offer. So, suppose we have:

ALN: “[Bidding] $3.85 for three.”
CAM: “Five [offered] at $3.85.”

A momentary pause, and then,

CAM: “Sell ’em!”
At this point, in the normal course of things, ALN acknowledges CAM and we’d say that a trade has occurred. A floor reporter overhearing the events would key in the price, and “$3.85” would be broadcast to the world.

For ALN and CAM, though, the dialog continues. Each will report the trade to the Exchange, their respective firms, and, unless they are trading for their own accounts, their customers. The transaction then moves into clearing (mutually confirming the terms of the trade), and settlement (transferring ownership and payment in a manner that is legal and irrevocable). Trades that don’t clear, because of some discrepancy in price, quantity or identity of buyer or seller, are bounced back to the traders for resolution.

It all seems quite simple and straightforward. What could go wrong?

3.2. Reputation

In the normal course of events, ALN responds to CAM’s “Sell ‘em,” with a confirmation that they have a deal at $3.85. Failure to confirm a transaction is a trading infraction (Rule 514A.4). Suppose, though, that instead of confirming, ALN shrugs and says, “Okay, but my price is $3.80.”

CAM: “But you were just bidding $3.85!”

ALN: “That’s history. My customer just changed his bid. You were too slow.”

What do the rules say in a case like this? Once a bid or offer is out of my mouth, for how long can other traders presume that it is available? Interestingly, the formal rules are silent on this point. We have instead a convention, that is, a practical guideline, which says something like “as long as the breath is warm.”

That helps a bit. If the elapsed time between ALN’s bid and CAM’s “Sell ‘em” is a half-second, then we side with CAM. If the time is five minutes, then we’d probably say that CAM should have at least confirmed that ALN’s bid was still available. But what if the elapsed time were something in between, like two or three seconds? Perhaps there should be a rule that specifies a precise presumed duration, but it is probably not practical for traders in a fast crowded pit to manage stopwatches on top of everything else.

As a practical matter, the situation can be handled by appealing to reputation. On the floor we’re face to face. The floor is not an anonymous market. A trader knows, for better or worse, the identities of her counterparties. If ALN backs away from his bid once, that is something that could happen to anyone. If ALN backs away twice, CAM will look for someone else to trade with. If ALN habitually fails to honor his bids and offers, he will find that the floor can be a very lonely place, where it can be very difficult to trade.

Reputation, behavioral expectations based on a presumption of repeated and ongoing interaction, provides a cohesiveness to the market. Small misunderstandings stay small, larger differences are avoided. Reputational enforcement of trading norms arises in a great many situations. Reputation also, on the other hand, can facilitate collusion. We noted earlier the “private club” aspect of Rule 520. This is easier to sustain when members know each other, go along, and get along.

3.3. The transition to electronic markets

We’ve now covered enough of the basic floor trading process to set the stage for the modern computerized market. But for the moment, let’s put ourselves in the visitors’ gallery of a floor market around 1985. As we watch the traders on the floor, we wonder, “what will the computerized market of the future look like?”
On my computer, the icon for a “folder” looks like a traditional paper file. My folder icons are arranged for display in a view called a “desktop”. The form of these icons intuitively suggests what they represent and how they are used. This visual correspondence is pervasive in our personal electronic devices.

When securities markets were going electronic, the principle of visual correspondence suggested that a computerized market might resemble very closely the floor market that it replaced. In the late 1980’s, the Chicago Board of Trade (a rival of the CME) unveiled a system, developed jointly with Apple, called Aurora. The New York Times reported (Berg (1989)):

> Like video games, Aurora will rely on computer graphics to display images of living creatures on the screen of traders’ computer terminals. In particular, the Board of Trade has tried with Aurora to replicate the “open outcry system,” under which traders shout buy and sell orders to one another in a large, frequently crowded trading ring.

With Aurora, which will operate only when the Board of Trade is closed, traders will be given a computer keyboard and screen. Displayed on that screen will be a large square that is supposed to represent a trading ring. And inside the square will be small circles and squares representing buyers and sellers.

Each of these small circles and squares will have some form of identification so users of the system can know who is who. When a buyer wants to buy futures contracts from someone offering them for sale, he will use a pointing device known as a “mouse” to designate a particular seller on the screen. Then he will execute the trade by entering a buy command at his keyboard.

Aurora was never implemented.
Chapter 4. Limit order markets

When our markets became computerized, they did not take the form of an app that visually simulated a trading floor, or anything like it. Instead they drew on a very particular aspect of a floor market: the limit order book. The system that evolved now governs most trading in equities, options and futures contracts.

Recall that a limit order specifies buy or sell, quantity, and a limit price. “Buy two July wheat, limit $4.40” is an instruction to buy (long) two July wheat futures contracts at a price no higher than four dollars (per bushel). In a floor market, the order would be communicated to a floor broker (exchange member). This member would then act as an agent for the order. He would “work” the order, representing it on the floor, bidding as if he were the actual submitter.

A broker with many customers might have accumulated many limit orders, and he would need to maintain a record of those that have not yet been executed (or cancelled). This collection is his limit order book (or simply, “his book”), and it would have originally been kept on paper.

A broker’s book might be quite full, with orders to buy priced “near the market” (that is, close to the current bid) and orders priced “away from the market” (well below the current bid). It might also have orders to sell, near and away from the market (close to or far from the current offer). In working these orders, the broker has to pay attention to the market activity, standing ready to bid or offer on behalf of an order.

Suppose that the $4.40 limit buy order is given to the broker when July wheat is $4.50 bid, offered at $4.51. The order is priced away from the market, and the customer would not expect an immediate execution. Now suppose that during the trading session the price falls, with bids and offers stepping downwards, so that an hour after the order is submitted, the offer is two
contracts at $4.39. At that point, the broker should step forth and shout "buy them," executing his customer’s order.7

In a futures market, the broker would typically be handling only wheat limit orders. By remaining in the wheat pit, he’d be in a position to monitor the market. A broker on the floor of a stock exchange, however, might receive limit orders in different stocks. If those stocks are traded in different areas of the floor, following the market might be difficult. When she runs over to listen to stock A’s bids and offers, he’s too far away to hear stock B’s activity.

There are several solutions. She might give the orders to another broker, at least during the time she has to step away. Alternatively, the members might collectively decide to rotate the trading. (“Now we’re all going to trade Stock A ... And now we’re all going to trade stock B.”) Finally, the exchange might decide to have one limit order book in the stock, managed by one person who always remains in the trading area.

Each solution has been used effectively, and we’ll subsequently consider some examples, but at the moment, we’ll focus on the last suggestion. At the New York Stock Exchange, there was one limit order book in each stock, and it was managed by an exchange member called the specialist. On the floor of the Tokyo Stock Exchange, the book was managed by an exchange employee called the saitori.

Traditionally the book was maintained in pencil-and-paper (New York) or on a chalkboard (Tokyo), but the present day book is computerized, a dynamic data structure that gets revised as orders arrive, get executed, or cancelled. A market can be organized so that all activity flows through the book. We’ll now look at how such a market operates.

4.1. Basic limit order processing

Let’s start with an order, “Buy 200 MSFT limit 25,” (“Buy 200 shares of MSFT, but don’t pay more than $25 per share.”) When this order arrives at a limit order market, there will first be an attempt to match it. A match (also called a trade, execution, or fill) occurs when price of the arriving order meets or crosses the price of a pre-existing (“standing” or “resting”) order.

Suppose that it is very early in the day and the MSFT book has one resting order: “Sell 200 MSFT limit 25.”

- A new arriving order, “buy 200 MSFT limit 25” is marketable. There is a trade at 25. The price is determined by the resting order. An arriving order, “Buy 200 MSFT limit 500,” would still result in a trade at 25.
- Quantity is determined by the smaller of the buy and sell quantities: “Buy 300 MSFT limit 25” results in a trade of 200 shares (the offer amount). “Buy 50 MSFT limit 25” causes a trade of 50 shares (the buy amount).

For a given incoming marketable order there might be multiple resting limit orders that are valid candidates for execution. Who gets priority? In most markets, the first priority is price. A buy order with a relatively high price is said to be (relatively) aggressive. (“The buyer is willing to pay more.”) A sell order with a relatively low price is aggressive. (“The seller is willing to accept less.”) More aggressive limit orders have priority over less aggressive orders.

7 The worst outcome is when an inattentive broker fails to execute the order, and price subsequently moves away. Suppose that the price reaches a low of $4.39, but subsequently rebounds to $4.70. A customer who’d entrusted “buy limit $4.40” to a broker would probably assume that she’d bought at $4.40 a contract that was now worth much more. If the broker failed to execute (“missed the market”), the customer would have a very legitimate grievance.
After price, however, markets vary in their secondary priorities. These priorities are usually established with the intent of promoting the exchange’s perceived liquidity, enhancing the attractiveness of the exchange as a desirable trading venue.

The most common secondary priority is time: the order that arrives earlier is executed before the later arrival. This principle rewards a bidder and offerer for stepping in quickly with their best price. “Time” is generally defined as when the order was received by the market.

Most markets accept hidden orders. These orders reside in the book, and are available for execution, but are not displayed. When such orders are allowed, visible orders usually have priority over hidden orders, even if they arrived later. A displayed limit order is like an advertisement for the market, and so priority for these orders therefore makes sense from the exchange’s viewpoint.

Price, visibility, and time (of order receipt), in that order define the most common priority scheme. To see how these work, consider the following examples

**Example 1. Organization and display of the bid book**

Suppose that the following buy orders arrived in sequence.

<table>
<thead>
<tr>
<th>Price</th>
<th>Visibility</th>
<th>Time</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.05</td>
<td>Hidden</td>
<td>9:30</td>
<td>Amy</td>
</tr>
<tr>
<td>20.04</td>
<td></td>
<td>9:31</td>
<td>Brian</td>
</tr>
<tr>
<td>20.04</td>
<td></td>
<td>9:32</td>
<td>Chao</td>
</tr>
<tr>
<td>20.05</td>
<td></td>
<td>9:33</td>
<td>Dmitri</td>
</tr>
<tr>
<td>20.03</td>
<td></td>
<td>9:34</td>
<td>Esteban</td>
</tr>
<tr>
<td>20.06</td>
<td>Hidden</td>
<td>9:35</td>
<td>Florio</td>
</tr>
</tbody>
</table>

Here, we identify the orders by trader names, but this is simply for our own convenience in referencing them. Real markets do not display this information, and sometimes might not even possess it. We’re not showing quantities here because they won’t affect the ranking. The default for visibility is “displayed,” so we omit it, flagging the visibility of the order only if it is hidden.

To build the buy side of the limit order book (the bid book) we first sort by price:

<table>
<thead>
<tr>
<th>Price</th>
<th>Visibility</th>
<th>Time</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.06</td>
<td>Hidden</td>
<td>9:35</td>
<td>Florio</td>
</tr>
<tr>
<td>20.05</td>
<td>Hidden</td>
<td>9:30</td>
<td>Amy</td>
</tr>
<tr>
<td>20.05</td>
<td></td>
<td>9:33</td>
<td>Dmitri</td>
</tr>
<tr>
<td>20.04</td>
<td></td>
<td>9:31</td>
<td>Brian</td>
</tr>
<tr>
<td>20.04</td>
<td></td>
<td>9:32</td>
<td>Chao</td>
</tr>
<tr>
<td>20.03</td>
<td></td>
<td>9:34</td>
<td>Esteban</td>
</tr>
</tbody>
</table>
Next, orders at the same price are ranked by visibility:

<table>
<thead>
<tr>
<th>Price</th>
<th>Visibility</th>
<th>Time</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.06</td>
<td>Hidden</td>
<td>9:35</td>
<td>Florio</td>
</tr>
<tr>
<td>20.05</td>
<td></td>
<td>9:33</td>
<td>Dmitri</td>
</tr>
<tr>
<td>20.05</td>
<td>Hidden</td>
<td>9:30</td>
<td>Amy</td>
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<tr>
<td>20.04</td>
<td></td>
<td>9:31</td>
<td>Brian</td>
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<tr>
<td>20.04</td>
<td></td>
<td>9:32</td>
<td>Chao</td>
</tr>
<tr>
<td>20.03</td>
<td></td>
<td>9:34</td>
<td>Esteban</td>
</tr>
</tbody>
</table>

The ranking by visibility changes the relative positions of Dmitri and Amy. At this point we are done. Because the initial set was ordered by arrival sequence, Brian’s order and Chao’s, which have the same price and visibility, are correctly ranked.

The best bid on the book is Florio’s 20.06. If any executions in this set were to occur, Florio’s order would come first. The market’s bid quote only reflects displayed orders. Florio’s order could not be shown. The market’s bid is therefore 20.05. This is also described as the top of the market’s bid book. To reiterate, a market’s quotes only reflect the visible orders.

Example 2. Organization and display of the ask book

Suppose that we are given the following sequence of sell orders.

<table>
<thead>
<tr>
<th>Price</th>
<th>Visibility</th>
<th>Time</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.20</td>
<td></td>
<td>9:30</td>
<td>Gregori</td>
</tr>
<tr>
<td>20.18</td>
<td></td>
<td>9:31</td>
<td>Haley</td>
</tr>
<tr>
<td>20.18</td>
<td>Hidden</td>
<td>9:32</td>
<td>Inez</td>
</tr>
<tr>
<td>20.10</td>
<td>Hidden</td>
<td>9:33</td>
<td>Jing</td>
</tr>
<tr>
<td>20.15</td>
<td></td>
<td>9:34</td>
<td>Kala</td>
</tr>
<tr>
<td>20.18</td>
<td></td>
<td>9:35</td>
<td>Lou</td>
</tr>
</tbody>
</table>

The ask book is constructed the same way as the bid, with the most aggressive offers at the top. Here is the result:

<table>
<thead>
<tr>
<th>Price</th>
<th>Visibility</th>
<th>Time</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.10</td>
<td>Hidden</td>
<td>9:33</td>
<td>Jing</td>
</tr>
<tr>
<td>20.15</td>
<td></td>
<td>9:34</td>
<td>Kala</td>
</tr>
<tr>
<td>20.18</td>
<td></td>
<td>9:31</td>
<td>Haley</td>
</tr>
<tr>
<td>20.18</td>
<td></td>
<td>9:35</td>
<td>Lou</td>
</tr>
<tr>
<td>20.18</td>
<td>Hidden</td>
<td>9:32</td>
<td>Inez</td>
</tr>
<tr>
<td>20.20</td>
<td></td>
<td>9:30</td>
<td>Gregori</td>
</tr>
</tbody>
</table>

Note that Inez loses time priority to Lou because her offer is hidden. Here, as in the case of the bid book, the highest-ranked order is hidden. The market’s ask quote is Kala’s offer at 20.15.
Presentation styles

On screens, the bid and ask books are often shown side-by-side, with the bid side on the left. Following this convention, the orders discussed above would be shown as:

<table>
<thead>
<tr>
<th>Bids</th>
<th>Offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Visibility</td>
</tr>
<tr>
<td>20.06</td>
<td>Hidden</td>
</tr>
<tr>
<td>20.05</td>
<td>9:33</td>
</tr>
<tr>
<td>20.05</td>
<td>Hidden</td>
</tr>
<tr>
<td>20.04</td>
<td>9:31</td>
</tr>
<tr>
<td>20.04</td>
<td>9:32</td>
</tr>
<tr>
<td>20.03</td>
<td>9:34</td>
</tr>
</tbody>
</table>

This is what would be available to an official observer at the market. Traders would not see the hidden orders, or any indication that they existed. An external display would show:

<table>
<thead>
<tr>
<th>Bids</th>
<th>Offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Visibility</td>
</tr>
<tr>
<td>20.05</td>
<td>9:33</td>
</tr>
<tr>
<td>20.04</td>
<td>9:31</td>
</tr>
<tr>
<td>20.04</td>
<td>9:32</td>
</tr>
<tr>
<td>20.03</td>
<td>9:34</td>
</tr>
</tbody>
</table>

Alternatively, limit order books are sometimes shown vertically, with offers on the top and bids on the bottom. In this arrangement, the set of orders would be displayed as:

<table>
<thead>
<tr>
<th>Bids</th>
<th>Offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Visibility</td>
</tr>
<tr>
<td>20.20</td>
<td>9:30</td>
</tr>
<tr>
<td>20.18</td>
<td>Hidden</td>
</tr>
<tr>
<td>20.18</td>
<td>9:35</td>
</tr>
<tr>
<td>20.18</td>
<td>9:31</td>
</tr>
<tr>
<td>20.15</td>
<td>9:34</td>
</tr>
<tr>
<td>Offers</td>
<td>20.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bids</th>
<th>Offers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Visibility</td>
</tr>
<tr>
<td>20.06</td>
<td>Hidden</td>
</tr>
<tr>
<td>20.05</td>
<td>9:33</td>
</tr>
<tr>
<td>20.05</td>
<td>Hidden</td>
</tr>
<tr>
<td>20.04</td>
<td>9:31</td>
</tr>
<tr>
<td>20.04</td>
<td>9:32</td>
</tr>
<tr>
<td>20.03</td>
<td>9:34</td>
</tr>
</tbody>
</table>
Note that with vertical arrangement, the positioning of the offers is reversed, to keep the price direction consistent.8

4.2. Interactions with incoming orders

The last section briefly described simple executions. When the book has multiple orders, things can get more involved. Consider the following book (presented vertically):

<table>
<thead>
<tr>
<th>Visibility</th>
<th>Price</th>
<th>Quantity</th>
<th>Submitted</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50.12</td>
<td>1,000</td>
<td>9:30</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>50.11</td>
<td>500</td>
<td>9:32</td>
<td>Bill</td>
</tr>
<tr>
<td>Hidden</td>
<td>50.10</td>
<td>200</td>
<td>9:30</td>
<td>Gina</td>
</tr>
<tr>
<td>SELL</td>
<td>50.10</td>
<td>400</td>
<td>9:31</td>
<td>Amy</td>
</tr>
<tr>
<td>BUY</td>
<td>50.05</td>
<td>1,000</td>
<td>9:30</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>50.04</td>
<td>500</td>
<td>9:32</td>
<td>Ellen</td>
</tr>
<tr>
<td></td>
<td>50.03</td>
<td>400</td>
<td>9:31</td>
<td>Fred</td>
</tr>
</tbody>
</table>

Example 1

Now suppose that at 9:40 Hari submits an order, “Buy 200, limit 50.10.” This will execute against Amy’s order, leaving 200 shares remaining on Amy’s order.

Example 2

Now suppose Hari’s order had been instead, “Sell 1,200 limit 50.04.” Because this sell order is priced below the best bid, it will at least partially execute. In this case, 1,000 shares executes against David, at $50.05; 200 shares execute against Ellen at 50.04. Ellen has 300 shares remaining, and 50.04 becomes the market’s bid quote. An order that executes at multiple prices is sometimes said to walk through the book.

Example 3

Hari (at 9:40): “Sell 1,200, limit 50.05.” Now, 1,000 shares will execute at 50.05 against David, but the next available bid, at 50.04, is worse than the Hari’s limit price. So Hari’s remaining 200 shares are added to the book on the sell side:

---

8 The Rotman Interactive Trader has both types of display. The Book Trader panel is side-by-side; the Ladder Trader panel is vertical.
<table>
<thead>
<tr>
<th>Visibility</th>
<th>Price</th>
<th>Quantity</th>
<th>Submitted</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50.12</td>
<td>1,000</td>
<td>9:30</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>50.11</td>
<td>500</td>
<td>9:32</td>
<td>Bill</td>
</tr>
<tr>
<td>Hidden</td>
<td>50.10</td>
<td>200</td>
<td>9:30</td>
<td>Gina</td>
</tr>
<tr>
<td></td>
<td>50.10</td>
<td>400</td>
<td>9:31</td>
<td>Amy</td>
</tr>
<tr>
<td>SELL</td>
<td>50.05</td>
<td>200</td>
<td>9:40</td>
<td>Hari</td>
</tr>
<tr>
<td>BUY</td>
<td>50.04</td>
<td>500</td>
<td>9:32</td>
<td>Ellen</td>
</tr>
<tr>
<td></td>
<td>50.03</td>
<td>400</td>
<td>9:31</td>
<td>Fred</td>
</tr>
</tbody>
</table>

**Example 4**

To the original book, Hari submits: “Buy 500, limit 50.11.” Now, 400 shares will execute against Amy at 50.10. Then 100 shares execute against Gina, also at 50.10. The new book is:

<table>
<thead>
<tr>
<th>Visibility</th>
<th>Price</th>
<th>Quantity</th>
<th>Submitted</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50.12</td>
<td>1,000</td>
<td>9:30</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>50.11</td>
<td>500</td>
<td>9:32</td>
<td>Bill</td>
</tr>
<tr>
<td>SELL</td>
<td>50.10</td>
<td>100</td>
<td>9:30</td>
<td>Gina</td>
</tr>
<tr>
<td>BUY</td>
<td>50.05</td>
<td>1,000</td>
<td>9:30</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>50.04</td>
<td>500</td>
<td>9:32</td>
<td>Ellen</td>
</tr>
<tr>
<td></td>
<td>50.03</td>
<td>400</td>
<td>9:31</td>
<td>Fred</td>
</tr>
</tbody>
</table>

The market’s new ask quote is 50.11.

From the trader’s perspective, executions against hidden orders are a bit unexpected. Based on what he could see, Hari would have expected the last 100 shares of his order to execute at 50.11, paying up for those shares. His actual outcome is better: he buys everything at the lower price. Bill is less pleased. Based on what Bill could see, he might have thought that he was next in line after Amy. Gina’s hidden order displaces his interest.

**4.3. More complex interactions and order qualifiers**

Sometimes qualifications are added to orders. Suppose that the current book is:

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
<th>Entered at</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.12</td>
<td>1,000</td>
<td>9:30</td>
<td>Cathy</td>
</tr>
<tr>
<td>50.11</td>
<td>500</td>
<td>9:32</td>
<td>Bill</td>
</tr>
<tr>
<td>SELL</td>
<td>50.10</td>
<td>9:31</td>
<td>Amy</td>
</tr>
<tr>
<td>BUY</td>
<td>50.05</td>
<td>9:30</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>50.04</td>
<td>9:32</td>
<td>Ellen</td>
</tr>
<tr>
<td></td>
<td>50.03</td>
<td>9:31</td>
<td>Fred</td>
</tr>
</tbody>
</table>

If Gina sends an order, “buy 500 shares, limit 50.10,” she gets a *partial fill*. She buys 400 shares @ $50.10 from Amy. Her remaining 100 shares are added to the book. They are added on the bid side (because she’s a buyer). The market’s new best bid is hers, $50.10. The best offer is now Bill’s, $50.11.
One common qualification is immediate or cancel (IOC). If Gina’s order were stamped IOC, she still would have bought 400 shares, but her unexecuted remainder would not have been added to the bid side. Even if she would be willing to buy 100 more shares at $50.10, she does not want to make her interest visible.

If Gina’s order had been stamped all or nothing (AON), she would not have bought 400 shares. In most systems, her order would still be considered active, though. If Amy (or some other buyer) adds 100 more shares to the offer side at $50.10, Gina’s order would then execute for 500 shares. Gina might use an AON if she were concerned that any execution would be perceived as a signal that more shares were sought.

The fill or kill (FOK) qualification is a combination of IOC and AON. If Gina’s order is marked FOK, nothing happens: the order can’t be executed immediately in full, and is therefore cancelled. These qualifiers (IOC, AON, FOK) lie at one end of a continuum that includes conditional orders and complicated algorithms, to which these notes will later return.

4.4. Alternative priority rules

Although price/visibility/time priority is most common, markets sometimes experiment with and adopt alternatives.

Some markets give priority to larger orders. Large orders advertise the exchange as a place that can handle large volumes. This same goal is often accomplished by using pro rata allocations. In a pro rata system, all limit orders at a price split execution quantities in proportion to their sizes, irrespective of when the orders were submitted. For example, suppose that there are two limit orders bidding $100, and that the sizes of these orders are 100 and 900 shares. If a marketable order arrives, say “sell 200 shares limit $100”, the 100-share bidder actually receives 20 shares, and the 900-share bidder receives 180 shares.

Some current interest centers on a different kind of time priority. The usual time priority is first-come-first-served, based on when the order was received. Limit orders have another attribute called time in force (TIF). If the order is not executed, how long is it considered valid? In the days of floor markets, the default was good-till-cancelled (GTC), and orders could sit on the book for many months. Nowadays, the usual convention is end-of-day. At the end of the day, all orders are cancelled, and the book starts the next day empty. But the trader can also submit an explicit TIF, like one minute, or ten seconds. As we’ll see later, today’s markets receive some limit orders that are cancelled almost immediately. These orders are sometimes considered problematic because their brief duration can confuse other traders. To discourage short-lived orders, some exchanges are considering giving priority to orders based on their commitment to a reasonably long time in force.

4.5. From floors to books

Trading procedures in modern limit order markets rely on automatic execution (“auto ex”): if an incoming order can be executed, it is executed, immediately. The brokers on most floor markets accepted electronic entry of orders, and computerized maintenance of the book. But they correctly saw that automatic execution threatened their survival, and sensibly resisted it.

In 1992 on the occasion of the New York Stock Exchange’s Bicentennial, the Exchange’s Chairman William H. Donaldson commented, “It’s safe to say that the securities market of the future won’t be an unmanned spacecraft, run by computers. It will more closely resemble a Starship Enterprise -- computerized beyond our wildest dreams -- but, in the American tradition, with the intelligence and the soul of human judgment on the bridge,” (Widder (1992)).
Floor markets managed the transition to limit order markets with varying degrees of pain. Many non-US stock exchanges seemed to fare well. The Toronto and Tokyo exchanges, and the Paris Bourse readily adopted the electronic limit order books. The U.S. futures exchanges built computerized systems that were initially used in off-hours, when the floor was not operating. Although the Chicago Board of Trade’s Aurora System did not work as planned, the Chicago Mercantile Exchange’s Globex limit order market functioned well. The Board of Trade, the Merc, the New York Mercantile Exchange and a few other floor-based US futures markets combined, and the Globex system is now used by all of them. Duncan MacKenzie provides an in-depth history of the Merc’s transition (MacKenzie (2013)).

The transition for US equity markets was more complicated, involving competing markets and a fair amount of regulatory pressure. At the New York Stock Exchange, full automatic execution did not occur until the adoption of the SEC’s Reg NMS in 2005. Presently, almost all trading at both the NYSE and NASDAQ occurs through their limit order books.

4.6. Further reading

Bessembinder, Panayides and Venkataraman (2009) discuss the usage of hidden orders. The limit order market is usually referred to by economists as a continuous double auction market. Economists usually attempt to characterize the equilibrium behavior of the market. Since the number of potential participants is large, and the set of strategies that any one agent might deploy is rich and varied, however, these models are difficult to solve, and even modest results are hard-won. See Glosten (1994); Goettler, Parlour and Rajan (2005); Hollifield, Miller, Sandas and Slive (2006); Parlour (1998); Sandas (2001). Physicists tend to model limit order markets using parsimonious statistical models of order arrival. These models are more successful at generating order-price behaviors that resemble what we see in real markets, but they do not (by design) allow us to predict how traders’ strategies might evolve in response to, say, a regulatory change. See Bouchaud, Farmer and Lillo (2008); Bouchaud, Mézard and Potters (2002); Huang, Lehalle and Rosenbaum (2015); Smith, Farmer, Gillemot and Krishnamurthy (2003).

Summary of terms and concepts

Operations of a limit order book; priority (price, visibility, time); undisplayed/hidden orders; matching (execution) procedures; “top of the book”; “walking through the book”; order qualifiers (IOC, AON, FOK); market orders, marketable and non-marketable limit orders.

Problems and exercises

Problem 4.1 Constructing the book under price-time priority. Build the bid side of the book from the following limit buy orders.

<table>
<thead>
<tr>
<th>Priority (1=top)</th>
<th>Trader</th>
<th>Arrival Sequence</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy</td>
<td>1</td>
<td>100</td>
<td>20.02</td>
<td></td>
</tr>
<tr>
<td>Brian</td>
<td>2</td>
<td>400</td>
<td>20.05</td>
<td></td>
</tr>
<tr>
<td>Chad</td>
<td>3</td>
<td>200</td>
<td>20.07</td>
<td></td>
</tr>
<tr>
<td>Dana</td>
<td>4</td>
<td>100</td>
<td>20.06</td>
<td></td>
</tr>
</tbody>
</table>
### Problem 4.2
Constructing the book. Under price-visibility-time priority, build the offer side of the book from the following sell orders.

<table>
<thead>
<tr>
<th>Priority (1=top)</th>
<th>Trader</th>
<th>Arrival Sequence</th>
<th>Quantity</th>
<th>Price</th>
<th>Display?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amy</td>
<td>1</td>
<td>100</td>
<td>10.10</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Brian</td>
<td>2</td>
<td>400</td>
<td>10.02</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Chad</td>
<td>3</td>
<td>200</td>
<td>10.01</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Dana</td>
<td>4</td>
<td>100</td>
<td>10.02</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Emily</td>
<td>5</td>
<td>300</td>
<td>10.01</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Frank</td>
<td>6</td>
<td>300</td>
<td>10.10</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Gina</td>
<td>7</td>
<td>200</td>
<td>10.00</td>
<td>N</td>
</tr>
</tbody>
</table>
Problem 4.3 Order outcomes. The state of the book is:

<table>
<thead>
<tr>
<th>Trader</th>
<th>Sequence</th>
<th>Price</th>
<th>Quantity</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>Irina</td>
<td>5</td>
<td>16.00</td>
<td>1,000</td>
<td>Y</td>
</tr>
<tr>
<td>Gina</td>
<td>2</td>
<td>15.07</td>
<td>500</td>
<td>Y</td>
</tr>
<tr>
<td>Esteban</td>
<td>1</td>
<td>15.07</td>
<td>100</td>
<td>Y</td>
</tr>
<tr>
<td>Dmitri</td>
<td>4</td>
<td>15.05</td>
<td>300</td>
<td>N</td>
</tr>
<tr>
<td>Sell</td>
<td>Alice</td>
<td>3</td>
<td>15.05</td>
<td>200</td>
</tr>
<tr>
<td>Buy</td>
<td>Bruce</td>
<td>2</td>
<td>15.03</td>
<td>400</td>
</tr>
<tr>
<td>Hans</td>
<td>1</td>
<td>15.01</td>
<td>300</td>
<td>Y</td>
</tr>
<tr>
<td>Cho</td>
<td>3</td>
<td>15.00</td>
<td>200</td>
<td>Y</td>
</tr>
<tr>
<td>Frank</td>
<td>4</td>
<td>15.00</td>
<td>300</td>
<td>N</td>
</tr>
<tr>
<td>Jing</td>
<td>5</td>
<td>14.50</td>
<td>2,000</td>
<td>Y</td>
</tr>
</tbody>
</table>

For each of the following incoming orders, describe the outcome (executions if any, and any changes in the prices, quantities and trader names at the BBO). The incoming orders represent different scenarios (not sequential arrivals).

a. Kathy: “Buy 300, limit 15.05.”

b. Lane: “Buy 600, limit 15.10.”

c. Maureen: “Buy 600, limit 15.05.”

d. Ollie: “Sell 500, limit 15.00.”

e. Petra: “Sell 500, limit 15.02.”

f. Rama: “Sell 500, limit 15.02, IOC.”

g. Sydney: “Sell 800, limit 15.01, FOK.”

Problem 4.4 A market receives the following buy orders. How are they ranked? That is, in what order will they be executed (1=first, 2=second, ..., 5=last)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>$10.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$10.05</td>
<td>11:02</td>
<td>1,000</td>
<td>H</td>
<td>Beth</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$10.05</td>
<td>11:10</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$10.10</td>
<td>11:08</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$10.20</td>
<td>11:08</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td></td>
</tr>
</tbody>
</table>

Problem 4.5 A market receives the following buy orders. How are they ranked? That is, in what order will they be executed (1=first, 2=second, ..., 5=last)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy</td>
<td>$20.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$20.00</td>
<td>11:02</td>
<td>100</td>
<td>D</td>
<td>Beth</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$20.40</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td></td>
</tr>
<tr>
<td>Buy</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td></td>
</tr>
</tbody>
</table>
Problem 4.6 A market receives the following sell orders. How are they ranked? That is, in what order will they be executed (1=first, 2=second, ..., 5=last)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sell</td>
<td>$20.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.00</td>
<td>11:02</td>
<td>100</td>
<td>D</td>
<td>Beth</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.45</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
</tr>
</tbody>
</table>

Problem 4.7 A market receives the following sell orders. How are they ranked? That is, in what order will they be executed (1=first, 2=second, ..., 5=last)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:01</td>
<td>200</td>
<td>H</td>
<td>Andy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:02</td>
<td>1,000</td>
<td>D</td>
<td>Beth</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:08</td>
<td>300</td>
<td>H</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
</tr>
</tbody>
</table>

Problem 4.8 A market receives the following sell orders. How are they ranked? That is, in what order will they be executed (1=first, 2=second, ..., 5=last)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:01</td>
<td>200</td>
<td>H</td>
<td>Andy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.30</td>
<td>11:02</td>
<td>1,000</td>
<td>D</td>
<td>Beth</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.45</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
</tr>
<tr>
<td></td>
<td>Sell</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>H</td>
<td>Emma</td>
</tr>
</tbody>
</table>

Problem 4.9 The book in XYZ is

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

Wendy enters an order: “Buy 500 shares limit 10.06”. What executions occur; what are the new bid and offer quotes?
**Problem 4.10** The book in XYZ is

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

At 10am Yuri submits an order: “Buy 500 shares limit 10.05.” What executions occur? What does the new book look like?

**Problem 4.11** The book in XYZ is

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

At 10am Zelda submits an order, “sell 600 shares limit 9.98.” What executions occur? What does the new book look like?

**Problem 4.12** The book in XYZ is

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

At 10am Arturo submits an order, “sell 600 shares limit 10.01 IOC.” What executions occur? What does the new book look like?
Chapter 5. Multiple markets

A market can be organized so that by law or established custom, all trading in a security is consolidated, and occurs through a single exchange. Formerly, this meant that all trading occurred in a single physically-convened market (one trading floor). Nowadays, “consolidated” usually means that trading happens in one central computer system usually a consolidated or centralized limit order book (CLOB).

Many present-day regulators, though, are reluctant to give one exchange a monopoly on trading. Allowing multiple exchanges results in a fragmented market. A fragmented market can simply result from having multiple limit order books. It may also involve alternative (non-limit-order) mechanisms, in which case the market is also considered to be a hybrid.

The US equity market presents an excellent example of a fragmented market.

5.1. US trading venues

Table 5.1 lists the largest venues and their trading volume (in million shares) on a typical recent day. Volume is reported separately for each listing venue. A listing for a US stock used to confer near-exclusive trading rights. If a stock were NYSE-listed, almost all of the trading would occur on the NYSE. Nowadays though, there are many places where a trade might occur. In view of the fact that not all of them are exchanges, these places are called trading venues or (in US regulation) market centers or trading centers. The trading venues differ by fee structure and trading protocols. These differences are important, and we will eventually discuss most of them. For the moment, though, the most important facts are: there are many venues and they are very competitive. You might recognize some of the names of the trading venues in the table, but a few are probably unfamiliar.
Table 5.1. Trading volume (million shares) on Thursday, September 3, 2015

<table>
<thead>
<tr>
<th>Executing exchange</th>
<th>NYSE</th>
<th>Nasdaq</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Volume (shares)</td>
<td>Percent</td>
</tr>
<tr>
<td>New York</td>
<td>888,211,019</td>
<td>25.5%</td>
</tr>
<tr>
<td>Chicago</td>
<td>32,179,766</td>
<td>0.9%</td>
</tr>
<tr>
<td>NYSE Arca</td>
<td>268,314,314</td>
<td>7.7%</td>
</tr>
<tr>
<td>Nasdaq</td>
<td>483,244,025</td>
<td>13.9%</td>
</tr>
<tr>
<td>Finra/Nasdaq TRF</td>
<td>1,028,036,386</td>
<td>29.5%</td>
</tr>
<tr>
<td>PSX</td>
<td>34,570,784</td>
<td>1.0%</td>
</tr>
<tr>
<td>NYSE MKT</td>
<td>4,054,051</td>
<td>0.2%</td>
</tr>
<tr>
<td>Boston</td>
<td>64,346,462</td>
<td>1.8%</td>
</tr>
<tr>
<td>BATS</td>
<td>220,062,627</td>
<td>6.3%</td>
</tr>
<tr>
<td>BATS Y</td>
<td>141,654,672</td>
<td>4.1%</td>
</tr>
<tr>
<td>EdgeA</td>
<td>83,144,604</td>
<td>2.4%</td>
</tr>
<tr>
<td>EdgeX</td>
<td>244,826,872</td>
<td>7.0%</td>
</tr>
<tr>
<td>Composite (total)</td>
<td>3,488,591,531</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Source: Wall St. Journal Online (Market Data Center, U.S. Stocks, Markets Diary)

For all listings, the largest entry is “Finra ADF”. Finra (or FINRA) stands for the Financial Industry Regulatory Authority. “ADF” stands for “Alternative Display Facility”. This system provides a trade reporting channel for venues that aren’t exchanges. This is a catchall category that covers trades occurring by many diverse means, but by most accounts, the largest contributors to ADF-reported volume are “dark pools”. We will discuss the operations of dark pools in Chapter 8. They are regulated, but in comparison with most markets they have low transparency.

5.2. Trading in fragmented markets.

Traders face great challenges in navigating a fragmented market. Different markets might have different prices. Who, at the moment, is posting the highest bid, or the lowest offer price? If the investor wants to make her own bid or offer (using a limit order), where and how will it be advertised? If someone sees her price, where can they send an order to trade against it?

Around 2005, the US adopted an overarching set of rules governing trading in US equity markets. Labeled “Regulation NMS” (for National Market System) and often simply referred to as “Reg NMS”, it establishes a framework that ties the separate markets together in a fashion sometimes called “virtual consolidation”.

In the language of Reg NMS, any place (or system) where a trade might occur is termed a market center. Although the various market centers often trade the same securities, they differentiate themselves according to trading rules and procedures (protocols) or by fees charged.
The centers are linked by *market information systems, access systems, and routing systems*. Market information systems communicate trade (last-sale) reports, current quotes and other information from the market centers to users. Access and routing systems go in the other direction, transmitting users’ orders to the market centers.

For New York and American Stock exchange-listed issues, the most important market information systems are the *Consolidated Trade System* (CTS) and the *Consolidated Quote System* (CQS). CTS consolidates reports of trades (wherever they occur); CQS consolidates and broadcasts each market center’s best bid and offer (BBO). Similar (but independent) systems exist for NASDAQ-listed securities. Vendors such as Bloomberg and Reuters purchase these feeds and redistribute the data to their customers. Of particular interest in the quote data are the highest bid at any given time, the *National Best Bid*, and the lowest ask price, the *National Best Offer*. The *National Best Bid and Offer* (NBBO) are important benchmarks for brokers and traders.

The market information systems are one-way; they do not provide the means for the investor to send in an order to execute against the NBBO. They are also broadcast systems. A message is not targeted to a specific recipient. They do not enable a market center to report the outcome of a received order back to its originator.

To accomplish these functions, investors rely on *access systems*. Access systems link brokers to market centers, and link the market centers to each other. There is little or no consolidation of these systems. Instead, there’s a collection of point-to-point communication systems that can convey executable orders and reports.

The systems that guide orders to the market centers where they are likely to be executed at favorable prices are called *routing systems*. Their intelligence comes from combining market information with situation-specific rules and practices. A customer order first arrives at the broker’s routing system, which may send it to another broker or a market center or another based on where the stock is listed, who is showing the best bid and offer, and the market center’s relationship with the broker. The receiving broker or market center may send it on to another, and in this fashion an order can make multiple hops before it arrives at its final destination. The routing is usually transparent (particularly to the retail customer), but it can take a bit of time. Customers can bypass the routing process by directing their orders to a particular destination. The downside is that the decisions made in the routing system often work to the customer’s advantage, and the receiving market might not be able to execute the order.

### 5.3. The NBBO (National Best Bid and Offer)

The current NBBO is generally indicated prominently on our trading screens. It is computed by the Consolidated Quote System and broadly disseminated. In many situations, though, such as transaction cost analysis, we need to determine the NBBO at some precise time in the past. How is this computation performed?

Consolidated quote data consist of a series of time-stamped records each of which contains a bid, an offer, and an exchange identifier. The records usually contain other information as well: the size (number of shares) at the bid, the size of the offer, and various modifiers (condition codes), but the bid, offer and exchange are the most important fields.

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9 The website for Essex Radez, a financial data provider, is a good online source of information about the various market information systems ([www.essexradez.com](http://www.essexradez.com)).
An exchange (or similar quoting venue) enters a new record into the stream whenever some feature of its quote changes (or is cancelled). This new record replaces all information on the previous record, and it is presumed valid until the next update. The previous record is valid until the new record arrives.

To determine the NBBO at a given time, we need to determine which exchanges were quoting, and when the most recent update for each exchange occurred. To determine the set of exchanges, we usually need to work forward from the start of the day. To determine the most recent update for each exchange, we work backwards from our reference time.

Table 5.2 Example computation of the NBBO

<table>
<thead>
<tr>
<th>Quote record</th>
<th>Computations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>Bid</td>
</tr>
<tr>
<td>9:31</td>
<td>70.00</td>
</tr>
<tr>
<td>9:32</td>
<td>70.05</td>
</tr>
<tr>
<td>9:33</td>
<td>69.90</td>
</tr>
<tr>
<td>9:34</td>
<td>70.00</td>
</tr>
<tr>
<td>9:35</td>
<td></td>
</tr>
<tr>
<td>9.50</td>
<td>70.10</td>
</tr>
</tbody>
</table>

Table 5.2 provides, on the left-hand side, a sample record of quotes. Suppose that we want to determine the NBBO at 9:35. First, we scan from the start of the day to determine which exchanges were actively quoting. Next, we set up a table for the bids and a table for the offers, with a heading corresponding to each exchange. Then, for each exchange, starting from 9:35 we scan backwards to determine each exchange’s current bid and offer. These are given in the italicized row at 9:35. The NBB is the highest bid, equal to 70.00; the NBO is the lowest offer, equal to 70.10. Both A and B are at the best bid; A is alone at the offer.

Note that in this determination that we are determining the max and min (highest and lowest) across exchanges (horizontally) at a point in time. We are not determining the max and min across time (vertically). The highest value across time for the bid, for example, is 70.05. This is not the NBB as of 9:35, though, because the 70.05 originated from exchange B at 9:32, and B superseded this price at 9:34 with a bid of 70.00.

5.4. Market-wide priority practices

The priority rules in a single limit order book are straightforward: price, visibility, and time. But how do these priority rules play out when there are multiple exchanges – and multiple limit order books. How do priority rules play out among multiple markets?

Most importantly, there is no overall coordination mechanism that consolidates the individual books so that we effectively have a single book where price, time and visibility priorities are observed. The priority rules that hold in a single book do not prevail across markets.
The following situations can happen:

- Violation of price priority. A limit order to buy at a price 100 on exchange A might be executed even though there is, at the same time, a limit order to buy at a price of 101 on exchange B.
- Violation of visibility priority. An undisplayed order at a price of 100 might be executed on exchange A even though there are quantities visible at 100 on exchange B.
- Violation of time priority. A limit order to buy at a price of 100 that was entered at 10:00 AM on exchange A might be filled before an order to buy at 100 that was entered at 9:30 AM on Exchange B.

In a single book, knowing that our order will always get executed before other orders that are priced less aggressively, entered subsequently, or not displayed encourages us to promptly post visible and aggressively priced limit orders. If these priority principles are violated cross-market our incentives are reduced.

The violation of price priority described above is called a *trade-through*. The person bidding at 101 on exchange B is traded-through when there’s an execution at 100 on exchange A. The seller on exchange A is disadvantaged because she sold at 100 when she could have sold at 101. The buyer on exchange B is disadvantaged because he is deprived of an execution on terms that would have been acceptable to him. Only the buyer on exchange A is better off.

When all trading occurs face-to-face on a floor market, the occurrence of a trade-through, and the identities of the parties to it are usually clearly evident. Suppose that Alice bids 100, Brian bids 101 and Cathy sells to Alice at 100. Brian can observe and protest. One remedy is as follows: Cathy’s sale to Alice stands, but Cathy also “owes Brian a fill,” that is, Cathy must find shares to sell to Brian at 101.

### 5.5. Order protection under Reg NMS

The self-policing of trade-throughs that arose naturally in most floor markets did not survive the transition to dispersed electronic markets. Network latencies (delays) coupled with the rapid pace of trading make it difficult to determine exactly what the bids and offers actually were at the precise instant that a trade occurred.

Trade-through protection is nevertheless generally thought to be such a desirable feature of a market that some attempt to preserve and enforce it is warranted. The most important current rule in this respect is the SEC’s Regulation National Market System (“Reg NMS”). We’ll defer a full discussion of this rule to Chapter 18, but right now we’ll examine the *order protection rule* (also called the “trade-through rule”) component of the regulation.

The Reg NMS order protection rule defines a class of protected bids and offers, practices that guard against trade-throughs of these orders, and assigns responsibility to market centers. The rule is sometimes described as prohibiting trade-throughs. This would imply, though, that if we observed one, someone would have violated the rule. This is not the case. Trade-throughs continue to occur, for some very good reasons, as we shall see. The rule does not implement “trade-through prohibition,” but we believe that with the rule we have fewer of them than we’d experience without a rule.

To be protected, an order must be visible (not hidden), at the market center’s best bid or offer (not necessarily the National Best Bid and Offer), and accessible (available for automatic execution). Orders priced away from the market’s BBO and hidden orders (even if they are superior to the BBO) are not protected.
But even for protected quotes, trade-throughs are not prohibited. The rule is worded so that markets shall “establish, maintain, and enforce written policies and procedures reasonably designed to prevent” trade-throughs. In practice, this means that a market must route quantities sufficient to avoid trade-throughs, based on what it can see. The rule essentially calls for a good faith attempt.

As an illustration, Table 5.3 gives the bid books in two markets. Order submission times aren’t relevant in this discussion, so we’ve omitted them.

<table>
<thead>
<tr>
<th>Bid</th>
<th>Trader</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Bid</th>
<th>Trader</th>
<th>Quantity</th>
<th>Visibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.40</td>
<td>Amy</td>
<td>200</td>
<td>Hidden</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.39</td>
<td>Alan</td>
<td>300</td>
<td>Display</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50.38</td>
<td>Ava</td>
<td>500</td>
<td>Display</td>
<td>Beth</td>
<td>100</td>
<td>Display</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Anna</td>
<td>200</td>
<td>Hidden</td>
<td>Ben</td>
<td>200</td>
<td>Display</td>
<td></td>
</tr>
<tr>
<td>50.37</td>
<td>Arnold</td>
<td>300</td>
<td>Display</td>
<td>Beverly</td>
<td>300</td>
<td>Display</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Alma</td>
<td>100</td>
<td>Display</td>
<td>Belle</td>
<td>100</td>
<td>Display</td>
<td></td>
</tr>
</tbody>
</table>

The top of A’s visible book is Alan’s bid of 50.39. This is protected. (Amy’s bid is higher, but it is hidden, and therefore not protected.) The top of Market B’s visible book is 50.38. Both Beth’s and Ben’s orders (a total of 300 shares) are protected. Assuming that these are the only two markets, the NBB is 50.39. Note that Beth’s and Ben’s orders are protected even though they aren’t priced at the NBB.

In the example, suppose Dana sends to Market B an order to “sell 100, limit 50.37.” An execution against B’s bid book (against Beth’s order at $50.38) would trade through the protected bid on market A. So B won’t allow this to happen.

Beyond attempting to avoid the trade-through, though, Reg NMS imposes no further requirements. Depending on Dana’s instructions (and Market B’s default practices), Dana’s order might be routed to Market A for execution, or cancelled. If Dana has the full picture of the market, of course, she (or her broker) will send her order to market A, where she knows that she can get a fill no worse than $50.39. But in a rapidly moving market, she (and her broker) might not be aware of this opportunity.

Executions against unprotected orders can sometimes lead to apparent violations of the rule. This can sometimes be confusing. Suppose that Market B receives an order to sell 600 shares limit 50.37. Any execution against its own book would cause an execution at 50.38. This would trade through A’s protected bid at 50.39 (Alan’s order), and so would be forbidden. In order to execute even part of the order, B would have to send (route) some portion of it to A. But how much?

In this situation, the rule requires B to route (to A) a quantity equal to the amount of A’s protected bid, that is, 300 shares. Assume that B does this, and executes the remainder against its own book. The resulting outcome is:

- On exchange A, 200 shares execute at 50.40 (against Amy’s hidden order), and 100 shares execute at 50.39 (against Alan). Alan has 200 shares remaining to buy, limit 50.39.
On exchange B, 100 shares execute against Beth (at 50.38), and 200 shares execute against Ben (also at 50.38).

Now this looks like a trade-through. Alan’s entire order was considered a protected bid, and since 200 shares remain unexecuted, these too are protected. Didn’t B’s executions at 50.38 trade through Alan’s remaining protected shares? Technically, B’s executions might well be considered trade-throughs. The order protection rule, however, only requires market B to make a good faith effort to avoid a trade-through. In this situation, it is considered sufficient for B to attempt to execute against A’s protected orders. This is what B has done. From B’s perspective, market A could have executed the 300 shares routed to it against Alan’s entire order. From market A’s perspective, of course, this is not possible: A must satisfy the hidden bid at a better price. But this is not market B’s problem or responsibility.

Here’s a related situation. Suppose that B receives an order to sell 1,000 shares limit 50.37. It could, as indicated above, route 300 shares to A, and execute the remaining 700 against its own book. But suppose instead that in a burst of generosity B sends 500 shares to A, and executes 500 shares against its own book. The resulting executions would be:

- In market A, 200 shares trade at 50.40 (against Amy), and 300 shares trade at 50.39 (against Alan). The best bid remaining on A’s book is 50.38 bid for 500 shares (Ava).
- In market B, 100 shares trade at 50.38 (against Beth), 200 shares at 50.38 (against Ben), and 200 shares at 50.37 (against Beverly).

This is all completely consistent with the rules. Now it could be argued that after A’s executions, Ava’s bid has become the best, and is therefore considered protected. From this perspective, B’s execution at 50.37 is a trade-through. But think for a moment. In a series of executions (like the one on A) that walks through the book, each execution is occurring at a price that is, at the moment of execution, “the best”. If this is considered sufficient for protection, then we’re really extending protection not just to orders at the top of the book, but to all orders in book. This is beyond the intent of the rule.

One way of rationalizing the situation is to group the events in two steps. When a market receives an order, it executes the order against its book, possibly against hidden orders, possibly at multiple prices. Then, after any executions have occurred, the market posts its best bid and offer based on its remaining visible orders. Only at this point are these bids and offers considered to be protected. They are not considered protected while the market is processing the original order.

Under the rules, the primary responsibility for avoiding trade-throughs falls on the market centers, who must constantly monitor each other’s protected quotes. All this monitoring and checking, of course, can slow down the processing of an order, often to the submitter’s disadvantage. For this reason, Reg NMS allows an alternative procedure whereby the order submitter can perform the cross-market check and avoid trade-throughs by routing orders to multiple markets simultaneously. The use of such intermarket sweep orders is discussed in Section 18.2.

5.6. Summary

When there are multiple market centers, trading strategies become more complex. The basic make-or-take decision faces the additional question of “where?” (the routing problem). Priorities and procedures that are clear in the context of a single market become much less so with multiple destinations. In U.S. equity markets, the trade-through protection afforded under Reg NMS is a partial remedy, but this does not apply to other markets and other securities.
Is this complexity really necessary? Would trading be simpler if all activity was forced into a single well-managed system? The benefits of simplicity are indeed appealing, but there is a cost. The single system would be a monopoly, an entity protected and privileged under law. Experience with near-monopolies in trading suggests that they are costly, inefficient and reluctant to innovate. Chapter 18 provides further discussion.

Summary of terms and concepts

Fragmentation and consolidation; types and purposes of linkage systems (market data/information, access, routing); direct (market) access; the Consolidated Trade System; the Consolidated Quote System; the National Best Bid and Offer (NBBO); trade-through; trade-through prevention provisions of Reg NMS;

Problems and exercises

Problem 5.1 Ticker EAS is traded on the Gulf, Pacific and Atlantic stock Exchanges. The sequence of bids in the consolidated record is:

<table>
<thead>
<tr>
<th>Time</th>
<th>Exchange</th>
<th>Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00:01</td>
<td>Atlantic</td>
<td>23.33</td>
</tr>
<tr>
<td>10:00:02</td>
<td>Pacific</td>
<td>23.32</td>
</tr>
<tr>
<td>10:00:03</td>
<td>Gulf</td>
<td>23.34</td>
</tr>
<tr>
<td>10:00:04</td>
<td>Pacific</td>
<td>23.40</td>
</tr>
<tr>
<td>10:00:05</td>
<td>Gulf</td>
<td>23.45</td>
</tr>
<tr>
<td>10:00:06</td>
<td>Pacific</td>
<td>23.44</td>
</tr>
<tr>
<td>10:00:07</td>
<td>Atlantic</td>
<td>23.43</td>
</tr>
<tr>
<td>10:00:08</td>
<td>Gulf</td>
<td>23.40</td>
</tr>
</tbody>
</table>

Construct the NBB at each time.

Problem 5.2 Protected quotes. Exchanges A and B have the following limit orders (displayed/visible, unless otherwise indicated).

<table>
<thead>
<tr>
<th>Price</th>
<th>Exchange A</th>
<th>Exchange B</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.10</td>
<td>100 sh, hidden</td>
<td></td>
</tr>
<tr>
<td>50.09</td>
<td>300 sh</td>
<td></td>
</tr>
<tr>
<td>50.08</td>
<td></td>
<td>200 sh</td>
</tr>
<tr>
<td>50.07</td>
<td>400 sh</td>
<td>500 sh</td>
</tr>
</tbody>
</table>

a. Which orders are protected (against trade-throughs)?
b. If Exchange A receives an order, “Sell 500 shares, limit 50.07”, what are its obligations?
c. If Exchange B receives an order, “Sell 500 shares, limit 50.07,” what are its obligations?
Part II. Alternatives to Limit Order Markets

The electronic limit order book is probably the most widely used trading mechanism. There are nevertheless many securities and settings where it is augmented, or even displaced, by other mechanisms. Here, we’ll look at auctions, dealers, and dark mechanisms.
Chapter 6. Auctions

Auctions are widely used in markets for diverse goods and services. The more familiar auctions involve wine, art, and collectibles. In an art auction, buyers compete with each other, bidding higher prices until only one buyer remains. This person buys the item at his last bid price. Christie’s and Sotheby’s conduct art auctions in which most of the buyers are physically present. Their bidding is coordinated by an auctioneer, who identifies the highest bid (and the person who made it), suggests higher prices, and generally tries to maintain interest, excitement and drama.

Most art auctions are open outcry (also called English) auctions: bidders can hear the bids of others. In other versions, sealed bids are used: bids are collected in sealed envelopes, one bid per buyer; the envelopes are opened; and the highest bid is selected. The art auction is an ascending auction: a bid must be higher than the one that preceded it. Alternatively, in a descending auction, the price is initially set very high, and this price is reduced until one buyer claims it by calling out “mine”. The art auction is a seller’s auction (one seller and many potential buyers). Many public agencies use buyer’s auctions, where suppliers compete by offering lower prices. Klemperer (2004) discusses the alternative formats and their economic principles.

Although auctions are generally viewed as reliable, robust, and fair, they are subject to disruption by a number of possible manipulative strategies. A seller might use a shill to pose as an aggressive buyer, driving up the bids of others. Buyers can collude (agree beforehand) not to bid against each other.

Securities auctions differ from art and wine auctions in many ways, but many issues of design and concerns about manipulation are common to both.
6.1. Primary market auctions

The primary market refers to the process by which a security is initially sold (“placed”) by the issuer. It typically involves many identical items (bonds or shares of stock), one seller (the issuer), many potentially buyers, and the transaction is usually accomplished all at once. An auction would seem to be a sensible format.

A bid specifies price and quantity, such as, “$50 per share, for 100 shares.” The bids are ranked in price priority (highest bids have highest priority). This defines a demand function that resembles a staircase, descending to the right. For these orders:

<table>
<thead>
<tr>
<th>Trader</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alan</td>
<td>2</td>
<td>$29</td>
</tr>
<tr>
<td>Beth</td>
<td>3</td>
<td>$28</td>
</tr>
<tr>
<td>Cam</td>
<td>1</td>
<td>$27</td>
</tr>
<tr>
<td>Dana</td>
<td>4</td>
<td>$26</td>
</tr>
<tr>
<td>Ed</td>
<td>2</td>
<td>$25</td>
</tr>
<tr>
<td>Fiona</td>
<td>3</td>
<td>$23</td>
</tr>
</tbody>
</table>

The corresponding demand function is:

The supply is however many units (bonds, shares, whatever) we have to sell. For this sort of auction, the supply does not depend on price, so it is graphed as a vertical line. The figure indicates a supply of nine units.

The supply and demand functions cross at a price of $26. All buyers who bid above this price (Alan, Beth, and Cam) have their orders satisfied in full. Dana gets a partial fill: she gets three of the four units that she wanted. An auction such as this is used by the Federal Reserve Bank of

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10 In contrast, most limit order markets, of the sort described in Chapter 3, are secondary markets. That is, they involve trade between a buyer and seller, neither of whom is the issuer of the security.
Auctions

New York to sell U.S. Treasury securities. Auctions are sometimes used for equity initial public offerings (IPOs). The largest one to date is Google’s IPO. But this was a rarity.

6.2. Opening and closing auctions

Regular trading hours for US stocks are 9:30 to 16:00 Eastern Time. There are certainly many ways to trade outside of this period, but even with these opportunities, many investors prefer the regular session.

For a security that normally exhibits low trading activity, it may not be necessary to have a special opening procedure. There may be few orders entered prior to 9:30, and no opportunities for matches until later in the day. It is also possible that as we approach 16:00, the book has few orders and a wide spread, with no strong interest on the buy or sell side of the market. Often, though, there is strong trading interest at beginning and end of the session, with many buyers and sellers who want to trade at the open or at the close. There are sensible reasons for this.

At the open, there are often numerous and large orders that have accumulated overnight. This is particularly true when there’s been an overnight or pre-opening news announcement.

Trading interest at the close tends to be even stronger:

- Redemptions and purchases of mutual fund shares are based on net asset values, which generally represent closing prices. Index funds also need to ensure that their composition closely matches that of the index at the close.
- Index futures contracts and other cash-settled derivatives are usually settled at closing index prices (although some use opening prices). Index arbitrageurs seeking to unwind their stock positions will usually need to sell or cover their shorts at closing prices.
- A wide range of index derivative products, such as leveraged and inverse ETFs seek to deliver some multiple of the close-to-close index return.

The need for special opening or closing procedures might not be obvious. Why don’t we simply turn on the limit order book at 9:30 and turn it off at 16:00? If we did this, we’d expect to see enormous order flow at these times. Aside from the strains that this might put on the networks, there might be hundreds of trades at different prices occurring nearly simultaneously. This volatility goes contrary to the outcomes we expect from a fair and orderly market.

To ensure that everyone receives the same price, the mechanism must consolidate the total buying and selling interest. The auction used in this context is a double-sided auction.

The single-price double auction (SPDA) is simple, in principle. Buyers enter their demands; sellers enter their offers. The system computes the supply and demand curves in real time, as the orders arrive and change. The state of the market can also be communicated back to the buyers and sellers: either the full supply and demand curve, or (more likely) an indication of the clearing price. When everyone is through entering and modifying their orders, the system finds the price and quantity at which supply and demand are equal.

We’ll now look a bit closer at this process. First there is a period of order accumulation. Buyers and sellers enter limit (priced orders). Some markets also accept orders without prices, designated as market-on-open (MOO) and market-on-close (MOC) orders.

Buy orders are ranked by price, high to low (essentially willingness to pay). If the rules permit market orders, they are effectively assigned an infinite price. The ranked orders are placed on the horizontal (quantity) axis, and cumulated. This defines the demand curve (in reality, a step function).
For example:

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Bid</th>
<th>Quantity</th>
<th>Cumulative Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brian</td>
<td>$10</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Beth</td>
<td>$8</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Ben</td>
<td>$6</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Bev</td>
<td>$4</td>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Seller</th>
<th>Quantity</th>
<th>Asking</th>
<th>Cumulative Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seth</td>
<td>3</td>
<td>$2</td>
<td>3</td>
</tr>
<tr>
<td>Sara</td>
<td>5</td>
<td>$4</td>
<td>8</td>
</tr>
<tr>
<td>Sasha</td>
<td>4</td>
<td>$6</td>
<td>12</td>
</tr>
<tr>
<td>Sam</td>
<td>4</td>
<td>$10</td>
<td>16</td>
</tr>
</tbody>
</table>

These are graphed as:

To clear the market, the supply and demand functions are combined and we search for the point at which the trading volume is maximized: the point that lies farthest to the right, in the set of all intersection points. As usually drawn in introductory economics class, the demand and supply are smoothly sloping curves, and the intersection consists of a single point. With discrete prices and quantities, though, these curves are actually staircase (step) functions, and determination of the clearing price can be a bit more involved.

To start with, at any price, we can read off the supply and demand. The smaller of these quantities defines the matched volume. For example, at a price of $9, demand is 4 units (Brian), supply is 12 units (Seth, Sara, and Sasha), and the matched volume is 4 units. The unmatched remainder is the imbalance. The imbalance is signed (as buy or sell). At a price of $9, the imbalance is 8 units, on the sell side. At $6, demand=14, supply=12, matched volume=12, there's a buy imbalance=2.

To determine the clearing price, we first find the price that maximize matched volume. (An exchange, after all, is in the business of trading, and higher volume corresponds to higher revenues.) In this example, that price is $6, for which the matched volume is 12. Immediately above $6, the matched volume drops to 10 units (as Ben drops out). Immediately below $6, matched
volume drops to 8 units (as Sasha drops out). In this case, maximization of matched volume defines a unique clearing price. This need not always be the case, however. Consider:

Here, all prices between $3 and $8 have the same matched volume – 8 units. So to handle these cases, we need a supplementary rule, which is: minimize the net imbalance. What if even after applying this supplementary rule, we are still left with multiple possible clearing prices? At this point, there is some variation in the rules. One common rule is to minimize the distance from some reference price. In the case of an opening auction, for example, we might pick the price closest to yesterday’s closing price. Minimizing the price change from a previous close is consistent with most investors’ preferences for lower price volatility.

Although simple in principle, real-world implementations face certain complexities, to which we turn next.

6.3. Clearing time

An open-outcry art auction usually doesn’t have a fixed stopping time. The bidding simply continues until only one bidder is left. The auctioneer calls out, “Going once, ..., going twice, ..., going three times, ... SOLD!” unless, of course, someone jumps back in with a higher bid.

Online auctions, though, and most opening auctions have a scheduled ending deadline, a fixed time, announced in advance. In many situations, though, a hard deadline simply induces traders to hold off entering their orders until shortly before the deadline.

To deal with the last-minute pile-up, online auctions often extend deadlines until a period of time (e.g. ten minutes) has elapsed with no changes in the bid. In an opening security auction, for example, we can imagine a similar procedure: waiting until the orders rest without modification for a decent interval before clearing the market.

In a securities market, though, information, prices, and traders desires are evolving constantly. On Friday, May 18, 2012, the public offering of Facebook stock was conducted, and the stock began continuous trading. The New York Times reported, “As NASDAQ’s systems were setting Facebook’s opening price, a wave of order modifications forced the exchange’s computers...
into a loop of constant recalculations. The firm was forced to switch to another system, knocking out some orders and delaying many trade confirmations."

A hard deadline may also make the market susceptible to manipulation. In the main example of this chapter, Brian wanted to buy 4 units, limit $10. Suppose that in addition to this bid, Brian makes a fictitious and deceptive bid of $11 for 10 units. With Brian’s two bids, the supply and demand functions look like this:

What happened to the other bidders? Beth, Ben and Bev simply got discouraged. Believing that Brian’s aggressive bids were likely to determine a final price of $10 or $11, well above their own limits, they did not bother to bid. Then, one instant before the deadline, Brian cancels his $11 order, leaving the market as:

Brian buys his shares at lower cost (that is, $4, instead of the $6 he would have payed earlier).
In online auctions, this is known as bid-shielding. When bidding opens on the sterling silver tea set, a bidder enters a price of $10, and then (via an accomplice or a separate online identity) bids $10,000. As the $10,000 establishes the high bid, no lower bids will be accepted. Immediately before the deadline, the $10,000 bid is cancelled, leaving the $10 bid the winner.

To discourage bid shielding and similar tactics, a market may randomize the clearing time. Even a small amount of uncertainty may be enough to discourage manipulations based on last-instant moves. There is some danger that the fictitious, manipulative bid (Brian’s $11 bid, for example) might actually be executed, with Brian paying a higher price for much more than he really wanted.

The London Stock Exchange describes their randomization procedure with a particularly thoughtful discussion (London Stock Exchange (2000)). Other markets that employ randomized clearing times include: the Tel Aviv Stock Exchange (Hauser, Kamara and Shurki (2012); Tel Aviv Stock Exchange (2013); Euronext Amsterdam Instinet (2010a); Euronext Lisbon Instinet (2010b); and Xetra Deutsche Börse Group (2011)).

In the U.S., however, exchanges don’t generally use random deadlines. Instead, they attempt to stabilize the price determination process by restricting the information available to auction participants and also by restricting order entry and modification immediately prior to the clearing.

We’ll examine the NASDAQ opening auction (“cross”). The open is timed for 9:30am. From roughly 7:00am onwards, traders can enter orders. These orders may be marked “on open” to ensure that they are held unexecuted until the open. On-open orders must be received before 9:28, and may not be cancelled. Beginning at 9:28, NASDAQ transmits matched volume and imbalance information. Between 9:28 and 9:30, NASDAQ accepts a special kind of order called imbalance-only. Imbalance-only orders are only in the direction of minimizing an existing imbalance. Similar procedures are used in the closing auction.

### 6.4. “Marking the close” / “Banging the close”

The closing price of a security is very important. It is used as a reference price for determining mutual funds’ net asset values (NAVs), the prices at which new shares are created and old shares are redeemed. It may be a reference price for cash-settled derivative contracts. It may be referenced to determine the acquisition price in a corporate takeover. Finally, margin calculations are often performed on the basis of closing prices, to determine whether the owner of a margined position has to put up additional cash.

In the usual art or collectible auction, anyone who deliberately overbid would simply be driving up the price he paid, with no obvious benefit. (If I pay $1 Million for painting when the next highest bid is $100,000, can I claim that I own a painting worth $1 Million?)

In the closing auction for a security, though, I may not care very much about overpaying if I have a much larger interest in some related transaction. In less-actively traded securities, for example, there might not be very many orders going into the closing auction, and entering one more large buy or sell order might move the closing price by a lot.

The litigation files of the Securities and Exchange Commission contain numerous instances in which an investor, facing a margin call that would require him to put up cash that he didn’t have, placed orders in the final moments of trading to affect the closing price. This is called “marking” or “banging” the close. It is illegal.
6.5. Use of auctions for illiquid securities
During regular trading hours, virtually all stocks in the US trade in a continuous market. Bids and offers may be placed, and matches may occur at any time.

There are strong considerations that favor continuous trading:
- News arrives continuously. Speculators want to trade on news.
- Hedgers often take a position in a security to offset risk. When they can’t trade, the risk must be borne.
- The management of a listed company might like the visibility that comes with belonging to a recognized index. Index membership is usually restricted to stocks that trade continuously.

But if trading interest in the stock is low, traders might not bother to participate in a continuous market. Why go to the trouble of entering, monitoring, and modifying a bid (as market conditions change), and exposing it to being picked off by low-latency traders, if it is unlikely that it will be hit by someone we might actually want to trade with?

To have any volume, a market needs to attract marketable orders. To do this, it needs reasonably priced limit orders. But traders won’t bother to post limit orders unless there’s a good flow of marketable orders. Below some critical mass, the book will simply be empty. There’s no trade and the market doesn’t get any trading revenue.

Auctions concentrate buying and selling interest: everyone comes together at the same time. The increased participation can support more trade and better prices. For these reasons, many markets use periodic call auctions for illiquid stocks, and don’t allow the stocks to trade via the continuous mechanisms.

The Euronext markets (Paris, Brussels, Amsterdam and others) run call auctions twice a day in illiquid securities (usually at the regular market open and close). These securities do not trade in the continuous market.

This chapter has examined auctions as a supplement to limit order books. Hendershott and Madhavan (2015) discuss their supplemental use in dealer markets.

6.6. High-frequency auctions
The continuous limit order book rewards speed. Traders compete to be the first to act on new information. The rewards to the winners here are sufficiently large that substantial resources are spent gaining advantages of mere milliseconds. Some view trading at this time scale as providing few benefits to overall social welfare, while encouraging an expensive “arms race” among technologically adept participants (Budish, Cramton and Shim (2015); Du and Zhu (2014); Schwartz and Wu (2013)).

An auction with no time priority concentrates buying and selling interest at a single point in time, and nullifies the reward for being first. It has been suggested that one possible way of keeping this property and also maintaining the essential advantages of continuous trading is to organize the market as a sequence of auctions with minute (or even sub-minute) periodicity.

6.7. Further reading
of empirical papers analyze opening auctions in stock markets (see Amihud and Mendelson (1991), and references therein).

### Summary of terms and concepts

Single-price call auctions, and how they’re run; manipulation in auctions; randomization; Nasdaq opening procedures (reference price, matched volume, imbalance); imbalance-only orders.

### Problems and exercises

**Problem 6.1 Clearing.** In an opening cross, the system receives the following on-open orders:

<table>
<thead>
<tr>
<th>Trader</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>Buy 200 shares, limit 20.10</td>
</tr>
<tr>
<td>Amy</td>
<td>Buy 200 shares, limit 20.00</td>
</tr>
<tr>
<td>Cathy</td>
<td>Buy 100 shares, limit 19.95</td>
</tr>
<tr>
<td>Brian</td>
<td>Buy 200 shares, limit 19.90</td>
</tr>
<tr>
<td>Ira</td>
<td>Sell 100 shares, limit 19.80</td>
</tr>
<tr>
<td>Gabe</td>
<td>Sell 200 shares, limit 19.90</td>
</tr>
<tr>
<td>Haley</td>
<td>Sell 300 shares, limit 19.95</td>
</tr>
<tr>
<td>Fiona</td>
<td>Sell 100 shares, limit 20.10</td>
</tr>
</tbody>
</table>

a. Construct the supply and demand schedules. At an indicative price of 20.00, what is the matched volume? What is the imbalance (direction and size)?

b. The clearing price is set to first maximize the matched volume and then minimize the net imbalance. What is the indicative clearing price? What are the matched volume and imbalance at the clearing price?

c. At the clearing price, who trades? What are the buyers’ and sellers’ trading profits?

d. Now suppose that prior to the single-price clearing, Brian and Gabe met and traded. (Brian bought 200 shares; Gabe sold 200 shares at 19.90.) Brian and Gabe don’t submit orders to the opening auction, but all other orders remain the same.

   i. What is the new clearing price?

   ii. What are the total buyers’ and sellers’ trading profits?

   iii. What is the efficiency of the new allocation relative to the allocation determined in parts b and c?
Problem 6.2 From orders entered for an opening auction, an exchange constructs the following demand and supply schedules. What is the market clearing price? At the clearing price, what is the matched volume? The buy or sell imbalance? The NYSE disseminates information based on a reference price that may differ from the clearing price. At a reference price of 9.70, what is the matched volume? The buy or sell imbalance?
Problem 6.3 The Back to the Futures Exchange is considering opening with a single-price double auction format. From the limit orders submitted to its brokers at the start of today’s trading, the exchange constructs the following demand (solid line) and supply (dashed line) curves. The price increment is $1 and fractional quantities are not allowed.

a. From this diagram, Beth’s limit order is buy/sell (circle one) ___ contracts, limit $____; Sara’s order is buy/sell (circle one) ___ contracts, limit $___.

b. At an indicative price of $27, the matched volume would have been ___ contracts and there would have been a buy/sell (circle one) imbalance of ___ contracts.

c. The exchange plans to maximize matched volume, and if two prices have the same matched volume, the clearing price will minimize the net imbalance.
   i. Under this rule, the clearing price would have been ___ with a matched volume of ___ contracts and there a buy/sell (circle one) imbalance of ___ contracts.
   ii. The total buyers’ profits are ___; the total sellers’ profits are ____.

d. When floor trading actually started today, Dave bought two contracts from Seth at $28. Following Dave and Seth’s trade, if Sara is the seller in the next trade, who are her potential counterparties? That is, among the remaining buyers (Dara, Brian, Beth, Belle, Bev, and Bonnie), which of them could trade with Sara at a price that would not result in a loss for either side?

e. Suppose that at the start of trading, the only three trades are the following

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Price</th>
<th>Quantity</th>
<th>Seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>$28</td>
<td>2</td>
<td>Seth</td>
</tr>
<tr>
<td>Dara</td>
<td>$27</td>
<td>3</td>
<td>Sasha</td>
</tr>
<tr>
<td>Brian</td>
<td>$26</td>
<td>2</td>
<td>Sam</td>
</tr>
</tbody>
</table>

What is the efficiency of these trades, relative to the proposed single-price auction?
Chapter 7. Dealers and Dealer Markets

The last chapter examined the use of auctions in two roles. Auctions can assist limit order markets (in opening and closing continuous trading sessions). They can also serve as the sole trading mechanism, as in the case of the periodic call auctions used for securities with low natural trading interest. The present discussion of dealers takes a similar approach. Like auctions, dealers can both assist limit order markets, and also serve as the sole or dominant trading mechanism.

The starting point of our discussion is the definition of a dealer and the basic conflict of interest that informs their activities. We next describe the roles of two kinds of dealers that have historically played central roles in U.S. equities markets: the exchange specialists and the NASDAQ market makers. In the landscape of today’s equity markets, though, these traditional dealers play a much smaller part. Technological forces have shifted the dealer functions from specialists and market makers to proprietary (“prop”) trading firms.

We are still grappling with the aftermath of this transition. The traditional dealers were heavily monitored and regulated; the newer ones are presently subject to much less oversight. The market-making process has been broadly implicated in several recent market “stress” events (the flash crash of May 10, 2010, and the near-failure of Knight Capital in 2012). These events have raised regulatory concerns, and historical parallels are evident. How did we expect the traditional market makers to behave, and should their replacements assume similar responsibilities?

Once we move beyond equities, we encounter markets where dealers are central. The markets for foreign exchange (FX), so-called over-the-counter derivatives (swaps and off-exchange options), corporate bonds, and so forth, are dominated by their dealer networks. These markets have not for the most part experienced the technology revolution that has reshaped the exchange markets. Are they likely to be transformed, or will they persist as something close to their present form?
7.1. What is a dealer?

A dealer is a financial intermediary who specializes in serving as a *counterparty* to customer trades: buying when the customer wants to sell, and vice versa. In this role, a dealer may also be described as a market-maker or liquidity-provider.

The function of a dealer is very different from that of a broker. A broker represents a customer order, conveying it to the market, and acting as its agent. A dealer executes the order against his own account.

Despite this difference, it is sometimes natural for the same person or firm to serve in both dealer and broker capacities. To see how this might happen, suppose that the NBBO is 20 bid, offered at 21 and the customer sends in an order to buy limit 21. The broker can pass the order to a market center posting the NBO and get a fill at 21.

But suppose that the broker (or someone at his firm, e.g., a proprietary trading desk) wants to sell the stock and is willing sell at a price better than 21, say 20.90. It would be very natural for the broker to sell directly to the customer.

Although this situation arises quite naturally, it places the broker in a position of conflicted interests. Acting as a broker, his obligation is as agent to his customer buyer, attempting to get the lowest price possible. But acting as a dealer, he wants to get the best terms of trade for himself, selling to the customer at a price as high as possible. In the above example, the broker is willing to accept 20.90, but the customer is willing to pay up to $21. Why not sell to the customer at 20.99? At 21?

The process of simultaneously representing customer orders and trading on one’s own behalf is called dual trading. In all markets it is highly regulated. But we’ll shortly see that there are a lot of gray areas.

7.2. Dealers in limit order markets

In our earlier discussion of limit order markets, the bid and offer books were generally assumed to be full of potential buyers and sellers. This is not always the case. As conditions change, limit orders may need to be repriced. This process requires ongoing monitoring and adaptation to new information. If the probability of execution is low, the cost of monitoring and updating may be too high to justify exposing a limit order in the first place. There are no bids or offers. Potential buyers and sellers checking the market’s quotes find nothing, and eventually they stop checking.

If there were even one bid and one offer, a potential trader might enter their own limit order. With sufficiently aggressive limit orders, a marketable order will eventually arrive and we’ll have an execution. In this sense, a bid and offer can encourage other traders to enter and sustain a market going forward. As an exchange’s revenue depends on trading volume, it is in the exchange’s interest to facilitate this process. The exchange may therefore appoint someone to make the market. This agent is variously called a designated market maker (DMM, the most common term), a liquidity provider or (formerly, on US exchanges) a specialist.

When there are insufficient customer limit orders, the DMM is required to post his/her own orders, priced aggressively enough to make a tight spread. The definition of “tight” here depends on the market, the security, and market conditions. This is DMM’s primary responsibility, but there may be others.

While there is a consensus that DMMs can usefully augment limit order book, there is less agreement about how they should be incentivized and regulated. The responsibility to make a market is often burdensome (in a volatile market, for example). What do the DMMs get in return? How should they get compensated?
Sometimes the arrangements are direct. In the Euronext markets, DMMs can be paid by the listing company. The argument in favor of this practice is that the beneficiaries of the DMM are the shareholders of the listed company, so they (through the company) are the logical ones to bear the cost. The argument against is that the arrangement links a large trader (the DMM) closely with management (who are likely to possess inside information). By reason of this moral hazard problem, U.S. regulators have traditionally opposed subsidized market makers. Recently, however, the practice is being permitted for certain Exchange Traded Products, which are less subject to insider trading concerns.11

More typically, however, DMMs are compensated indirectly, through a variety of subsidies, rules and other practices that attempt in some broad fashion to enable them to recover the costs of the services they provide. These mechanisms are rarely isolated features or privileges. They are instead deeply embedded in the specification of the market-maker’s role. This will become clearer as we turn to the discussion of U.S. exchange specialists and NASDAQ market makers.

7.3. Exchange Specialists

The New York Stock Exchange was historically a floor market. According to legend, the specialist system began in the 19th century when a member suffering from a broken leg decided to remain in one spot and specialize in trading a small number of stocks. Whatever the truth of this story, the acceptance and adoption of this system must have followed from stronger economic forces at work. In any event, by the advent of federal regulation in the 1930s, the specialist system was well established at the NYSE and other stock exchanges. Around 2005, the role of the specialist was redefined and they were renamed designated market makers (DMMs).

Our discussion will start with an examination of the specialist system in the last half of the 20th century, when floor markets were still widespread. Why start here? The specialist of this era was a single person placed in an extraordinarily powerful position, at the center of trading activity. This power, and the rules designed to monitor and shape it, represented a balance of public and private interests involving certain principles and trade-offs that have informed regulatory debates well into the era of high-frequency trading. When we think about how today’s market makers should be regulated, the rules of the former specialist system are still a relevant touchstone.

The specialist (around, say, 1975)

The specialist was an independent trader, buying and selling on his own account (not an Exchange employee). Each listed stock generally had one specialist, but a specialist could handle multiple stocks. Since the NYSE accounted for the preponderance of trading volume in its listed companies, the specialist truly stood at the center of the trading process. The specialist was broadly charged with “maintaining a fair and orderly market,” but was also subject to a wide range of specific affirmative (positive) and negative obligations.

- **Making a market.** The specialist was expected to always post a bid and offer, not necessarily for large sizes, but with a spread that was reasonably (given the characteristics of the stock) narrow. This is the essential expectation, of course, on all market makers.

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11 Exchange traded products (ETPs) include exchange traded (mutual) funds, like the SPY, and also exchange traded notes (ETFs and ETNs). An ETP usually represents a portfolio of stocks, and its value depends on readily observable market prices. In the case of ETPs, the DMMs are paid by the sponsor of the product, the firm that constructed it and manages it.
• **Avoiding “destabilizing trades.”** The specialist was not supposed to trade actively, that is, by hitting/taking public limit orders. The ideal was neutrality, bringing buyers and sellers together in a way that encouraged the emergence of the “natural” price.

• **Public priority.** A specialist was supposed to yield to customer bids and offers at the same price. For example, if the specialist were bidding 50, and a customer were to enter a limit order to buy at 50, the customer’s bid would have priority over the specialist’s. Nowadays we think of the limit order market as a self-contained trading mechanism. But in a floor market, a limit order book is simply a collection of orders written on pieces of paper. Each order needs an advocate, an agent, to represent it in the trading crowd. The specialist was this agent. Public priority follows from the belief that an agent should not trade ahead of those whom he is representing.\(^{12}\)

• **Crossing public orders.** If market buy and sell orders arrived simultaneously, the specialist was supposed to match them directly, generally at the midpoint of the bid and offer.

• **Price continuity.** Large price swings are sometimes viewed as evidence of a chaotic market. To avoid these jumps, the specialist was required to bridge large changes by a series of trades at intermediate prices. These were often unprofitable, especially when the gap was large.

The specialist was sometimes described as a monopolist. In fact, his market power was more circumscribed. The principle of public priority meant that customer bids and offers were often effective competition against his own. As a result, the specialists’ trades did not generally constitute the preponderance of trading activity. Most trades were customer-to-customer, without the direct involvement of the specialist.

The specialist also enjoyed certain advantages. As the agent for the limit order book, he knew the contents of the book. As he was prohibited from disclosing the book to others, this constituted a distinct edge. In his role as agent for incoming market orders, the specialist also enjoyed a first mover advantage. That is, when a market order arrived, he could decide whether to take the other side of the order or let the order execute against the book. This was not quite a right of first refusal because public priority might compel him to offer a better price. For example, if there were a customer limit order offering stock at $50 and a market buy order arrived, the specialist could not sell to the market order at $50. He would have to offer a better price, say, $49 7/8. (Until roughly the end of the 20th century, the tick size in U.S. equity markets was $1/8 = $0.125.)

The **NYSE’s Designated Market Makers**

By the 1990s, the specialist system was coming under pressures of technology, competition, and increased regulatory scrutiny. In the early 2000’s, the NYSE fundamentally changed its structure. The specialist was replaced by a designated market maker (DMM). Two prominent responsibilities remain. The DMM must maintain a fair and orderly market, and post a bid and an offer. Other rules are quite different. The specialist is no longer the agent for the limit order book, and the principle of public priority has been replaced by **parity** (essentially that the specialist can share executions alongside the limit order book).

\(^{12}\) The practice of agents (brokers) trading in advance of their principals (customers) is called front-running. In many circumstances front running is against market rules. Nowadays, however, the term is more broadly used to describe any situation where someone is trading in advance of another trader’s order. This might not be desirable, but if there is no agency duty, it is not usually deemed illegal.
NASDAQ Market Makers

NASDAQ's roots were in a dispersed network of loosely-linked ("over-the-counter", OTC) brokers and dealers. Over time, the network was computerized and the role of market makers was formalized.

Some NASDAQ members were classified as order entry firms. An order entry firm could accept customer orders, but it could not trade against them as principal. Instead, the order entry firm would typically pass the orders to a NASDAQ market maker for execution. A NASDAQ market-maker in a particular stock would meet certain minimum capital markets agree to make and maintain a bid and offer in that stock. A NASDAQ-listed company had to have at least two market makers. A large stock (like Microsoft) might have over fifty. A market-making firm might have its own retail arm, but others, variously called "wholesalers" or "OTC market-makers" would specialize in handling the orders sent to them by other firms.

This industrial structure is still a pretty good description of the industry. Most firms familiar to retail investors (like Charles Schwab or E-TRADE) are primarily order entry firms. The wholesalers include the large sell-side banks (such as JP Morgan, Credit Suisse, and Goldman Sachs), and firms that are more focused on market-making (such as Automated Trading Desk, Cantor Fitzgerald, and Knight Capital).

NYSE-listed and NASDAQ-listed stocks trade in a similar fashion, but this was not historically the case. Prior to the 1990s NASDAQ did not have a central limit order book. Customer bids and offers were not displayed, and direct trade between a buyer and seller was rare. The handling of customer limit orders became a point of regulatory conflict. Eventually the SEC’s order handling rules compelled NASDAQ market-makers to yield to public orders, display them, and make them available for execution (U.S. Securities and Exchange Commission (1996)).

7.4. De facto market makers ("market makers in fact, if not in name")

Customer bids and offers are competition for a dealer's own quotes. The principles of public priority and the order handling rules attempt to protect the customers' interests. In a historical context, this was sensible. Members of the floor-based exchanges enjoyed a centrality to the market process that often worked to the disadvantage of off-floor customers. These customers were generally viewed as long-term investors. Who could hope to make short-term proprietary trading profits in competition with the floor?

When orders were communicated verbally or over slow networks, a human market maker on a floor market enjoyed a latency (speed) advantage over public bids and offers. With the advent of low-latency computer systems, though, the competition from off-floor bids and offers became much stronger.

The competition between the (human) market makers and off-floor customers reached a tipping point with the introduction of Reg NMS (2005). As discussed in Section 5.5, Reg NMS mandates trade-through protection of bids and offers, subject to certain restrictions. It only applies to the top of the visible book (a market center's best bid and offer). An additional restriction, though, concerns access. For a bid or offer to be protected, it must be immediately accessible for execution. The bids and offers of non-automated markets or market participants are not covered.

This elevated the relative status of customers who invested in the fastest technology. Their bids and offers largely supplanted those of the traditional market makers. These customers became the new "de facto" market makers. The trend was not limited to the US. Menkveld (2013) examines the entry of a new electronic market (CHI-X) into Europe: "[Three] events coincided:
[the] new market’s take-off, the arrival of a large HFT [high frequency trader], and a 50% drop in the bid-ask spread.”

Beginning in 2007, the US entered a financial crisis in which one major investment firm (Lehman) failed, others were merged into stronger partners (sometimes under regulatory pressure), and government guarantees were deemed necessary to restore confidence in the financial system as a whole. During this crisis, the markets for some securities (notably mortgage-backed and auction-rate) performed poorly, with greatly reduced liquidity and availability.

The equity markets, however, appeared to function well. In the face of unprecedented volume and volatility, these markets continued to operate smoothly. Spreads widened somewhat in recognition of the higher risks, but limit order books did not suddenly become empty. It was almost always possible to trade.

Despite this generally satisfactory performance, concerns arose. It was observed that the de facto market makers had no formal affiliation or obligations. They generally supplied liquidity, but this provision was opportunistic. There was no penalty for withdrawal in a volatile market. Nor were there prohibitions against destabilizing trades. The 2010 SEC Concept Release on Equity Market Structure noted (U.S. Securities and Exchange Commission (2010)):

The use of certain strategies by some proprietary firms has, in many trading centers, largely replaced the role of specialists and market makers with affirmative and negative obligations. Has market quality improved or suffered from this development? How important are affirmative and negative obligations to market quality in today’s market structure? Are they more important for any particular equity type or during certain periods, such as times of stress? Should some or all proprietary firms be subject to affirmative or negative trading obligations that are designed to promote market quality and prevent harmful conduct? Is there any evidence that proprietary firms increase or reduce the amount of liquidity they provide to the market during times of stress?

The Concept Release was published in January of 2010. A few months later, the flash-crash of May 6, 2010 provided some direct evidence. Around 2:30pm the US broad market indices dropped around 5% in a few minutes, and then rebounded a similar amount also in the span of a few minutes. A Joint CFTC-SEC report analyzes the event (U.S. Commodities Futures Trading Commission and Commission (2010)). “The de facto market makers, it should be emphasized, did not trigger or cause the decline.” The report nevertheless notes:

[At about 2:45pm] based on interviews with a variety of large market participants, automated trading systems used by many liquidity providers temporarily paused in reaction to the sudden price declines observed during the first liquidity crisis. These built-in pauses are designed to prevent automated systems from trading when prices move beyond pre-defined thresholds in order to allow traders and risk managers to fully assess market conditions before trading is resumed.

After their trading systems were automatically paused, individual market participants had to assess the risks associated with continuing their trading ...

Based on their respective individual risk assessments, some market makers and other liquidity providers widened their quote spreads, others reduced offered liquidity, and a significant number withdrew completely from the markets. Some fell back to manual trading but had to limit their focus to only a subset of securities as they were not able to keep up with the nearly ten-fold increase in volume that occurred as prices in many securities rapidly declined.
HFTs in the equity markets, who normally both provide and take liquidity as part of their strategies, traded proportionally more as volume increased, and overall were net sellers in the rapidly declining broad market along with most other participants. Some of these firms continued to trade as the broad indices began to recover and individual securities started to experience severe price dislocations, whereas others reduced or halted trading completely.

Many over-the-counter ("OTC") market makers who would otherwise internally execute as principal a significant fraction of the buy and sell orders they receive from retail customers (i.e., "internalizers") began routing most, if not all, of these orders directly to the public exchanges where they competed with other orders for immediately available, but dwindling, liquidity.

In summary, while traditional exchange specialists were expected to maintain a bid and an offer, their successors weren’t and didn’t. While specialists were discouraged from selling into a declining market (trading in a destabilizing fashion), the newer firms were not so constrained and took advantage of this possibility. Retail orders became like hot potatoes, as firms that would normally give executions re-routed them to other venues.

7.5. Dealer markets

The discussion above focused on dealers in the context of a limit order market. In this capacity, they are supplemental. They post bids and offers, and may even possess technological advantages over the customers in doing so. The structure of the limit order market, though, still provides opportunities for direct customer trade.

The dealer can also be the defining feature of a market. In a dealer market (also called a quote driven market), the dealer network is the sole mechanism. The important dealer markets include: foreign exchange (FX), over-the-counter derivatives, swaps, government bonds, and corporate bonds. Measured by notional trading volume, these markets are substantially larger than the equity markets.

As an example of a trade in a dealer market, a hedge fund wishing to buy Euros may contact the FX desk (of a large banks) and ask for a two-sided market: the dealer’s bid and ask quotes. The quotes are often oral, and good only at the time they are made. The quotes are “take it or leave it”. The dealer/customer arrangement is sustained by reputation: The dealer will always make a market. The customer must (at least sometimes) trade.

Dealers are linked by computers and telecommunications, but there is little centralized coordination. Customers have access only to dealers with whom they’ve previously established a relationship. A dealer is always the counterparty to a customer trade.

Dealers will usually disseminate indicative bids and asks widely (on Bloomberg, or financial web sites, for example). These are mainly advertising. Firm quotes (against which a customer can actually execute an order) are generally given only to customers with whom the firm has a pre-existing relationship, and often only in response to inquiry.

Generally, trades are not publicly reported. For investors who are accustomed to the comprehensive last-sale reporting available in most equity markets, this may come as a surprise. There is no consolidated feed, for example, that reports trades in foreign exchange or US government bonds.

A dealer making a market (even against random, uninformed customer orders) will over time build up large long or short positions. Dealers lay off positions (against other dealers) in an interdealer market. Only dealers can directly participate in the interdealer market. Prices are
generally not visible to non-dealer customers. The arrangement is sometimes described as an “inner” and “outer” market. Figure 7.1 shows the connections that might exist in such an arrangement.

Figure 7.1. Topology of a dealer market

Several mechanisms facilitate trade in the interdealer market. The simplest is direct negotiation: the € dealer at bank X may contact the dealer at Bank Y, and ask for a quote. Bank Y will try to accommodate the request because over the course of the next hour or day, it is likely that at some point the situation will be reversed and Bank Y will be making the request. The communication usually takes place via secure instant messaging links. Voice brokers are intermediaries who negotiate trades between dealers, but without disclosing either party’s identity. Finally, there is extensive use of order book markets. These are virtually identical to the systems used in equities, but entry is much more restricted. Very few customers have access privileges, and access fees are high.

Customers can often obtain access to web-based systems that have the look and feel of an order book market. The screens will show bids and offers, and it’s possible to click and obtain an automatic execution. These systems do not, however, constitute a window on the interdealer market or any other comprehensive activity. They are simply platforms where the customers of a particular dealer can execute against the quotes posted by that dealer. These quotes typically track those in the interdealer market, but are usually outside of (and therefore inferior to) them.
7.6. Market segments

**Government bonds**

Traditionally dominated by dealers, but (relatively) open order books have become more important. These books are not open to retail customers, and there are no publicly available last sale prices. European government bonds are generally traded on the MTS system. U.S. Treasuries trading is concentrated on two systems eSpeed (a subsidiary of NASDAQ) and BrokerTec (a subsidiary of ICAP). Fleming, Mizrach and Nguyen (2014) discuss BrokerTec.

On October 15, 2015, the U.S. Treasury market experienced an episode of high volatility. Since this did not seem to be caused by any major news announcement, attention turned to the trading process. The events are discussed in a joint interagency report that drew input from the Federal Reserve Bank, the Treasury Department, the SEC and the CFTC (see U.S. Department of the Treasury (2015)). The Treasury Dept. subsequently initiated a request for information (Treasury (2016)), and appears to be considering recommendations for trade reporting.

**Corporate bonds**

In fundamental structure the US corporate bond market is a dealer market. Nevertheless, under SEC pressure, the industry has adopted last-sale reporting. All bond trades must be reported within fifteen minutes to the FINRA-operated Trade Reporting and Compliance Engine (TRACE). Data are available through private vendors and on the FINRA website (http://cxa.gtm.idmanagedsolutions.com/finra/BondCenter/Default.aspx). The market has long been viewed as one due for an expanded role of limit order platforms. These now appear to be viable and some brokers have started consolidating their bids and offers. Harris (2015) discusses the current trading environment. Sirri (2014) provides a parallel analysis of the municipal bond market.

**Foreign exchange (currency, FX)**

By some measures of trading volume, FX constitutes the largest financial market. Traditionally it has been a dealer market, with two main interdealer limit order markets (run by Reuters and ICAP EBS). Multi-dealer electronic platforms have made inroads, however, and non-dealers are gaining access to the interdealer markets through prime brokerage arrangements. See Evans and Lyons (2002); Karnaukh, Ranaldo and Söderlind (2015); King, Osler and Rime (2012); Lyons (2001).

7.7. Further reading

Summary of terms and concepts

The broker/dealer conflict of interest; designated market makers; contract market makers; NYSE specialist affirmative obligations (price continuity, maintaining a narrow bid and ask spread, crossing public buyers and sellers, public priority) and negative obligations (trading in a destabilizing fashion); Designated market makers (DMMs); parity; operations of dealer markets; interdealer markets; sales and position traders.

Problems and exercises

Problem 7.1 Are the following statements about dealer markets generally true or false?
   a. Trades reports are consolidated and sold to vendors for real-time redistribution.
   b. Customer limit orders are displayed to all market participants.
   c. Dealer markets dominate in the trading of currency, bonds and swaps.
   d. When a dealer simultaneously receives buy and sell orders from customers, government regulations require them to pair off the customers against each other (let the customers trade directly with each other).
   e. Customers can see and execute against bids and offers in the interdealer market.

Problem 7.2 The principle of public priority prevents an NYSE specialist (market maker) from ___ (True or False).
   a. Bidding or offering more aggressively than public customers.
   b. Buying or selling to public customers.
   c. Buying at a price bid by a public customer.
   d. Lifting a customer offer.

Problem 7.3 An NYSE specialist was required to ___ (True or False)
   a. Always quote a bid and an offer.
   b. Always ensure that the day’s trading volume in a stock was non-zero.
   c. Support the stock price so that it didn’t decline below the day’s opening price.
   d. Avoid losses when trading on the Exchange’s account.

Problem 7.4 True or False
   a. Dealer markets dominate trading in currency, bonds and swaps.
   b. Dealer markets facilitate direct trade between customers.
   c. By law, dealers must quote the same bid and offer to all customers.
   d. A customer who doesn’t want to “take” a dealer’s bid or offer can “make” their own bid or offer in the interdealer market.
   e. In a dealer market, a customer can generally judge the reasonableness of the dealer’s bid or offer by comparing the bid or offer to the last reported trade price.
Chapter 8. Dark Markets

“Darkness” generally refers to pre-trade quote transparency. In US equity markets, we infer the occurrence of a dark trade when:

- An execution is reported (“prints”) at a price inside of the National Best Bid and Offer.
- An execution prints at the NBB or the NBO, but it occurs at a market center whose quote was not (at time of execution) at the NBB or NBO.
- An execution is reported on FINRA’s ADF (Alternate Display Facility).

It is important to emphasize that a dark trade does not go unreported. The darkness reflects the fact that neither the buyer nor the seller is posting a visible bid or ask. Post-trade, the execution is reported like any other.

Dark executions typically arise from one of following mechanisms:

- A hidden ( undisplayed) limit order in a limit order market (like Nasdaq) that also handles visible orders.
- A NASDAQ market maker trades against a customer order at the NBBO at a time when the MM’s own quotes are behind the NBBO.
- The trade occurs in a crossing network or dark pool that posts no quotes of its own, but matches buyers and sellers at prices determined by the NBBO.

The second and third of these mechanisms probably account for much greater trading volume than the first. They also differ from the first in one other important respect. Whereas the hidden limit order is executed at its own limit price, the MM and the crossing network/dark pool trades occur at prices determined by the NBBO. This reliance will turn out to be problematic.

Before considering dark mechanisms in detail, we turn to the question of why they exist in the first place.
8.1. The logic of dark mechanisms.

A dark trade is defined by the unwillingness of either party to the trade to post a visible bid or offer. Why should this be? If buyer is willing to pay, say, $10, why not advertise the bid to attract sellers?

It’s useful to consider a more familiar situation. The markets in consumer electronics products are extraordinarily competitive: the product (a given model from a given manufacturer) is homogeneous, and the internet makes it easy to comparison shop. Many retailers advertise aggressive prices. Other retailers claim, “Bring us any advertised price, and we’ll match or better it. Guaranteed.” The situation is complex because it is unlikely that any two sellers are truly identical with respect to location, reputation, and many other small differentiating characteristics. We can still draw a few lessons by thinking about their strategies.

Firstly, the price matcher avoids the expense of determining and updating advertised prices. They are letting someone else “do the math,” and simply doing as much or as little as necessary to remain competitive. Secondly, in the presence of a price matcher, the advertiser has a reduced incentive to post an aggressive price. When the advertiser lowers the posted price, the additional customers will be split with the price matcher. Both of these considerations operate in securities markets. A seller who is willing to match the market’s best visible offer can avoid the calculations and judgments necessary to determine their own reservation price. From the viewpoint of a seller posting a visible offer, why post an aggressive (low) offer if we’re simply establishing a reference price for someone else’s trade? We’ll raise or keep the offer above our true reservation selling price.

We now turn to a description of the mechanisms.

8.2. Undisplayed limit orders

In a hybrid market (like the US) a limit order has no market-wide time-priority, and only partial market-wide price priority. Not only may a limit order lose priority to visible limit orders posted at other market centers, but executions may occur at their prices, via market makers and dark pools who are posting no visible quotes of their own. Visibility has its benefits (in making potential counterparties aware of the price), but in a hybrid market it also carries a cost. Hiding a limit order, or displaying it only for a brief interval, is one way to control this cost.

8.3. Dealers

When a broker receives a customer market order, he may, if his firm is a market maker in the stock, simply keep the order within the firm. The market-making desk or division can take the other side of the customer order. The main requirement is that the execution occur at or within the NBB or NBO. It is not necessary that the firm’s market-maker’s quote be at the NBBO. The execution must be reported, of course. The market maker will probably send the report to FINRA’s Alternative Display Facility (ADF). Orders executed within the firm are said to be internalized.

The market maker may also be receiving order flow from other firms. Much of this happens by pre-arrangement. An order-entry firm may routinely send (or preference) its customer orders to a particular market-maker with the understanding that the orders will receive the NBBO or better.

It was noted in the discussion of private information that less-informed order flow, such as that originating from retail customers, is more profitable for dealers. There is less chance of loss.
This is not merely an academic fine point. One aspect of the arrangement between order-entry and market firms involves payment: the order-entry firm receives a small consideration for each order that it sends.

Internalized and preferred orders never interact with other orders. They aren't generally conveyed to any central limit order book, for example, and so don't fully participate in the market. They receive a price that is set by others. In the price determination process, they represent buyers and sellers who are participating without a vote.

8.4. Point-in-time crossing networks and continuous dark pools

Crossing networks and dark pools are markets that are separate from, but still very reliant on, the lit market. Historically, point-in-time crossing networks came first. Many of them are still around, but some of them evolved into continuous crosses, and ultimately today’s dark pools.

Before exploring the differences, here are the points of similarity.

- The “darkness” is pre-trade only. Trades on dark pools have to be reported (and are visible on the Consolidated Trade System) just like all other trades.
- They don’t disseminate quotes or provide price discovery or determination. The prices for all trades are set by reference to prices determined in the lit market (usually the NBBO).
- They are regulated by the SEC as “Alternative Trading Systems” (ATSs).

Across these markets there are many variations in the trading procedures. The following describes typical systems. We’ll then discuss modifications and extensions.

Point-in-time crossing networks

Orders are submitted anonymously. They usually specify direction (buy or sell) and quantity (or sometimes a quantity range), but not prices. They are held in the system undisplayed. At some pre-scheduled time, the system goes through all of the orders and attempts to match (pair off) buyers and sellers. A buyer for 20,000 MSFT and a seller for 30,000 MSFT would be paired off for 20,000 shares, for example. Whenever there is a match, the execution price for the trade is determined by reference to the lit market. Depending on how the dark pool is designed, this price might be the prevailing NBBO midpoint, the day’s closing price or the day’s volume-weighted average price (VWAP). Of course, with VWAP or closing price, the price is not determined and the trade is not reported until the end of the day. Instinet runs 11 daily point-in-time crosses in US equities, and is also active in Europe and Asia; ITG Posit runs intraday NBBO midpoint crosses; The NYSE MatchPoint system runs a closing cross. Point-in-time matches are used by institutional investors who seek to trade in size. Reported trade sizes are large.

Trades occurring in crossing networks are sometimes described as “zero impact”. When the trade is reported, it can’t be determined whether the aggressor was a buyer or seller. Nothing is shown prior to execution. If there is no execution, nothing is shown.

Continuous dark pools

Consider the MSFT example, above. In a crossing network, the timing of order arrival doesn’t matter. Everyone must wait for the next scheduled match. In a continuous dark pool the system looks for a match whenever a new order arrives. If a match can be made, the trade is executed immediately. As in a crossing network, the execution price is set by reference to some price from the lit market.
How is this different from a limit order book in which all orders are hidden? Remember that in a limit order book, the execution price is set to the limit price of the resting order (the bid or offer that is already in the book). In a continuous dark pool, the trade price is set by reference to the lit market, typically the NBBO midpoint. As with the point-in-time crossing networks, nothing is shown unless there is an execution.

By some counts there are over fifty dark pools. Users often try them sequentially, favoring those that in the past have proven likely to provide fills (the dark pool routing decision).

Variations

Some systems allow traders to specify conditions that an execution must satisfy. The most important of these are:

- Minimum quantity. (“Don’t execute the trade unless it is for at least 10,000 shares.”)
- Limit price.

Note that when a limit price is specified, it restricts the allowable outcomes, but does not affect the execution price. For example, if “sell 1,000 shares limit $10.50” is an order resting in a conventional limit order book, then (if it is executed) the price will be $10.50. If the same order is sent to a typical dark pool, the execution price will be the NBBO midpoint at the time of the match, and if the NBBO midpoint is below $10.50 there will be no match.

8.5. Regulatory issues

Dark markets are sometimes viewed with suspicion. Confidentiality of customer orders is difficult to verify and several recent cases raise doubts.


  Pipeline Trading ... operated an alternative system (“ATS”), a private stock-trading platform commonly referred to as a “dark pool.” Pipeline held out its ATS as a “crossing network” that anonymously matched customers’ interests in trading large amounts of stock. However, Pipeline did not disclose to its customers that the overwhelming majority of the shares traded on its ATS were bought or sold by a wholly owned subsidiary of Pipeline [Millstream, the “Affiliate”].

- In the case of a Barclay’s dark pool, the New York attorney general alleged that although the firm reassured customers that they were being protected from predatory high frequency traders, the firm was actively soliciting high frequency traders to join the pool.
- UBS paid a $12 Million fine to settle an SEC complaint that in its dark pool UBS allowed high frequency traders and market makers to jump ahead of customer orders.

Domowitz, Finkelsteyn and Yegerman (2009) note that:

Even if the dark pool is operationally secure, other users may still be able to draw inferences via “sniffing” and “sniping”. These practices involve using small standing orders to detect larger incoming orders, or small marketable orders to detect larger standing orders.

Any system that matches buyers and sellers at some external reference price gives the users the incentive to manipulate that price. For example, a buyer sending an order to a dark pool knows that any execution will be priced at the NBBO midpoint. The buyer can lower the midpoint by
submitting a small aggressive sell limit order to a lit market. After achieving a dark pool execution, the sell limit order is cancelled. This practice, one type of "spoofing," has attracted regulatory and enforcement interest.

### 8.6. The interplay of dark and lit markets

Dark mechanisms are controversial. Aside from hidden orders, they assign prices using the quotes in the lit market, essentially “free riding” on the lit prices. While they provide liquidity, their contributions aren’t visible pre-trade.

An aggressive visible bid or offer is an advertisement that encourages potential counterparties to bid or lift it. If these counterparties can obtain the same price in a dark pool, their orders will migrate away from the visible market. With fewer traders posting orders in the lit market, the bid ask spread will become wider, and (with less participation) more variable. This in turn hurts not only the traders in the lit market, but those in the dark pool as well (since the dark pool relies on the lit market’s prices).

After extensive debate Canadian regulators adopted restrictive measures on dark trading. Generally, large dark executions (above $100,000) are permitted at or within the NBBO. Smaller orders can be executed in the dark only if there is price improvement relative to the visible bid or offer. For example, if the Toronto Stock Exchange bid in Toronto Dominion Bank (TD) is 82.00, a retail sell order could not execute below 82.01. That is, unlike in the U.S., it is not permissible for a broker dealer to simply match the 82.00 bid.

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**Summary of terms and concepts**

Dark trades vs. dark markets; lit markets; hidden orders; internalized executions; crossing sessions; continuous dark pools; midpoint matches; spoofing; leakage; pros and cons of dark trades; regulatory concerns; recent cases (Pipeline/Millstream; UBS; Barclays).

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**Problems and exercises**

*Problem 8.1* In each of the following situations, determine if the trade is regarded as “dark”, and why or why not? In all cases, the NBBO is 10.00 bid, offered at 10.10.

- **a.** A NASDAQ market maker in SPQR, whose own offer is 10.15, receives a customer order to buy 100 shares at the market. He routes the order to an exchange offering 10.10, where the order is executed at 10.10.

- **b.** A NASDAQ market maker in SPQR, whose own offer is 10.15, receives a customer order to buy 100 shares at the market. He sells 100 shares to the customer at the NBO, 10.10.

- **c.** The INET book receives an order to buy 100 shares limit 10.20. The order executes against a hidden limit order priced at 10.09.

- **d.** Pension fund LSM and mutual fund EFT are paired off in the Credit Suisse Crossfinder system. LSM buys 10,000 shares; EFT sells, and the trade is priced at the NBBO midpoint of 10.05.
Part III. Information and efficiency

Changes in securities prices are driven mainly by new information. The competition to act on the new information leads to market prices that incorporate the information, a desirable property known as market efficiency. The trading process is critical in the transition to the new price. It matters very much whether the information is public (known to all) or private (known to a select few). Insufficient public information impairs one of the key benefits of a financial market, its ability to produce price signals that can guide real investment. Too much private information can result in a market so overshadowed by mutual distrust that nobody will trade. The nexus of financial markets and information is an important concern for law and regulation. These laws determine the type and quantity of information that is produced, how that information is disseminated, and what kinds of information are deemed for purposes of trade to be illegal.
Chapter 9. Public Information

Investors form beliefs about securities’ values largely on the basis of public “common knowledge” information. This information set is extremely broad, ranging from public fundamental information of obvious relevance (such as the firm’s financial statements) to more diffuse information that might affect investor sentiment (such as a political development in a distant country). Prices generally adjust to reflect changes in this information, and trades are often a part of the adjustment process.

The doctrine that a security price fully (and, on average, accurately) reflects all available public information is one form of the efficient market hypothesis. Violations of market efficiency raise the possibility of “incorrect” market valuations, and trading strategies that can profit by exploiting these errors. Therefore, an alternative (and more provocative) form of the efficient market hypothesis conjectures the impossibility of consistently outperforming the average investor (“beating the market”).

Can all security price changes be attributed to value-relevant information? Of course not. The large and growing academic field of behavioral finance has cataloged many kinds of behavior that aren’t (at least not obviously) “rational”. For example, Hirshleifer and Shumway (2003) note: “Psychological evidence and casual intuition predict that sunny weather is associated with upbeat mood. … Sunshine is strongly significantly correlated with stock returns … These findings are difficult to reconcile with fully rational price setting.” Warren Buffet, one of the most successful investors of the modern era, has been quoted: “I would be a bum on the street with a tin cup if markets were efficient.”

The economic force driving market efficiency is competition among investors to accurately assess and interpret the information available to them. Even if we agree with Warren Buffet and
reject the pure form of the hypothesis, we can certainly observe this competitive process at work.\textsuperscript{13}

The dynamics of price adjustment to new public information are most clearly demonstrated in a situation where there is a scheduled release of information.

\textbf{9.1. Scheduled public announcements}

The security with US ticker symbol SPY is an exchange-traded fund (ETF). Its assets are shares in other stocks. The portfolio is designed to mirror the S&P Composite Index of 500 stocks. From an investor’s viewpoint, it provides a convenient and low-cost way to hold the index portfolio. It is also attractive from a trader’s perspective. Whereas a standard (open-ended) mutual fund can only be purchased or sold at daily closing prices (the net asset value), the SPY can be traded intraday, just like any other stock. The market for the SPY is extremely liquid. The bid-ask spread is generally $0.01; sizes at the bid and ask are large; trading volumes are large.

Figure 9.1 depicts trade prices for SPY on April 15, 2011. During many periods throughout the day, successive price changes are small and random (13:30 to about 15:00, for example). In the figure, at 8:30 am, there is a sudden jump in the price. What happened?

Economic analysts in the US government and elsewhere measure economic quantities that reveal the current state of the economy and suggest its evolution in the near future. Inflation figures, industrial production, housing starts and the like are among the useful indicators. Estimates are usually released on a well-publicized schedule. “Economic calendars” that list upcoming announcements are featured on many financial web sites, including the Wall Street Journal online and \url{www.briefing.com}. April 15, 2011 was a particularly busy day. Table 9.1 gives a partial record.

To connect the news to the price movements, note first that the announcement may incorporate information previously known. The information content of the announcement must be assessed relative to what market participants already knew and their previously formed beliefs. “At $1.50 per share, earnings rose 10% from the same period last year.” This sounds positive, but if market participants were expecting earnings of $2.00, the announcement conveys bad news. An old Wall Street adage holds that “the baby is born.” (Birth is the predictable outcome of a previously observed condition.)

For each statistic, the table gives the released estimate, and the pre-release consensus of surveyed analysts. At 8:30, there were two announcements: the consumer price index and the Empire Manufacturing Survey. The CPI figures came in on the consensus value – no surprise there. The Empire release, though, was substantially higher than the consensus value.

The Empire Manufacturing Survey is a monthly product of the Federal Reserve Bank of New York. Manufacturing firms in New York (the “Empire State”) are surveyed about general business conditions. The direction of the surprise suggests that business conditions will be better than previously thought. Production, sales, and presumably earnings will be higher.

\textsuperscript{13} Lasse Pedersen’s \textit{Efficiently Inefficient} is a modern and accessible discussion of why markets are almost, but not quite, completely efficient (Pedersen (2015)).
Figure 9.1 Trades in the composite index SPDR on April 15, 2011.

The data are “thinned” for clarity. Each point marks the price of the last trade in a ten-second interval. Vertical lines demarcate the traditional trading hours (9:30am to 16:00pm).

Table 9.1 Public announcements on April 15, 2011.

<table>
<thead>
<tr>
<th>Scheduled release time (Eastern)</th>
<th>Release statistic</th>
<th>For</th>
<th>Actual</th>
<th>Briefing.com Consensus</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30</td>
<td>Consumer price index (CPI)</td>
<td>Mar</td>
<td>0.50%</td>
<td>0.50%</td>
</tr>
<tr>
<td>8:30</td>
<td>Empire Manufacturing Survey</td>
<td>Apr</td>
<td>21.7</td>
<td>15</td>
</tr>
<tr>
<td>9:15</td>
<td>Industrial Production</td>
<td>Mar</td>
<td>0.80%</td>
<td>0.60%</td>
</tr>
<tr>
<td>9:15</td>
<td>Capacity Utilization</td>
<td>Mar</td>
<td>77.40%</td>
<td>77.40%</td>
</tr>
<tr>
<td>9:55</td>
<td>Michigan Consumer Sentiment</td>
<td>Apr</td>
<td>69.6</td>
<td>66.5</td>
</tr>
</tbody>
</table>

Source: briefing.com
Figure 9.2 provides more detail in the neighborhood of the 8:30 announcement. The entire time span covered by the plot is thirty seconds. Prior to the announcement, there are a few trades at the bid, and a few at the ask. Neither the bid nor the ask makes a major move, but since the bid drops by a few pennies, the bid-ask spread widens.

Immediately after 8:30, more marketable buy orders arrive. They take out the standing sell limit orders at $131.10, leaving $131.11 as the newly exposed ask. Additional marketable buy orders exhaust the quantities at $131.11, and those at $131.12. The next most aggressive sell orders are priced at $131.19, a jump of seven cents. (It turns out that there was a “hole” or “air pocket” in the book.) A buy at $131.19 leaves the ask at $131.20. While these orders were walking through the ask side of the book, the bid remained steady at $131.06.

The development around 8:30:05 is interesting. Suppose that someone wants a quick sale. They could hit the bid at $131.06, but pattern of recent trades suggests that the market is moving up. So they try a limit sell order priced at $131.12. Sometimes limit orders can sit unexecuted for hours. But this one is priced so aggressively that it is hit within a fraction of a second.

Figure 9.2. Quotes and trades for SPY around 8:30 on April 15, 2011
After this, the flow of marketable buy orders continues, pushing prices (on bids, asks, and trades) higher. Then some sellers enter the market, and prices drop a bit. The whole set of events has played out in about fifteen seconds, and the net price change is about ten cents. All of this analysis, of course, refers to a particular example. Which features generalize?

Generally, prior to a scheduled public announcement, trading volume drops, and the bid-ask spread widens. There are many reasons for this, some of which we’ll investigate later. But for the moment, it suffices to note that the period immediately subsequent to the announcement is likely to have high volatility. Someone who buys or sells prior to the announcement is bearing high risk.

In the adjustment to the new information, trades (executions) are not a necessary feature. The bid and ask may rise or fall together, bracketing the market’s new estimation of the security’s value. More often than not, though, trades do occur following a news announcement, and volume is often high. There are several reasons for this.

- Traders might disagree about the importance of the information.
- Traders who took established a position with the intent of betting on the impact of the announcement will unwind.
- Any kind of an announcement brings the stock to people’s attention.

The process of arriving at the new price, involving a complex interplay of bids, asks, and trades, is called price discovery. The term “discovery” emphasizes that the outcome is unknown. Although everyone might agree that the news is positive, no individual trader knows the economic value of the news. This value can only be established collectively. In economic terms, the market aggregates the heterogeneous beliefs of the participants.14

Because major announcements often induce volatility, they are generally scheduled outside of regular trading hours (before the official market open or after the close). Of course, the force of this practice has declined over time, as trading activity has spread beyond regular hours, but the timing persists. If a company decides that an announcement must be made during regular trading hours, the company will usually notify the listing exchange. If the news is major, the listing exchange will halt trading immediately prior to the announcement.

When US companies release information, they are for the most part constrained by the Securities and Exchange Commission’s Regulation FD (“Reg FD;” “Full Disclosure”). The force of this rule is that disclosures must be made public in a way that ensures that the information is available to everyone at the same time. For example, it typically prohibits management from holding private conversations with favored shareholders to give them advance knowledge of important developments.

Reg FD does not apply to the release of information by people or entities that aren’t connected to the company (if any) that’s the subject of their comments. This encompasses independent research firms, doing company-specific or market-wide research.

As an interesting example, the Michigan Consumer Sentiment index is compiled by a unit of the University of Michigan. According to a Wall Street Journal article, “[An] early look at the consumer-sentiment findings comes from Thomson Reuters Corp. The company will pay the University of Michigan $1.1 million this year for rights to distribute the findings, according to the university. ... In turn, Thomson Reuters's marketing materials say the firm offers paying clients an ‘exclusive 2-second advanced feed of results...designed specifically for algorithmic trading.’ Clients who pay a subscription fee to Thomson Reuters, which for some is $5,000 a month plus a $1,025 monthly connection charge, get the high-speed feed at 9:54:58 a.m. Eastern time. Those

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14 Lee, Mucklow and Ready (1993) discuss the market dynamics around earnings announcements.
who pay for Thomson Reuters’s regular news services get the report two seconds later,” (Mullins, Rothfeld, McGinty and Strasburg (2013)).

9.2. Unscheduled announcements

It is easy to think of information, such as that arising from natural disasters, that cannot be scheduled ahead of time. Most of these shocks will be broad, affecting large portions of the world economy. But unscheduled surprises can also happen in ways that are very specific to a particular company.

Acorda Therapeutics (ticker symbol: ACOR) is a NASDAQ-listed pharmaceutical firm. Figure 9.3 describes trade prices on April 14, 2011. Until about 13:10 the stock trades in a narrow range around $21.20 per share. Shortly after 13:10, the price suddenly jumps.

Figure 9.3 Trading in Acorda, April 14, 2011
In this case, the news came not from the company itself, but from an analyst following the company. RBC Capital Markets is an investment bank affiliated with RBC (the Royal Bank of Canada). According to a posting on Xconomy (www.xconomy.com), “RBC Capital Markets speculated in a report that the patent on the company’s multiple sclerosis (MS) drug, dalfampridine (Ampyra), might extend for longer than initially expected.” The news is unambiguously positive, and it appears to have caught traders completely by surprise.

The price discovery process is much rougher than that for the Empire announcement discussed above. The Empire announcement moved the price of the SPY by about ten cents, and the adjustment process was over in fifteen seconds. Here, the price oscillates wildly between about $26 and $28. The price chart stops at about 14:00. At this point, NASDAQ halted trading in the security.

A pre-scheduled news release allows all potential buyers and sellers of security to coordinate their trading activities. It focuses attention, saying in effect, “pay attention to your news feeds”. Absent a scheduled news release, the number of people following the market in any given stock might be very low. Without strong participation, the price discovery process may be extremely volatile.

A surprise information event usually sets up a race. Limit orders pre-existing in the book are, relative to the new information, mispriced. Alert traders will try to hit the mispriced side of the book (in the ACOR case, the ask) before the limit orders can be cancelled or repriced. This is sometimes called “picking off stale limit orders.” Traders pursuing pick-off strategies typically work off of “low-latency” news feeds. One offering claims:

“The Dow Jones Elementized News Feeds are an ultra-low latency, XML-tagged, machine-readable news data feeds that deliver economic indicators and corporate news, with a corresponding elementized archive, into quantitative models and electronic trading programs. These innovative feeds revolutionize how newsflow can be interpreted and give firms an enhanced news source for analyzing and identifying trading, investing and hedging opportunities—while moving on information in milliseconds.”

Clearly, not everyone gets the same “public information” at the same time. In the interest of fairness, some regulators and market operators believe that when a significant event is pending or when a significant surprise has just occurred, the market should be closed, until most market participants have received the news.

9.3. Trading halts, circuit breakers, and price limits

At its best the trading process involves many participants reacting thoughtfully and deliberately to the unfolding of public information. The ideal of a “fair and orderly” market comes to mind. While the meaning of these two words might be debated without end, it must be admitted that situations arise which are by common agreement anything but. Such instances make a case for temporarily closing the market.

This can happen by a variety of mechanisms. Trading halts are generally news-related, reflecting information originating from or in relation to a specific company. Circuit-breakers and price limits are triggered by market price movements.

In the US stock market, the primary listing exchange has the responsibility of declaring a halt, which is communicated to market participants by a message sent over the quote stream. Trading is typically halted when a news announcement is pending, or is in process. Halts are common. Table 9.2 gives a sample.
A halt should only last long enough to ensure widespread dissemination of accurate information. Once this has occurred, the market can be reopened. This typically happens using a single-price auction run by the primary listing exchange (see Chapter 6).

<table>
<thead>
<tr>
<th>Halt Date</th>
<th>Halt Time</th>
<th>Issue Symbol</th>
<th>Issue Name</th>
<th>Reason Code</th>
<th>Resume Quoting</th>
<th>Resume Trading</th>
</tr>
</thead>
<tbody>
<tr>
<td>01/25/2012</td>
<td>15:04:42</td>
<td>NUVA</td>
<td>NuVasive Inc</td>
<td>T1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>01/25/2012</td>
<td>09:19:08</td>
<td>GLRE</td>
<td>Greenlight Capital Re, Ltd.</td>
<td>T3</td>
<td>12:45:00</td>
<td>12:50:00</td>
</tr>
<tr>
<td>01/25/2012</td>
<td>09:00:38</td>
<td>PNNW</td>
<td>Pennichuck Corporation</td>
<td>T12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>01/25/2012</td>
<td>08:10:27</td>
<td>INCB</td>
<td>Indiana Community Bancorp</td>
<td>T3</td>
<td>08:40:00</td>
<td>08:45:00</td>
</tr>
<tr>
<td>12/19/2011</td>
<td>13:29:39</td>
<td>FEED</td>
<td>AgFeed Industries, Inc.</td>
<td>T12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11/29/2011</td>
<td>07:01:23</td>
<td>BQI</td>
<td>Oilsands Quest Inc</td>
<td>T2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The reason codes are: T1 (a news release is pending); T2 (a news release is in process); T3 (trading will resume shortly); T12 (NASDAQ is requesting additional information).


Circuit breakers are market-wide trading halts. The current SEC information bulletin states (U.S. Securities and Exchange Commission (2016b)):

A cross-market trading halt can be triggered at three circuit breaker thresholds—7% (Level 1), 13% (Level 2), and 20% (Level 3). These triggers are set by the markets at point levels that are calculated daily based on the prior day’s closing price of the S&P 500 Index.

A market decline that triggers a Level 1 or Level 2 circuit breaker before 3:25 p.m. will halt market-wide trading for 15 minutes, while a similar market decline “at or after” 3:25 p.m. will not halt market-wide trading. A market decline that triggers a Level 3 circuit breaker, at any time during the trading day, will halt market-wide trading for the remainder of the trading day.

The market-wide circuit-breakers were first instituted in response to the October 1987 “market break,” a sudden price decline across virtually all stocks. On May 6, 2010, a similar event occurred (the “flash crash”), but all stocks were not affected (U.S. Commodities Futures Trading Commission and Commission (2010)). In response, the SEC proposed single-stock circuit breakers, which were subsequently refined into the present limit up-limit down (LULD) rules.

Briefly, the limit-up/limit-down rule sets a price band which, once hit, triggers a five-minute “pause”. At the end of the pause, the primary listing exchange reopens the stock. For most stocks, the band is ±5% of a reference price. The reference price is the average trade price over the previous five minutes, or (if there are no trades) the previous reference price.
The details are somewhat complex, but a sketch of the process is as follows. When the bid (NBB) drops below the reference band, it becomes non-executable. It is still disseminated, but it can’t be hit. If the offer (NBO) then drops to the lower limit of the reference band, the market is said to enter a Limit State, and the offer is marked as a Limit State Quotation. This starts a 15-second clock. If within this time the Limit State Quotation is executed (or cancelled), the market exits the Limit State. If the exit doesn’t happen, trading is paused for five minutes. At the end of the pause, trading is reopened on the primary listing exchange. (See U.S. Securities and Exchange Commission (2012a); (2013b); (2013c).)

Trading halts and price limits are controversial. The arguments against halts and limits are:

- A trade is a voluntary act between two consenting parties, with no effects on anyone else. From a Libertarian perspective, a trading halt is an arbitrary prohibition.
- Markets are most efficient when prices reflect a free flow of information. Trading halts impede the market’s reaction and therefore impair efficiency.

The arguments for:

- A sudden news announcement is unfair to limit-order traders. It can make what was a relatively level playing field very uneven. It is better to halt trading, at least for a few minutes, to allow everyone the chance to see and react to the new information.
- The loss in efficiency from a short trading halt might be very small. (Does it really matter if the information reflected in prices is a few minutes old?)
- The market is more liquid if the bid-ask spread is tight, i.e., if buyers and sellers are posting aggressive limit orders. If a trader thinks he’ll be picked off, he’ll price his order less aggressively. If everyone does this the bid-ask spread widens.

The effects on market volatility are complex and ambiguous. Obviously, while a trading halt is in effect reported market prices don’t change. Perceptions of fundamental values, however, can’t be prevented from changing, and during a halt they are based on less information than usual. Moreover, if traders believe that a trading halt is imminent, they might accelerate their trading plans, aggravating volatility.

In recent years, however, market volatility caused by wayward algorithms has emerged as the overriding concern. In the absence of any other practical way to contain these programs, price-based halts remain regulators’ first choice.

9.4. Small, diffuse news

To this point the discussion has focused on significant news announcements. The day’s activity for SPY depicted in Figure 9.1 certainly contains a few such periods, where there is a jump in price that can be attributed to a particular news item. Often, though, market information consists of small, frequently-arriving news bits. Each individual item is of minor importance, but over time the accumulations can become large enough to notice, and to move prices. During these periods, successive price changes are small and random (13:30 to about 15:00, for example).

The view that all information effects are small, random, frequent and cumulative over time, leads to a statistical construct known as a diffusion model of prices. This model is very useful mathematically and appears in financial theory and practice in many places, notably the Black-Scholes options pricing framework.

Now how does small information actually get reflected in the bid, ask or trade price of SPY? What actually happens? If we really believe that all information is reflected in the security price, then even something like a hailstorm in one Kansas wheat field should in some small way contribute to the value of SPY. Breaking down the causal chain leads to many questions that extend
Public Information

beyond the SPY market. First, how, and in what sense, do SPY traders become aware of the hailstorm? In fact, they probably don’t have any direct awareness. This information will most likely come to their attention indirectly, perhaps as a result of a price move in some other market (like the wheat futures contract).

The price plot also possesses several other distinctive features.
• There are periods when the price seems to bounce around some constant value, without moving too far away.
• Some periods seem to exhibit momentum. That is, we get a long run of price drops or price increases.
• Large price changes are often followed by a "bounce back," a "correction" in the opposite direction.

In all of these instances, there is a local low price, at which we might hypothetically have bought, and a nearby high price at which we might have sold (or sold short). These trades would have been profitable. Unfortunately, we’re discerning these patterns after the fact. It is well-known that sample paths of random walks often possess these and similar features, merely by chance. A more meaningful question is whether or not these patterns are predictable and consistent. Attempts to find such patterns usually involve automated search and back-testing, and invariably fall well short of establishing definitive results.

Summary of terms and concepts

Market efficiency; limit-up/limit-down; circuit breakers; timing of public announcements; trading halts; re-openings; Reg FD.

Problems and exercises

Problem 9.1 It is 1pm. After the market close, Briarthorne is planning to announce an accounting irregularity that calls last quarter’s financial statements into question. Briarthorne might move the announcement earlier, and during trading hours, if
   a. The primary listing exchange believes that the announcement would increase trading volume.
   b. A hurricane has hit Florida, and Briarthorne believes that this news would distract attention from their announcement.
   c. There is evidence that the news may already have been leaked.
   d. Briarthorne wants to discourage traders from selling the stock short.

Problem 9.2 U.S. government economic statistics are often announced when the market is open. Immediately prior to an announcement, the bid-ask spread usually ___ and trading volume ___.
   a. Widens; rises
   b. Widens; drops
   c. Narrows; drops
   d. Narrows; rises
Problem 9.3 Limit-up/limit-down procedures in US equity markets.
   a. Set price limits for individual stocks. These limits can trigger trading halts.
   b. Govern when a market center can accept limit buy and sell orders.
   c. Limit retail investors’ profits and losses over the day to ±5% of the prior day’s closing price.
   d. Specify limits such that all trades occurring outside of those limits can be broken (cancelled) by either the buyer or the seller.

Problem 9.4 Circuit breakers in US equity markets.
   a. Force orders to be routed from dark to lit markets.
   b. Are triggered only when the S&P index declines by a pre-set amount.
   c. Are triggered when the S&P index declines or increases by pre-set amounts.
   d. Are pre-emptively used in advance of US Presidential news conferences.
Chapter 10. Securities Class Action Law Suits.

The last chapter looked at how markets react to newly arriving information. But the importance of public informational efficiency goes beyond the market adjustment process, a dynamic that mostly plays out over a few minutes. The connection between market prices and information is crucial to investors who use the information and those (usually corporate managers and accountants) who produce it. This chapter takes a brief look at the role of market efficiency in the regulation of information and the resolution of disputes involving information. The subject is a complex one, however, and this overview will just touch on a few of largest concepts.

In the early 20th century, there was little national securities regulation. Most U.S. rules were adopted by the individual states. Then, as now, government was greatly concerned with protecting investors from unscrupulous promoters of dubious investment schemes. Because they sought to eliminate the sale of securities based on empty promises, the state laws that emerged were (and still are) known as “Blue Sky Laws”. Protecting the public from bad investments, though, turned out to be much more complicated than it seemed at first. Could government really evaluate the merits of new securities? By the time that the first federal securities laws were being framed in the 1930s, it seemed better to settle for a more modest aim: full disclosure. The difference between merit and disclosure is important. When Apple Computer first sold stock, they did not have to convince a regulator that the securities would be a good investment. They only had to establish that they were openly disclosing the risks and uncertainties.
The principle of disclosure arises in many places in U.S. regulation, but perhaps the most well-known statement occurs in the 1934 Securities Act (§240.10b-5):

It shall be unlawful for any person, directly or indirectly, by the use of any means or instrumentality of interstate commerce, or of the mails or of any facility of any national securities exchange,

(a) To employ any device, scheme, or artifice to defraud,

(b) To make any untrue statement of a material fact or to omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading, or

(c) To engage in any act, practice, or course of business which operates or would operate as a fraud or deceit upon any person,

in connection with the purchase or sale of any security.

This law has far reaching implications. Although it seems at first to deal only with public information (what we say or don’t say), it in fact underlies a lot of the private information (insider trading) laws we’ll be considering later. It is also worth pointing out that the 1934 Act applies mostly to stocks that are already trading (that is, in the secondary market). A similar and generally stronger provision of the 1933 Act applies to the initial public offering (the primary market). The wording of this law has been subjected to extremely careful examination and interpretation by lawyers, experts, and courts, and the volume of commentary is substantial. For the moment, though, it suffices to draw out one phrase, “It shall be unlawful ... to make any untrue statement of a material fact or to omit to state a material fact ...” Simply put: management has to tell the truth. What makes a fact “material”? One common formulation is that a reasonable investor would consider it important in considering the purchase or sale of the securities. That is, the difference between knowing it or not might cause an investor to view the stock very differently.

The disclosure obligation has some important exceptions. A firm need not publicly disclose its legitimate business secrets, such as technical information that might be useful to competitors. Activities that are questionable or even illegal are not necessarily exempt from the requirement. In September, 2015, a suit was filed against Volkswagen AG, alleging failure to disclose facts connected to the emission-testing defeat devices. The alleged failure to disclose (to investors) the activity is an offense quite distinct from the activity itself.

As with many securities laws, the primary responsibility for disclosure enforcement lies with the SEC. The SEC, for example, recently announced an investigation of whether Chrysler Fiat was inflating its monthly sales figures, (Boudette (2016)). Rule 10b-5, however, also has a private right of action. People who bought or sold the securities can bring lawsuits on their own, seeking to recover damages from the firm, without relying on the SEC. This has given rise to a large number of securities lawsuits, sometimes known simply as “10b-5” cases.

To simplify matters somewhat, a 10b-5 situation arises when a corporation makes some misstatement (a lie or a lie of omission). An investor buys stock in the company. At some later time, the company admits the lie, and the stock price drops, leaving the investor with a loss.

The investor then sues the company. Crucially, at this point, the investor is suing not just to recover her own losses, but also on behalf of a class of all other purchasers in the same time period. This class action greatly magnifies the damages sought in the suit. The suit is brought in a Federal court. The investor is the plaintiff, and the company is the defendant. Both sides retain economic and accounting experts. The case then proceeds through the court system. Typically,
though, the final outcome is not a judicial verdict. Instead, both sides reach a settlement. In the settlement, the defendant company pays the plaintiff’s lawyers, the plaintiff’s experts, and damages to the investors.\textsuperscript{15} The sums involved can be large: Enron settled for over $7 Billion. The Securities Class Action Clearinghouse at Stanford University (in collaboration with Cornerstone Research) maintains a database of the filings (http://securities.stanford.edu).

\textbf{10.1. Informational efficiency}

The importance of efficiency in these cases can be seen by comparing things before and after the idea took hold. Although many ideas related to market efficiency go back a hundred years or more, it was formalized as an economic principle in the 1960’s and 1970’s. Shortly thereafter the idea entered the legal arena. In arguing the legal case for market efficiency, Fischel (1982) captures essence of the transition to our present view.

By the time of Fischel’s paper, the courts had adopted a set of criteria that 10b-5 plaintiffs had to satisfy:

- Materiality. Would a reasonable investor consider misstated or omitted fact important in making an investment decision?
- Reliance. Did the plaintiff actually rely on the misstated fact (or would have relied on the omitted fact) in making the purchase decision?
- Causation. Did the misstated/omitted fact cause the economic loss suffered by the plaintiff?
- Damages. What are the losses (dollar amounts) that can be attributed to the misstated/omitted fact?

These could be difficult things to establish. An investor might have, for example, relied on a thorough financial analysis of the company, perhaps using a valuation model in which the misstatement could be reduced to a single input. The damages could be calculated as the difference between the intrinsic values computed with the wrong and right inputs. But valuation models can be subjective, and it is likely that the defendant could find an expert who would determine the misvaluation to be negligible. Would every plaintiff have to produce her own financial analysis? Even if the analysis could be rigorously documented, would the damages for a purchaser of 100 shares, or even 100,000 shares, be enough to cover the attorney’s costs? These considerations established high barriers to the pursuit of these cases.

But if the market is efficient, all of these requirements can be established by appealing to the market price. A purchaser of the stock might not have used any sort of valuation model at all, but he certainly would have considered the market price of stock, at least at the moment when he traded. Assuming efficiency, this price would have reflected all public information, including the misinformation (alleged by the plaintiff). It is not necessary for the investor to demonstrate direct reliance on the misstatement. It suffices to establish indirect reliance, via the market price. In the new view, someone making a misstatement is causing a fraud on the market. This presumption of efficiency, and indirect reliance on the price, defines a broad class of investors, essentially anyone who purchases the stock while the misstatement is operative.

Market efficiency dramatically simplified the establishment of reliance. The concept also had implications for materiality, causation, and damages, but these results are not as sweeping. An

\textsuperscript{15} By way of disclosure, I was retained and paid as a consultant in one such case.
investor cannot simply claim, “I bought at the efficient price; I sold at the efficient price; the company lied; the company should reimburse my losses.” To establish materiality, causation and damages, the presumption of efficiency certainly helps, but it’s not enough.

The following discussion is drawn from Fischel (1982), Dunbar and Tabak (1999), and Feitzinger (2014).

10.2. The Framework

There is usually little argument about what the company told its investors: accounting statements, SEC filings, press releases and so forth are a matter of public record. The questions of what management actually knew and what should have been told to investors, on the other hand, are frequently at issue. Management’s information can sometimes be reconstructed from emails and other internal communications, so these things are often requested during the discovery phase of the legal process. Defining exactly what management should have disclosed and when is often a question of accounting practice, and expert opinions here can be clarifying. In any case, the information actually available to investors vs. the information that should have been available must be established at the outset. We’ll assume that the misstatement occurred on day 100, with the release of an erroneous accounting statement and ended on day 400, when the company admitted the error (the corrective disclosure).

The plaintiff class is all investors who purchased the stock over the class period, the span of time covering the misstatement. When the suit is initially filed it must name a real purchaser, identified by name. Other purchasers are included by reference, as in, “Joel Hasbrouck, and all others similarly situated.” The defendants comprise the corporation, and often managers and directors who are named individually. The defense usually immediately files a motion for dismissal, and the plaintiffs indicate how they will pursue their case. If the motion for dismissal is rejected, the case proceeds to the discovery phase. In discovery, the plaintiffs may obtain access to internal corporate documents such as emails and memoranda relevant to the case. The plaintiffs use this information to build an informational time line of what management knew, and what they told (or didn’t tell) the investors.

The principles of market efficiency come into play when we try to connect this information to prices. Ideally we’d like to assemble a picture similar to Figure 10.1. This graph shows the actual stock price and a price that we think would have reflected the correct information. Appealing to market efficiency, we might then assert that the actual stock price includes the effect of the misinformation, and the shaded difference between the two lines is a measure of the inflation due to misinformation. Suppose that the misinformation caused the value of the stock to be inflated by $3 per share, beginning on day 100 and ending on day 400.

If we could construct such a picture, we’d could determine the effect of the misstatements on any investor, based on when the investor bought and sold. Let’s look at some of possibilities. Suppose that Amy bought a share on day 290 for $23.61, and sold on day 450 for $13.03. She lost $10.58 on her investment, and since she bought at a price that reflected the error and sold after the error was corrected, she might well believe that she deserves to recover her entire loss. In fact, though, only $3 of her loss is actually caused by the error (and its subsequent correction). Her total loss was larger due to other factors besides the error.

Suppose instead that Amy had sold on day 350 for $21.27, for a loss of $2.34. This is a poor outcome, but none of the loss can be considered to have been caused by the reporting error. Essentially, she bought at an inflated price and sold at an inflated price.
Suppose that Brian purchased the stock on day 80 (prior to the error) for $14.12 and sold on day 450 for $13.03. He, too, has a loss (of $1.09), but since neither his purchase nor his sale occurred on a day when the stock price was inflated, he is not entitled to any damages.

Some investors might have benefitted from the error. Suppose that Cathy purchased stock on day 99 (before the error) for $14.30, and sold at on day 100 for $17.49. Her profit is $3.19, of which $3 is directly attributable to the error. Does she have to give up the $3? Usually, no: she was an innocent beneficiary of the error.

These examples are simplified because the price inflation attributable to the accounting error is a constant $3 and the start and finish times are precisely defined. In practice, though, the inflation might not be constant over time. The start and stop times might also be blurry. The accounting error might have arisen from an incorrect process that was adopted gradually, and the corrective disclosure might have occurred in the form of several pronouncements, spread out over days or weeks. Both the start of the error and the corrective disclosure are likely to be entangled with other announcements or market developments. In attempting to unravel these effects, where do we begin?

10.3. Event studies: an overview

Suppose that the corrective disclosure occurs in the form of a single announcement, released after the close of regular trading hours. Daily stock data usually comes in the form of closing prices, so the close-to-close return spanning the disclosure will run from the market close on day 399 to the closing price on day 400.

We can’t simply attribute the entire day’s return to the disclosure, however. Other information will have affected the price of the stock. Most stocks, for example, have returns that are driven in large part by the returns on the broader market. To separate the effect of the corrective
disclosure from the market return, a good place to the start is the market model, also known as the single-index model.

The market model is a statistical model that is based on simple linear regression. We’ll label the stock that we’re trying to model as stock “i”, and let “m” refer to the overall market (in practice, a market index, like the S&P Composite). Then the return for stock i on day t is $r_{it}$. It is related to the market return for the day by:

$$r_{it} = \alpha_i + \beta_i \times r_{Mt} + e_{it}$$

This breaks $r_{it}$ into three pieces. $\alpha_i$ is a constant “starting point” for the return. $\beta_i \times r_{Mt}$ is the market-related piece: $\beta_i$ is a multiplier, usually a number close to one. Although $\beta_i$ is constant for a given stock, it multiples $r_{Mt}$, which is random and different every day. The final piece, $e_{it}$, captures the part of the stocks return that is not related to the market. Company-specific news (like a corrective disclosure) shows up in $e_{it}$.

The market model is essentially a best-fit line through a scatterplot of returns. To construct the scatterplot, we first compile a sample of returns, observations for $r_{it}$ and $r_{Mt}$ for a number of days. The pair of returns on day t defines a data point on a graph in which $r_{Mt}$ is on the horizontal axis and $r_{it}$ is on the vertical. The alpha is the intercept of the best-fit line; the beta is the slope. Figure 10.2 depicts an example.

---

**Figure 10.2**

![Scatterplot of returns](image)
The single-index model is usually introduced in connection with portfolio theory, the science of combining many risky securities (like stocks) to achieve the best possible trade-off between risk and the return. In portfolio applications, we’re almost always primarily interested in the stock’s beta, which measures risk relative to the market.

For present purposes, though, it is more important to consider how the model can help attribute returns to information. In Figure 10.3 we drill down on one observation. All days have been grayed out, except for the one in which \( r_M = 4.0\% \) and \( r_i = 6.0\% \). The estimated regression equation essentially says that if the market return is four percent, then we’d expect the return on the stock to be

\[
Er_i = -0.013 + 1.352 \times .04 = .041 = 4.1\%.
\]

The actual return is \( r_i = 6.0\% \), so for that day \( e_i = r_i - Er_i = 1.9\% \). This is the unexplained return for the day. Or, more properly, it is unexplained by the market. It is therefore attributed to other influences, such as company-specific news announcements.\(^{16}\)

\(^{16}\) Now if we are ultimately interested in modeling the stock’s price (as in Figure 10.1), why don’t we start with a statistical model that is based on prices rather than returns? Analogously to the return model, for example, we might specify something like \( p_{it} = a_i + b_i p_{Mt} + z_{it} \) where \( p_{it} \) is the closing price of the stock and \( p_{Mt} \) is the level of the market index. The error in this regression, \( z_{it} \), might be interpreted as the price component that contains non-market information (or misinformation). It turns out in most cases that the relation between a stock’s return and the market return is much more stable than the relation between the stock’s and market’s prices. When the regression is specified in terms of prices, the estimates of the parameters (and residuals) are subject to large errors.
10.4. Building the event study

See “Performing the basic event study” in Dunbar and Tabak (1999).

The first step in estimating the value of the information is to determine an event window, a short period of time that corresponds to when the information would have affected the value of the stock. In the situation illustrated in Figure 10.1, the timing of the information is clear. The alleged misinformation started on day 100 and ended on day 400. Looking at the plot, there are two candidates. We might examine the return on day 100, when the price jumped in reaction to the misinformation, or on day 400, when the price dropped at the time of the corrective disclosure.

Typically, though, the misinformation involves a lie by omission. Negative information is withheld from investors, and then belatedly disclosed. Often the first indication of a problem is the price drop at the time of the corrective disclosure. There is no corresponding price jump at the time of the omission because there is nothing that would have caused the market to react. This changes the picture of the value inflation.

Figure 10.4. Withholding of negative information

![Graph showing stock price over time with start and end dates and labels](image)

In Figure 10.4, bad news should have been disclosed on day 100. The correct valuation should have dropped on that day. On day 400 we have a belated corrective disclosure. On day 100, though, there was no news, so the stock price simply continued on its path. Only on day 400 do we have a significant drop. In this case the event window would only include day 400.

In order to compute the residual on the day of corrective disclosure, $e_{400}$, we need estimates of $\alpha$ and $\beta$. These will be based on data compiled over an estimation window. The behavior over the estimation window should be representative of the stock’s normal co-movement with the market. Estimates are usually more precise when the sample is large, so there is a case for including all the available data. Companies change over time, however, as new products are introduced,
new competitors emerge, and so on. These real changes are often mirrored by changes in the stock price dynamics. This argues in favor of a sample that is restricted to data close in time to the event period, usually a smaller sample. In practice, these two conflicting goals must be balanced. It is not usually necessary to exclude the entire period of misinformation. After all, the market could not have been reacting to what wasn’t known.

In addition to the estimates of $\alpha, \beta$, and $e_{400}$ we must also allow for errors in these estimates. To judge how well the model fits the data, we first consider the regression coefficient of determination, the $R^2$. This number indicates the explanatory power of the model, and ranges from zero to one. An $R^2$ close to zero implies that the stock return and the market return are essentially uncorrelated, while an $R^2$ near one suggests that the stock and market are moving in near lock-step. We then construct confidence intervals around the parameters. A confidence interval for $\beta$, for example, is a range, centered around the estimated $\beta$ that contains the unknown true value for $\beta$. The regression model may be extended to incorporate industry effects and other types of announcements.

Summary of terms and concepts

Rule 10b-5 (not exact wording, but the content); material fact/materiality; reliance; causation; inflation (Feltzinger); damages under the “traditional” view (Fischel); material … damages under the market-efficiency view; fraud on the market; of materiality and reliance; corrective disclosure; the single-index stock return model and its use in damage calculation; event window; class period; how we identify the class; identification of injured buyers and sellers.

Problems and exercises

**Problem 10.1** A typical class-action securities lawsuit alleges misstatements that inflated the price of the stock. In such a case, purchasers of the stock recover damages in the form of payments made by ____.

- a. the people who sold them the shares
- b. the corporation itself
- c. the directors
- d. the employees who aided or abetted the misstatement.

**Problem 10.2** An investor class-action law suit alleges that management lied about the rate of recent sales growth. In this context, establishing “reliance” means that an investor must establish that ____.

- a. They relied on management’s misstatements in assessing the intrinsic value of the company.
- b. Management relied on the principle of market efficiency to ensure that the incorrect information would actually inflate the stock price.
- c. Investors who suspected misinformation relied on a liquid market when selling their holdings.
- d. They relied on the price, which reflected management’s misstatements.
Problem 10.3 In investor class-action law suits, a corrective disclosure is a public statement ___.
   a. By management that corrects inaccurate reports in the media.
   b. By the auditor that clarifies a company’s financial statements.
   c. By management that corrects previous misstatements by management.
   d. By the SEC that notifies investors that a company is under investigation.

Problem 10.4 In an investor class action lawsuit, the company announced on September 3, 2015 that previously-reported results for calendar year 2014 were overstated. In the event study used to quantify the economic value of the misinformation, the event window would be ___
   a. The period of time that contains the market’s reaction to the announcement.
   b. The period of time over which investors were at least partially misinformed.
   c. Calendar year 2014.

Problem 10.5 In a daily event study of a public announcement, we might want to begin the event period before the actual date of the announcement if
   a. We suspect that the announcement may have been leaked.
   b. We suspect that management might have deliberately delayed the announcement.
   c. The announcement was made 30 minutes after the close of normal trading hours (4:00pm).
   d. The announcement was made 30 minutes before the market opened at 9:30am.

Problem 10.6 In an investor class-action lawsuit, to establish causation and damages, we usually look for a sharp drop in the stock price at the time of the corrective disclosure. What other evidence might be used?
The next two questions refer to the following situation.

- APX first reported its 2013 IV (fourth quarter) earnings on January 29, 2014.
- On Monday, March 3, 2014, APX closed down 2% from the previous Friday. After the market close, APX discloses that previously reported 2013 IV results can’t be relied upon and are likely to be revised.
- On Tuesday, March 4, APX closes down 18% from the previous day. Also on Tuesday, after the market close, APX discloses that 2013 IV earnings were overstated by $0.30 per share.
- On Wednesday, March 5, APX closes down 8% from the March 4 close.

**Problem 10.7** Randolph and Mortimer Duke immediately file a law suit alleging that the accounting fraud cost them 28% of their investment in APX (the sum of the Monday, Tuesday and Wednesday declines). APX hires an expert. Using market data prior to June, 2014, the single-index model is specified for daily returns as

\[
 r_{APX} = \alpha_{APX} + \beta_{APX} r_M + e
\]

where the expert’s estimated parameters are \( \alpha_{APX} = 0; \beta_{APX} = 1.3 \). The daily returns on APX and the market (“M”) during the week are:

<table>
<thead>
<tr>
<th>Date</th>
<th>( r_{APX} )</th>
<th>( r_M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday, March 3</td>
<td>-2.0%</td>
<td>-4.0%</td>
</tr>
<tr>
<td>Tuesday, March 4</td>
<td>-18.0%</td>
<td>-5.0%</td>
</tr>
<tr>
<td>Wednesday, March 5</td>
<td>-8.0%</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Thursday, March 6</td>
<td>+3.0%</td>
<td>-1.0%</td>
</tr>
<tr>
<td>Friday, March 7</td>
<td>6.0%</td>
<td>5.0%</td>
</tr>
</tbody>
</table>

What case could the expert make to refute the Duke claim? What number might she suggest as an alternative to the 28% loss?

**Problem 10.8** The Dukes also present the following records of their purchases and sales:

<table>
<thead>
<tr>
<th>Date</th>
<th>Shares bought (sold)</th>
<th>Net holdings</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 1, 2013</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>December 29, 2013</td>
<td>(9,000)</td>
<td>1,000</td>
</tr>
<tr>
<td>January 5, 2014</td>
<td>5,000</td>
<td>6,000</td>
</tr>
<tr>
<td>February 1, 2014</td>
<td>2,000</td>
<td>8,000</td>
</tr>
<tr>
<td>March 7, 2014</td>
<td>(8,000)</td>
<td>0</td>
</tr>
</tbody>
</table>

On the basis of these records, they claim that their 28% loss applies to the 17,000 shares they purchased before or during the period affected by the alleged fraud. What alternative number might APX’s expert suggest?
Chapter 11. Private Information

11.1. Overview

The public-information version of the efficient market hypothesis usually strikes people as a reasonable first approximation. After all, if public facts support a unanimous public consensus on the value of a stock, then any deviations from this value will represent clear profit opportunities. Our belief in this principle is sufficiently strong that if we were to see Apple stock (generally trading in the neighborhood of $500 a share) offered at $100 a share (right this instant, take it or leave it), we might well pass. After all, which of the following scenarios is more likely?

- There has been a sudden news announcement that has devastated the value of Apple stock, and we haven’t yet checked the news websites, or
- Sellers of Apple stock have offered us an opportunity to exploit an obvious valuation error.

The second possibility is very unlikely. Yes, one does encounter obvious valuation errors from time to time, but they are very rare and usually for small amounts. The market forces causing prices to reflect public information are powerful ones.

But why should the price of a security reflect private, non-public information? The simplest answer is that anyone who possesses such information has a strong incentive to trade until the public price hits their private estimate of value. For example, suppose that the offer price for a share of a mining stock is $10 per share. One person (“Clarence”) determines that the company’s reserves actually suggest a value of $11. Clarence will buy, depleting limit orders on the offer side of the book, stopping only when the offer reaches $11 per share. Those setting the offer prices may be completely ignorant of the fundamental reason for the price runup (the higher reserves). They only see that a buyer (or buyers) is putting a value of $11 on the stock.

This explanation is a mechanical one, though, and it is far from satisfying. Once all of the offers on the book at $10 are taken, why don’t new ones come in to “refresh” the book? If the
company has made public no new information, why should sellers be reluctant or unwilling to offer shares at $10? Clarence makes a profit on all shares that he buys for less than $11, but the profit on a share purchased at $10 is $1, while a share purchased at $10.10 nets him only $0.90. Why doesn’t he wait until the offers come back in at $10? Below, we analyze the market’s reaction in detail.

For the moment, though, it is useful to emphasize two important things about the process. Firstly, it necessarily involves trading. Without the opportunity to actually purchase the shares, there would be no reason for the market prices to move. Secondly, for the trader to actually realize the profits, the information must be made public. Once the reserves are known to justify $11 per share, buyers will be willing to pay that amount (or something close), the informed trader will be able to unload his shares and realize a cash profit. A patient buy-and-hold investor with favorable private information can capture her advantage gradually over time, eventually realizing the benefit of the information in the stream of dividends that turn out to be higher than what would generally be predicted. But to a buy-and-sell trader, valuable private information is advance knowledge of public information.

The necessity of trade explains the framing of our insider laws and regulations. In the normal course of a firm’s activities, many people inside and outside of the company will be in possession of non-public information. They are simply restricted in the extent to which they can trade. The SEC states:

Illegal insider trading refers generally to buying or selling a security, in breach of a fiduciary duty or other relationship of trust and confidence, while in possession of material, nonpublic information about the security. (U.S. Securities and Exchange Commission (2013a))

To the extent that the law is followed, we would not expect prices to reflect illegal private information. Full compliance with the law is not, however, something that can be simply assumed by other traders in the market. We have many instances of violations, and presumably many instances of violations that weren’t detected. Furthermore, even when violations are detected and prosecuted, the counterparty to the insider’s trade has dim prospects of recovering losses.

The following discussion will show that private information affects markets in profound ways. Among the most important:

- Private information causes a spread between the bid and ask prices.
- Private information induces an order impact: orders that lift the ask ("buys") cause all subsequent prices, including bids to rise; orders that hit the bid ("sells") cause prices to fall.
- The order impact mechanism opens the market to manipulation.
- Trading against people who have superior information will usually lead to losses. Avoiding these losses motives market participants to expend effort to identify their counterparties.

To explain these effects, we first consider the person most directly affected by the private information: the dealer.

11.2. The dealer’s perspective.

When market participants differ in the amount and quality of their information, the market is said to exhibit asymmetric information. Asymmetric information is a feature of many current economic analyses, but one of the earliest descriptions of the problem actually focused on a dealer in the stock market (Bagehot (1971)). “Bagehot” was a pen name adopted by Jack Treynor,
a successful professional money manager. (The real Walter Bagehot was a 19th-century British writer.)

Treynor noted that many investors (retail and professional) appeared to lose money. He suggested that these people lost money because they incurred trading costs, that these costs arose when investors bought at the ask and sold at the bid, and that the reason for the bid-ask spread was the dealers’ dilemma caused by a few people with better (private) information.

Suppose, following Treynor, that the dealer faces two sorts of customers. Liquidity traders are driven by idiosyncratic factors unrelated to the fundamental value of the security. They buy because other activities left them with surplus funds to invest; they sell because there arose an unexpected need for cash. Any one of them is equally likely to buy or sell. A dealer’s trades against such customers can create uncertainty because on any given day liquidity buyers or liquidity sellers might be more numerous. But this inventory risk (or position risk) can be managed by prudently arranging offsetting trades. If a liquidity seller were to arrive, followed a short time later by a liquidity buyer, the dealer would capture the bid-spread. On average, therefore, the dealers’ profit on each liquidity trader is one-half of the spread.

The other group of customers, however, consists of informed traders. By whatever mechanism, legal or not, they simply possess superior information. Unlike the liquidity traders, they have no idiosyncratic trading motives. They will only buy, paying the dealer’s ask price, if they believe that the security is worth more than the ask price. They will only sell, receiving the dealer’s bid, if that bid is above their estimate of the security’s value. They always trade in the direction of their information, never selling when their news is good or buying when the news is bad. In a trade against an informed customer, the dealer loses on average.

The dealer, of course, would like to identify the customers. But this might be very difficult. Orders might arrive anonymously. Even if there is a name attached to the order, the customer might be informed for one set of securities but not another, or be informed in one type of situation. We’ll later encounter situations that might help a dealer make a better guess as to the type of customer, but ultimately the quality of the customer’s type is unknown. The dealer must act as if he faces a random mix of informed and uninformed customers.

In facing this random mix, the dealer’s expected losses depend on the likelihood that the next trader is informed, and the quality of that trader’s information. A population in which nine out of ten traders are informed imposes costs higher than if there were only one informed trader out of a hundred. The losses will also be higher if the trader’s information concerns an upcoming takeover announcement, instead of, say, a promotion of the third assistant sales manager at a regional office.

For the dealer to survive the losses to informed traders must be at least offset by the profits realized from the uninformed traders. The dealer accomplishes this by setting his bid-ask spread sufficiently wide: a high offer price and a low bid price. With a narrow spread the dealer can’t recover enough revenue on the uninformed trades to cover his losses to the informed.

Why should the dealer exercise any restraint at all? If the “efficient” price of the stock is $20, why not bid $15 and offer at $25, making a full $10 profit on each pair of uninformed buyers and sellers? Recall from the earlier discussion of dealers that they are constrained by competition and refusal. Competition comes from other potential dealers who offer their customers better terms of trade. Refusal refers to the ability of customers to decline a dealer’s bids and offers. There is a balance here, in that some customers will be slow to locate and switch to a more competitive dealer, and some customers may have such strong trading needs that they are inclined to take almost any price their dealer offers. But over time, competition and refusal impose substantive limits on dealers’ behavior.
11.3. A formal model of insider trading

What’s the simplest picture of insider thinking we might compose? We’d need a random outcome; an insider who knows the outcome in advance; and at least one other trader (a passive one) who is going to post the bid and offer prices that the insider will hit or lift.

For the outcome, let’s start with and end-of-day share value \( V \) that will be: \( V = \text{Low} \) or \( V = \text{High} \) with equal probability. \( V \) will be determined at the start of day, prior to trading, but whether it is \( \text{Low} \) or \( \text{High} \) is not generally known.

Preparing for the first trade of the day

Giving ourselves a role as the dealer or passive trader, we have to post a bid and an offer. If the insider is the only other trader in the market, we won’t even bother to show up. If \( V = \text{Low} \), the insider will hit any bid above \( \text{Low} \), making a profit for himself, and a loss for us. If \( V = \text{High} \), the insider will lift any offer below \( \text{High} \), and we lose again. The withdrawal of the passive trader is not only an modeling inconvenience, it is a practical problem as well. A market with “too much” private information is not sustainable.

If we consistently lose when trading with the insider, there has to be another trader (or traders) against which we’ll generally profit. So we introduce a group of traders that are generally described as liquidity, uninformed, or noise traders.

Although there are informed and uninformed traders, the first one of the day to arrive and maybe trade at the passive trader’s quotes is either one or the other, not some sort of blend. The selection device is a random draw. Let’s say that there is a 20% chance that the first arriving trader is informed.

Events occur in the following order.

1. Nature (the force of fate and chance) flips a coin: \( V = \text{Low} \) or \( V = \text{High} \). This outcome is
   known only to informed traders.
2. We post a bid and an offer (for one share).
3. The first trader arrives.
   a) With 20% probability, the trader is informed. If \( V = \text{High} \), he’ll buy at any ask price
      below \( \text{High} \); if \( V = \text{Low} \), he’ll sell at any bid above \( \text{Low} \).
   b) With 80% probability, the trader is uninformed. Uninformed traders buy or sell with
      equal probability.

Figure 11.1 describes the sequencing of the random events in the model. (The setting of bid and ask quotes is not indicated.) Now let’s think about our bid. All of the paths that end in “Sell” will hit our bid, but we can’t tell either before or after the trade which path brought us to that point.

We’ll pick some numerical values. Suppose that \( V \) is equally likely to be \( \text{Low} \) ($100) or \( \text{High} \) ($150). (Think of a stock that is presently at $100 per share. At the close of trading, the board will announce whether or not the firm has won a contract worth $50 per share.) And it’s given that the probability of an informed trader arrival is 0.20. These probabilities are shown in italics in Figure 11.1. They are transitional probabilities, reflecting the likelihood of moving from one box (“node”) to another.

From the transitional probabilities, we can compute the joint probability of a path of events that winds up at a particular node. For example, the probability of \( V = \text{High} \) and the arrival of an informed trader is:
The probability that \( V = \text{High}, \text{and} \) the trader is informed, \text{and} the trader hits our bid is:

\[
Pr(V = \text{High}, \text{Informed}) = 0.5 \times 0.2 = 0.10
\]

These joint probabilities are shown in bold in Figure 11.2. Note that the total probability of a buy is 0.5, as is the total probability of a sell.

---

**Figure 11.1 Sequencing of random events (transitional probabilities are in italics).**
Although a buy and a sell are equally likely, the paths leading to these events are not identical. Informed traders never buy on bad news, and never sell on good news. As a result, the trade direction tells us something. If our bid is hit, we learn something. The relevant conditional probability here is:

\[ P(V = \text{Low} | \text{Sell}) = \frac{P(V = \text{Low}, \text{Sell})}{P(\text{Sell})} = \frac{0.1 + 0.2}{0.5} = 0.6 \]

You can check that \( P(V = \text{Low} | \text{Buy}) = 0.4 \), \( P(V = \text{High} | \text{Buy}) = 0.6 \), and \( P(V = \text{High} | \text{Sell}) = 0.4 \).

Our revised beliefs affect our expectation of the firm’s value. Before observing the trade, we thought the firm was worth the unconditional expected value:

\[ E[V] = 0.5 \times \text{Low} + 0.5 \times \text{High} = 0.5 \times 100 + 0.5 \times 150 = 125 \]

The expected value conditional on a sell is,

\[ E[V | \text{Sell}] = 0.6 \times \text{Low} + 0.4 \times \text{High} = 0.6 \times 100 + 0.4 \times 150 = 120 \]

In this way, we (and other uninformed market participants) can “read” the order flow, to draw inferences about what the informed participants already know.
How should we set our bid price? If we are unconstrained, we'll set it low, to make as much profit as possible from the uninformed traders. But in reality, when we bid we are competing against other actual or potential bidders. The question then involves how high we should be willing to bid.

Suppose that before the first order arrives I'm bidding $121, and that's the best bid in the market. If someone hits my bid, I've bought the stock at $121. The problem is, given that someone hit my bid, I (and the rest of the market) think the stock is worth (on average) $120. I have a one-dollar loss.

If I lower my bid to $119, then if I'm hit, I'll have (on average) a $1 profit. But why should other traders let me capture that profit? Someone will bid $119.10, someone else will raise to $119.20, and so on, at least until the bid is $119.99 (one tick below $120). Anyone hit at that price will make an average $0.01 profit.

If anyone overbids, above $120, they'll be facing an expected loss. Would someone bother to bid exactly $120? Would someone go to the trouble of putting in a zero-expected-profit order just for the psychic joy of doing a trade? We probably shouldn't rule out the possibility. The point is that the bid is set pretty close to $E[V|Sell]$.

In this view, on average, bidders break even. Informed sellers make money, and uninformed sellers lose. Informed trading works like a tax on uninformed traders. The analysis on the offer side of the market is similar, leading to an ask that is close to $E[V|Buy]$.

When we vary the assumptions that go into this model, we get some interesting results.

- Increasing the relative proportion of informed traders in the population (from 20% to, say, 30%) causes the competitive bid to fall. The ask rises, and the bid-ask spread increases.
- We can raise the volatility of the security by increasing the distance between the low and high values. (For example, instead of Low = 100 and High = 150, let Low = 90 and High = 160.) If we do this, the bid-ask spread increases.

Real markets seem to show both effects. Immediately prior to a scheduled news announcement the spread tends to widen. This may also be the time when, in the process of releasing the news, there is the largest chance of a pre-announcement leak. We also find that when market volatility increases due to an intense flow of public news, spreads are higher.

The second trade and order impact

One of the mystifying things about the mining stock example was why limit order sellers did not refresh the orders depleted by the informed trader. With no news announcements from the company, and no apparent change in the company's fundamental economic prospects, why were they suddenly reluctant to sell at a price ($10) that seemed fair and reasonable only a few minutes earlier.

Almost all security markets exhibit something called order impact (or price impact, depending on whether we want to highlight the cause or the effect). This refers to a movement in market prices that is driven by and apparently caused by incoming marketable orders. For example, a sequence of incoming buy orders that successively lift the ask will move all market prices (bid, ask, and subsequent trade prices) upwards. In the case of the ask quote the reaction is almost mechanical: execution of the buy orders reduces and eventually exhausts the quantity at the prevailing ask price, exposing the next higher sell limit order in the book, and so on. But the bid and subsequent trade prices (even those resulting from marketable sell orders) will also generally be
higher. Often the price reverts, but this reversion is usually only partial. A portion of the order impact appears to be permanent.

Increased demand in any market, from candy bars to cars, is also generally associated with a price increase. So why is order impact in security markets remarkable? For one thing, there is an apparent disconnect between the relative size of the order and the price impact. For example, a firm might have 10 Million shares of stock outstanding. At a price of, say, $20 per share, the firm’s equity market capitalization is $200 Million. A 1,000 share marketable buy order might move the market share price upwards by $0.01. The total value of the order is about $20,000. Yet the market capitalization of the firm has increased by 10 Million x $0.01 = $100,000, about five times the total value of the order. It is as if an over-eager car buyer, accepting the dealer’s high initial “list price” offer, caused the price of that model to increase for everyone else in the world.

To answer these questions, we return to the simple model, and look at what happens next. Once the first trader arrives and bid is hit or the offer is taken, the probability of low and high values are revised. In setting the next bid and offer, the market looks ahead to the second trade. Suppose that the first trade was a “sell” (the bid was hit). The analysis of the second trade is formally identical to that of the first trade, but our initial assessment of the probability of a low value is 60%. The event tree is given in Figure 11.3.

*Figure 11.3. The tree for the second trade*

<table>
<thead>
<tr>
<th>Value Realization</th>
<th>Trader arrival</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low, 0.6</td>
<td>Informed, 0.12</td>
<td>Buy, 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell, 0.12</td>
</tr>
<tr>
<td>High, 0.4</td>
<td>Uninformed, 0.48</td>
<td>Buy, 0.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell, 0.24</td>
</tr>
<tr>
<td></td>
<td>Informed, 0.08</td>
<td>Buy, 0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell, 0</td>
</tr>
<tr>
<td></td>
<td>Uninformed, 0.32</td>
<td>Buy, 0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sell, 0.16</td>
</tr>
</tbody>
</table>

By summing the joint probabilities, you can see that \( P(Sell) = 0.12 + 0.24 + 0.16 = 0.52 \), and \( P(Buy) = 0.24 + 0.08 + 0.16 = 0.48 \). So on the second trade we don’t expect the incoming order flow to be symmetric. The probability of a low value conditional on a second sell is
\[
P(V = \text{Low}|\text{Sell}, \text{Sell}) = \frac{P(\text{Low}, \text{Sell}, \text{Sell})}{P(\text{Sell}, \text{Sell})} = \frac{0.12 + 0.24}{0.52} = 0.692
\]

So although we started believing that the probability of a low value was 0.5, once two sellers in a row have arrived, that probability is up to 0.692.

Conditional on observing the first sell, and then the second,

\[
E[V|\text{Sell}, \text{Sell}] = 0.692 \times \text{Low} + 0.308 \times \text{High} = 0.692 \times 100 + 0.308 \times 150 = 115.38
\]

Repeating the argument that we used above, this should equal the next (second round) bid price. On the offer side of the second market, if the incoming trade following the first sell is a buy, \(P[V = \text{Low}|\text{Sell}, \text{Buy} = 50\%]\), and \(E[V|\text{Sell}, \text{Buy}] = 0.5 \times \text{Low} + 0.5 \times \text{High} = 0.5 \times 100 + 0.5 \times 150 = 125\). With competitive sellers, this will be the new ask price.

This is what we believed before there was any trade. This is reasonable, if the initial seller is followed by a buyer, the order flow appears balanced and symmetric. It is one-sidedness in the order flow that reveals the private information.

This analysis also clarifies why bids and offers are not generally refreshed at the same level, once acted upon. Although there may be no new fundamental information released by or concerning the company, the fact of the trade itself (and the fact of its direction, buy or sell) enters the public information set. After the trade, the market has learned something.

**11.4. The dynamics of prices and orders**

We can use the model to study any sequence of bids and offers over any horizon. SimpleSequentialTradeModel02.xlsx (on the website) is set up to perform these calculations. Figure 11.4 depicts a graph for a sample sequence of twelve trades: three buys followed by nine sells. At each trade, the top of the vertical line is the ask immediately prior to the trade (and the bottom of the line is the bid). A dot marks the trade.
Figure 11.4. Sample price path

Although this is a graph for one particular sequence of trades, it illustrates some general features of the model and, we think, of reality.

Market prices (in the general sense of bids, asks and trades) follow the order flow. Buys drive prices up; sells drive prices down. This order (or price) impact is an important consideration in trading strategies. Order-splitting strategies divide a large order into multiple child orders, which are executed over time. It is an empirical fact that the child orders executed early in the sequence move the price to the detriment of the later child orders.

Since the order flow reflects on average the true value outcome, the private information is revealed in the market prices over time. We’d be more likely to observe a sequence of nine sells when the value is in fact Low.

Not only does the order flow give us a directional signal about the true value, it also reduces (over time) the uncertainty about the true value. After a period of one-sided order flow, the spread narrows.

11.5. Causation and detection

A customer lifts the ask (buys) and subsequent prices rise. If the customer had hit the bid (sold), subsequent prices would be lower. In this sense, incoming orders cause price movements. But the ultimate value of the security does not depend on the order flow. In our stylized model the value is randomly set at the start of the day. In real-world equity markets, the value is determined by the production, sales and costs of the underlying firm. Most changes in the firm’s ownership do not affect these things. (Exceptions involve activist investing, takeovers, and so on.)

The apparent impact of order flow on prices and the ultimate irrelevance of order flow for firm value seem completely inconsistent. On a deeper level, though, they agree. This is because
the effect of order flow comes about because it reveals, through the trading process, what has already occurred. Order flow is a signal, a window onto things that aren’t yet public.

This also explains the apparent paradox of scale that opened this chapter. Why can a trade cause a change in market capitalization that is much larger than the value of the trade? The answer is that the trade is a signal of value, not just for the shares involved in the trade, but for all shares held by everyone.

Can we identify insider trading after the fact by examining the trade-price record immediately prior to an information release? Perhaps. Some illegal insider trading has been detected in situations where the orders are unusually large, of localized origin (a particular broker or city), and overwhelmingly one-sided (on, of course, the profitable side). It is well to remember, though, that the dynamics that arise in the simple multiperiod model are a consequence of the market’s beliefs about the incidence of informed trading – not the actual incidence. If the market believes that the proportion of informed traders in the population is zero, an insider will enjoy an extremely deep market, and may trade extensively with little impact.

Statistical models of impact play a large role in the formation of dynamic trading strategies. Their estimation and uses are considered in Chapter 15.

11.6. Further reading

The academic literature on asymmetric information in securities markets is largely divided into two approaches. The sequential trade models (like the one considered in this chapter) view the market as a series of moves where at any given time one player is facing a decision. Some of the early key papers are: Easley and O’Hara (1987); (1991); (1992); Glosten and Harris (1988); Glosten and Milgrom (1985). Strategic trade models form the other main branch. In these models, one or more informed traders interact over time with market makers and uninformed traders. Mostly in these models it is the informed traders who are strategic, trading enough to make profits, but not so intensely as to move the market prices any more than necessary (as in the order splitting models of Chapter 16). Representative papers would include Foster and Viswanathan (1993); Holden and Subrahmanyam (1992); Kyle (1985); (1989); Spiegel and Subrahmanyam (1992); Subrahmanyam (1991). Back and Baruch (2004) provide an elegant synthesis that bridges the two approaches.

Summary of terms and concepts

Private vs. common values; symmetric and asymmetric information; Treynor/”Bagiehot” analysis of the bid-ask spread. The sequential trade model of the bid-ask spread and price impact; market failure.
Problems and exercises

**Problem 11.1** Retail order flow is more desirable to a market maker because, relative to institutions/institutional orders, ____.

a. Retail traders are less likely to be informed (in possession of non-public information).
b. Retail traders are more likely to use order splitting strategies.
c. Retail traders are more likely to direct their orders to specific trading venues.
d. Retail orders are usually subject to payment for order flow arrangements.

**Problem 11.2** At the end of the day, SLK stock will be worth 100 (an 80% probability) or 110 (a 20% probability). A market-maker believes that 90% of her incoming traders are uninformed, and equally likely to buy or sell; the remaining 10% know SLK’s end-of-day value. The market maker is trying to set his ask price. At what value of the ask does she just break even?

**Problem 11.3** Before posting her ask price, the market maker notices that the date is April 15. Uninformed traders are more likely to be selling stock to pay their taxes. She now figures that there is only a 30% probability that an uninformed trader will be buying. What is her revised ask price?

**Problem 11.4** Clarence is an accountant working for a client, the ABC corporation. At a meeting to discuss the upcoming annual report, Clarence learns that ABC will shortly be making takeover bid for XYZ. The accountant buys XYZ stock. In his insider trading trial, the defense presents expert evidence establishing that the impact of Clarence’s orders on the price of XYZ’s stock was exactly the same as the impact of all other orders in XYZ. The defense claims that if Clarence had in fact been trading on inside information, the impact of his trades would have been larger. Could the evidence be correct? And, if so, is the defense claim correct? Discuss.
Chapter 12. Insider Trading

Inside information is private information that has crossed the line into illegality. But where and how should that line be drawn?

12.1. The social value of informed trading

Informed traders live under a cloud of negative associations. The clearest examples of private information are the most extreme cases of illegal inside information. Furthermore, as in the model, the profits earned by the informed traders often come at the expense of the uninformed. Are the informed traders simply parasitic? Should the insider trading laws be broadened to include all private information, no matter how obtained?

Unfortunately, information is costly to produce. Securities analysts are highly educated professionals, and they rely on comprehensive (and expensive) data. Were they to cease their research, the market would be reliant on information produced by the listed company. If a modest profit at the expense of the uninformed is the price of independent research, perhaps private information is not as bad as we might have initially thought.

12.2. The social cost of informed trading

In our simple model of the last chapter, the profits of the informed traders come at the expense of the uninformed traders, that is, other investors. Market makers on average break even. In real world markets, of course, market makers may well fare better than simply breaking even, but there is no question that informed traders profit at the expense of the uninformed. So if, in a sense, the market is “rigged” against them, why do uninformed traders participate?

The uninformed might participate out of ignorance, of course. They might also believe themselves smarter than they in fact are. But even limiting our attention to the rational and sober, it is
a fact that uninformed traders participate because the pursuit of their long-term investment goals makes the stock market difficult to avoid.

Difficult, perhaps, but not impossible. Could the process become so stacked against public investors that they simply refuse to invest? Looking about the globe, we must admit this possibility. More than a few countries have no public capital markets. Investment is made either by the government, or by family firms.

Private information raises the possibility of market failure. Once the investing public has “lost confidence” in the securities market, it is very difficult to restart the market. Capital is not raised, risks are not hedged, and the entire economy suffers. The consequences of crossing this line are so costly that regulators often try to ensure that markets stay well to the safe side of it. This takes the form of required disclosure, and prohibitions against insider trading.

### 12.3. The boundary between public and private information

In the section on public information, we noted that information can only be considered public if it is widely available. An importance aspect of this availability is the delay or, in current jargon, the *latency* of the communication process. A trader who receives the information first, and has the opportunity to trade on it, is enjoying a brief interval where the public information is for all intents and purposes, private. The justification suggested above in favor of private information is most convincing when the private information arises from the analysis of fundamental information. It is less attractive if the private information is being produced by expenditure on a computer network that will allow the user to trade on the basis of a press release ten milliseconds before everyone else.

### 12.4. Manipulation

Trade not only facilitates the incorporation of private information into prices. It almost inevitably creates new private information. If the first trader in the market buys, the price goes up by the same amount whether or not the trader is informed. The market reaction is the same because nobody in the market can tell what group the incoming trader belongs to. The trader herself, though, does know (barring self-delusion) her group.

Suppose that the first trader of the day is uninformed and she buys. The prices rise. From the viewpoint of all uninformed traders in the market, this is a fair and appropriate reaction given the possibility of informed trader. All traders, that is, except one. The first trader knows she’s uninformed, and given that knowledge the price reaction is a mistake.

In general uninformed traders can move prices – not because they actually know anything, but because the market can’t tell whether they’re informed or not. Are there trading strategies where they can use this to their advantage? Can they drive up the price by buying, for example, and unload all of their shares at the top? In the simple model of this section, this strategy doesn’t work, but slight changes in the model can make it profitable.

The term “manipulation” does not have a universally agreed-upon definition, either in economics or in law. But from a rough “I-know-it-when-I-see-it” viewpoint, manipulations often involve market activity that moves the price to an “artificial” level, and trades that attempt to profit from this movement.

How far can an uninformed trader push the price, and for how long? Remember that the order impact occurs because the market reads the order as a signal of information that is private for the moment, but is soon expected to become public. An urgent flurry of purchases may appear
to originate from buyers anxious to establish their positions prior to a favorable news announcement. But if no public announcement is forthcoming, and particularly if company management denies the existence of any development that might explain the purchases, the share price will revert.

So suppose that events in fact play out as we suggest. An uninformed trader throws in a string of buy orders, at increasing prices. Then, after it is clear that no favorable developments are pending, the price falls back to where it was originally. Doesn’t this mean that the shares purchased at high prices are now worth less, and that that trader has incurred a loss? This might indeed be the case for the series of trades considered in isolation for everything else.

But market prices are sometimes used as reference prices, for determining the value of “cash-settled” derivative contracts or for computation of margin requirements. The trader might therefore have a strong interest in a higher or lower price at a particular instant, but not at other times. Suppose, for example, that a trader has established a highly levered long-margin position (with borrowed funds). The account is typically valued at closing prices for purposes of determining how much actual cash the trader must put up. By pushing up the price at the close, the trader can increase the apparent net worth in the position, and minimizing the additional cash contribution.

It is not the purpose of this discussion to certify whether or not this practice, a form of “marking the close,” constitutes manipulation in a formal economic or legal sense. Regulatory records, however, contain numerous instances where traders have settled the charges and paid penalties, in preference to asserting innocence at a trial.

12.5. US insider trading laws

Under US law, persons who trade while in possession of superior information might be subject to civil or criminal penalties. The relevant statutes (written laws) do not precisely define what constitutes illegal insider trading, however, so our practical sense of the term is the outcome of principles and precedents that have emerged over the years as the courts have decided actual cases. This evolution is still in process. In many areas technological advances have shaped and reshaped our notions of information, its economic value, and its property rights (who owns it). This is certainly true in how we think about insider trading.

Rule 10b-5 states that, “It shall be unlawful for any person ... to engage in any act ... that operates as a fraud or deceit upon any person in connection with the purchase or sale of any security.” Insider trading is subject to regulation because it is potentially fraudulent.

Who exactly is an “insider”? With respect to a particular firm’s securities, a corporate insider is a manager, officer or employee of the firm. In the normal course of business, we’d expect them to know more about the firm than most others who hold or trade the firm’s stocks or bonds. But the reach of the law extends beyond this group. Accountants, attorneys, consultants, and others who work with the firm may also learn about the firm’s activities. These outsiders may be considered constructive insiders, that is, those whose relations with the firm make them, at least temporarily, in a position similar to that of a manager, officer or employee. A tipper is someone who passes along information; a tippee is someone who receives it.

Information can be material in the everyday sense of “significant”. In this context, though, the courts have adopted a criterion that the information would likely be considered important by an investor buying or selling the stock. A considerable body of law has evolved to refine a more precise definition of the term.
Of course, small bits of information can add up to strong conclusions. This is sometimes called the mosaic principle. Takeover announcements, for example, are usually very material to the value of a target, and many insider trading cases have involved direct leaks of target identities and takeover timing. But public news releases, speeches, and so forth can also signal an acquirer’s intentions. Defendants sometimes advance the mosaic defense, that argument that a takeover was in fact deduced from disparate facts, hearsay and rumor. In commenting on a series of 2013 cases, however, Morrison-Foerster (2015) note that, “The ‘mosaic theory’ defense, while still viable in theory, met with no success in court because defendants who have asserted that they legitimately pieced together a cogent investment thesis from bits of immaterial nonpublic information faced direct evidence – from wiretaps or former co-conspirators – that provided far less innocent explanations for their purchases or sales of securities.”

The following discussion first discusses the rules for corporate insiders, and then considers the broader rules that might apply to anyone buying to selling security.

12.6. Special provisions for corporate insiders

Corporate insiders are prohibited from trading on the basis of non-public material information. They can’t sell short (thereby profiting from declines in the value of the firm). Profits realized from buying and selling within six months are considered short-swing profits, and must be given to the firm. They must report their trades to the SEC within two days.

Managers have all sorts of legitimate reasons to hold their stock. An ownership stake aligns their interests with those of the other shareholders. On the other hand, a stake that represents a large proportion of a manager’s wealth is undiversified, and therefore especially risky. Selling part of this investment will reduce the risk, and may be desirable even if the manager has an overall favorable view of the stock’s expected performance. To allow for these sorts of transactions, the law contains a safe harbor, a course of action that is presumed to protect a manager against prosecution. Specifically:

... [A] person’s purchase or sale is not “on the basis of” material nonpublic information if the person making the purchase or sale demonstrates that ... Before becoming aware of the information, the person had:

(1) Entered into a binding contract to purchase or sell the security,

(2) Instructed another person to purchase or sell the security for the instructing person’s account, or

(3) Adopted a written plan for trading securities;

These so-called “10b-5” plans are used for routine transactions.

12.7. Some key principles of US insider trading law

Although insider trading is not clearly defined in any rule, principles have emerged over time as cases have been argued.

Disclose or abstain

The principle of “disclose or abstain” holds that anyone in possession of non-public material information must disclose that information to a counterparty or simply refrain from trading. Since
it is not often practical in an anonymous decentralized security market even to identify one's counterparty (let alone disclose the information), the principle is effectively a prohibition.

In 1964, Texas Gulf Sulfur (a US mining company) determined that an area of Ontario where they'd been engaged in geological explorations was likely to be particularly valuable. Company personnel bought stock well before the news was made public. The SEC brought an insider trading case and won.

_Fiduciary duty of confidentiality_

A “fiduciary duty” is a legal obligation to act on behalf of someone else. The courts have held that a duty to disclose only exists if the information was obtained through a relationship of trust. This principle narrows the applicability of disclose or abstain.

In Chiarella v. U.S. (1980), Chiarella worked for a financial printer. In printing documents for corporate acquirers, he determined the identities of target firms. He purchased the stock prior to the announcement. Chiarella (an outsider) was convicted, but the Supreme Court reversed the conviction on the grounds that he had no fiduciary relationships to the target companies or their stockholders.

_Misappropriation of information_

Misappropriation involves taking something (information, in this case) and using it for trading purposes.

O'Hagan was a partner in a law firm, Dorsey and Whitney. The firm was advising Grand Metropolitan on a takeover of Pillsbury. O'Hagan was not working on this case, but became aware of the deal by seeing papers on another partner’s desk. He bought shares of Pillsbury, and profited when the takeover was later announced. Since Grand Met was not his client, there was no violation of attorney-client confidentiality. The information that he used in trading, though, was not his to use.

R. Foster Winans (ca. 1983) wrote a column “Heard on the Street” for the Wall Street Journal. Companies mentioned often experienced price moves after the column appeared. Prior to publication, Winans identified the stock stock others (including a stock broker), who went on to generate large trading profits. The SEC asserted that the information was the property of Winans’ employer (the Journal), and in tipping others he had misappropriated that information from the Wall Street Journal. The Court of Appeals agreed. Interestingly, if we grant that the Wall Street Journal owned the information, presumably it would have been okay for them to use the information in trading.

In using the Journal’s information for personal profit, Winans was breaching an employee’s _duty of confidentiality_. The duty of confidentiality can be critical in ascertaining misappropriation, so it’s important to note that it can arise in all sorts of business and personal relationships. From the Code of Federal Regulations, §240.10b5-2 _Duties of trust or confidence in misappropriation insider trading cases:

... (b) Enumerated “duties of trust or confidence.” For purposes of this section, a “duty of trust or confidence” exists in the following circumstances, among others:

(1) Whenever a person agrees to maintain information in confidence;
(2) Whenever the person communicating the material nonpublic information and the person to whom it is communicated have a history, pattern, or practice of sharing confidences, such that the recipient of the information knows or reasonably should know that the person communicating the material nonpublic information expects that the recipient will maintain its confidentiality; or

(3) Whenever a person receives or obtains material nonpublic information from his or her spouse, parent, child, or sibling; provided, however, that the person receiving or obtaining the information may demonstrate that no duty of trust or confidence existed with respect to the information, by establishing that he or she neither knew nor reasonably should have known that the person who was the source of the information expected that the person would keep the information confidential, because of the parties' history, pattern, or practice of sharing and maintaining confidences, and because there was no agreement or understanding to maintain the confidentiality of the information.

Tipper-tippee liability

Often, as in the Winans case, the source of the information does not trade directly, but passes the information along to someone else. This separation does not remove legal liability. There might still have been an initial breach of a duty of confidentiality. The tippee (recipient of the information) may still be liable if they knew (or should have known) about the breach.

Sometimes the information can pass through many people before someone actually trades on it. In an SEC case involving IBM’s takeover of Lotus, at least one person who traded only learned of the information after it had been passed on through seven other people (U.S. Securities and Exchange Commission (2001a)).

In 1973, Raymond Dirks was a securities analyst who learned about fraud in the Equity Funding of America (an insurance company). Dirks alerted his clients, and they sold before the fraud became widely known, and Equity collapsed. Dirks’ source of information was Secrist, a former Equity insider. The SEC initially censured (penalized) Dirks, but this was ultimately reversed by the US Supreme Court. In this case, the Court found that there had not been a breach of a fiducial duty because Secrist derived no benefit from his disclosures.

The question of benefit

It is a commonplace truth that we derive benefits from sharing information in our relationships. Recommendations and endorsements about everything from movies to medical providers get passed back and forth among family and acquaintances in the normal course of maintaining and strengthening the bonds. When we say that someone “owes us one” or that a good deed “went into the favor bank,” we’re using terms of commerce to describe a general system of social barter and exchange. Where, then, does valuable inside information fit within this scheme?

In 2012, Anthony Chiasson and Todd Newman were convicted of insider trading in a jury trial. The instructions to the jury limited their determination to whether or not Chiasson and Newman knew that the information on which they relied had been improperly disclosed. The question of whether the ultimate source of the information received a personal benefit was not, according to the instructions, relevant. In December 2014, a panel of justices from the 2nd US Circuit Court of Appeals reversed the conviction, however, arguing that the question of benefit was
an important one, and that “the mere fact of friendship” does not suffice to establish a presumption of such a benefit. In January, 2015, the US Attorney (Preet Bharara) filed for a full review of the case.

12.8. Practical advice

Many of us are or will be knowledge workers in a knowledge-based economy. Many careers involve the synthesis and use of information that is economically valuable, information that at times is non-public and material. While some situations may be fraught with ambiguity, there are several clear guideposts. Many of the situations discussed in this section fall under the broad umbrella of employee conduct. Apple’s Business Conduct Policy, for example, states (in part):

Never buy or sell stock when aware of information that has not been publicly announced and could have a material effect on the value of the stock. This applies to decisions to buy or sell Apple stock and to third party stock, such as the stock of an Apple supplier or vendor. It is also against Apple policy and may be illegal to give others, such as friends and family, tips on when to buy or sell stock when aware of material, nonpublic information concerning that stock.

Some of the FAQs address the kind situations that any of us might face.

I have stock in companies that do business with Apple. Is this a problem?

Probably not. However, it could be a concern if (1) you’re influencing a transaction between Apple and the company, or (2) the transaction is significant enough to potentially affect the value of your investment.

Does Apple’s policy apply to buying or selling stock in other companies?

Yes. For example, say you learn about a customer’s nonpublic expansion plans through discussions about hardware purchases. If you purchase stock in the customer’s company or advise others to do so, it could be viewed as insider trading.

Most regulated firms in the financial sector (such banks, brokerages, investment advisers) are required to have a designated compliance officer. The compliance officer should be consulted in any case of doubt.

Summary of terms and concepts

Disclose or abstain; fiduciary duty; tipper-tippee liability; misappropriation of information; short-swing profits; 10b5-1 plans and the criticisms; economic arguments and counter-arguments in favor of insider trading.

Cases: Cady, Roberts (disclose or abstain); Texas Gulf Sulfur (disclose or abstain); Chiarella (the requirement of a fiduciary duty); O’Hagan, also Carpenter [Winans] (misappropriation of information); Chiasson and Newman (the requirement of some element of payment).
Problems and exercises

Problem 12.1 In support of the idea that insider trading laws should be abolished, all of the following arguments are advanced, except ____.
   a. Insider trading increases the informational efficiency of stock prices.
   b. Insider trading is simply a way of compensating managers.
   c. Insider trading will encourage managers to quickly publicize specific corporate developments.
   d. Any limits to insider trading by managers are best addressed in their employment contracts.

Problem 12.2 Chiarella was a financial printer who learned the identities of target firms and bought their stock before the announcement of the takeover bid. He was ultimately ______, on the grounds that ____.
   a. Convicted; he misappropriated information from his employer (the printing company).
   b. Found innocent; he had no fiduciary responsibility to the target companies.
   c. Convicted; he was simply trading on the basis of material non-public information.
   d. Found innocent; his trades had no material impact on the bidding company.

Problem 12.3 O'Hagan was an attorney who learned the identity of a client’s takeover target, and bought shares prior to the announcement. On charges of insider trading he was ultimately ____ on the grounds that ____.
   a. Found innocent; he had no fiduciary obligation to the target company or its shareholders.
   b. Found innocent; he did not pay anything to acquire the information.
   c. Found innocent; he did not misappropriate any information from the target company.
   d. Convicted; he misappropriated information from this employer.

Problem 12.4 Raymond Dirks was a securities analyst and a tippee (someone who received information from a tipper, in this case Ronald Secrist). The SEC accused Dirks of insider trading in Equitable Funding, but the Supreme Court said that it wasn’t insider trading because ___.
   a. Secrist’s information concerned a fraud that the SEC should have detected, but didn’t.
   b. Secrist did not receive any personal benefit in exchange for passing along the information.
   c. Dirks was not did not benefit in any way from the information.
   d. Dirks did not owe any duty of confidentiality to Secrist.

Problem 12.5 Newman was convicted of insider trading on information that originated in the Dell investor relations department. In reversing his conviction, the appeals court noted that the SEC did not convincingly establish that ___. (Possibly more than one correct answer.)
   b. Newman was aware that the information was obtained in violation of a duty confidentiality.
   c. Newman knew that the tipper (at Dell) received some sort of benefit.
   d. Newman knew that the information was accurate.
Problem 12.6 Consider the following situation.

- IceMountain (ICE) and SlipperySlope (SSL) are both publicly-traded companies that manufacture snowboards.
- ICE has done extensive tests of both firms' products; this testing clearly indicates that ICE boards are better.
- Next week, Downhill Magazine will release its annual review of snowboards.
- Mike, the president of ICE, is certain that Downhill's review will confirm the ICE tests, and the price of SSL will drop. He sees an opportunity.
- He plans to sell short, in his personal account, 10,000 shares of his competitor, SSL, just before the review is released.

How might the principles of disclose-or-abstain, fiduciary duty, and misappropriation apply in this situation?
Part IV. The Basics of Algorithmic Trading

A trading algorithm is a recipe, a structured procedure for trading, that is generally implemented on a computer directly connected to the market. Algorithms ("algos") lie on a spectrum. At the simple end are qualified orders (like immediate-or-cancel) that are a bit more reactive and adaptive to market conditions. These can be classed as conditional orders. More complex algos, though, generally rely on assumptions about how market prices and orders evolve, and particularly about how our own orders will affect the prices. These assumptions take the form of dynamic statistical models. Once we have a statistical model, we can formulate a trading problem and determine the optimal strategy. We apply this approach to the classic order splitting problem.
Chapter 13. Complex Orders

Earlier we looked at qualifiers such as immediate or cancel (IOC), all or none (AON), fill or kill (FOK), that modified the handling of standard market and limit orders. Conditional orders are those for which activation or execution depends on some market event, like the stock price hitting a pre-specified level.

The dividing line between qualified and conditional orders is not a precise one. The complexity of an order or order strategy lies on a continuum. Labeling the orders discussed in here as “conditional” is simply a way to indicate that they are more complicated than the qualified orders discussed earlier, but less complicated than the multi-stage algorithms that will be discussed later.

The rules defining conditional orders are straightforward, and it is usually easy to identify one dominant appealing feature. On the other hand, it can be very difficult to decide which type is optimal in any given situation. Moreover, in the right circumstances, with the right knowledge, they can often be “gamed” (or “tricked,” to use a less sporting term). For each type, you might well ask, “How might I respond if my competitors (on the same side of the market) were using one of these algos?” Or, “… if my potential counterparties (on the opposite side of the market) were using one?”

13.1. Stop orders

A stop order is elected (that is, becomes active) when there is a trade at or through the stop price. The stop price is different from the limit price. A stop sell order is also called a stop loss order.

Example: “Sell 100 MSFT stopped at 24, limit 23” would typically be entered when MSFT is trading well above 24. When there is a trade in MSFT at 24 or below, the stop order is elected (becomes effective). It becomes “Sell 100 MSFT limit 23”. This is a stop limit order because it carries a limit price. Some markets accept stop market orders. Upon election, the order becomes “sell 100 MSFT at the market.”
A stop loss order might be used when an investor has an accumulated profit on a long position. The stock was purchased; the price went up, and the investor wants to keep at least some of the profit in the event that the price drops. (With a trailing stop loss order, the stop price is reset relative to the the highest price recently realized.) Alternatively, losses in a long margin position can mount rapidly, due to leverage. A stop loss order can provide an automatic exit.

A stop loss order is not really an order until the triggering event occurs and the order is elected. It is sometimes thought that a stop loss order ensures a sale at the stop price. In fact, there is no such certainty. A market stop loss order becomes a market sell order when there is a trade at or below the stop price. If the bid is falling rapidly, a market sell order might execute well below the stop price. If the bid has fallen through the limit price of the stop loss order, the order won’t be executed. Before it is elected, a stop order is not placed in the book, made available for execution, or accorded any time priority (except, perhaps, relative to other stop orders).

A stop buy order works in the opposite direction. “Buy 100 MSFT stopped at 28, limit 29” might be entered when MSFT is trading below 28. When there is a trade at 28 or higher, the order is elected and it becomes “Buy MSFT limit 28.” It may be an attractive option for an investor holding a short margined position. This investor faces the risk of a sudden price rise.

Stop orders are not displayed until they are elected. If a trader suspects that there are many stop-loss orders waiting at a particular price, he may aggressively short the security, driving the market down to the stop price. The election of these orders triggers a wave of selling that can quickly drive the price down further. The short seller then covers at the lower price, realizing a profit. This practice, known as “gunning the stops,” is usually considered manipulative, exposing the trader to legal and regulatory sanctions.

In this connection, the U.K. Financial Conduct Authority recently determined that five banks (Citibank, JPMorgan Chase, RBS, UBS, and HSBC) engaged in this practice in the FX market: “Traders [from these banks] shared the information … to help them work out their trading strategies. They then attempted to manipulate fix rates and trigger client “stop loss” orders.” Retail traders, incidentally, are sometimes believed to herd in the placing of their stops, grouping at “natural” price points (five and ten dollar multiples, for example).

### 13.2. Pegged orders

With a pegged order, the price is set relative to some other price (typically the NBO, the NBB or the midpoint). If the reference price changes, the order is repriced.

Example: If the market is $25 bid for 1,000; 2,000 offered at $25.10, we might see “Buy 500, pegged at the NBB less $0.05.” This order will initially be priced at 24.95, but if the NBB changes, the order will be repriced. It will execute only when there is sell order that walks through the book.

A repriced order is added to the book at the new price behind all orders previously entered at that price. Effectively, the time stamp on a pegged order is the time of the most recent repricing, not when the pegged order was initially submitted.

Once identified, a visible pegged order is an inviting target. Suppose that market $A$ has a buy order pegged to the NBB. A seller might enter an aggressive bid on market $B$ (to raise the NBB), send a marketable sell order to market $A$ (which would execute against the pegged order), and lastly cancel the bid on market $B$. This practice ("spoofing") is forbidden in most markets, but it may be difficult to detect. At least one exchange (BATS) requires pegged orders to be hidden.
13.3. Discretionary orders

This is a basically a limit order, but if the opposing quote gets within a specified range, the order is repriced to become marketable.

Example: If the market is $25 bid for 1,000; 2,000 offered at $25.10, we might see “Buy 500 limit pegged at the NBB, with discretion price $0.04 above the NBB.” This will initially go in as a limit order pegged at the NBB. If the NBB changes, the order will be repriced. But if there is ever an offer at or closer to NBB+$0.04, the buy order is repriced to be marketable.

If we suspect that potential counterparties (on the opposite side of the market) are using discretionary orders, we will manage our own orders less aggressively. For example, suppose that we’re buying and the market is $50.00 bid for 100 shares (ours), 1,000 shares offered at $50.10. If we are up against a deadline to accomplish our purchase, we might be inclined to reprice our order at $50.10, lifting the market offer. If we suspect that some of the offers are discretionary, though, we would instead raise our bid incrementally, in the hope of triggering one.

13.4. Reserve (“iceberg”) orders

A reserve order is partially hidden. The “tip” of the order is visible, the larger portion is hidden. When the visible part of the order is executed, it is “refreshed” from the undisplayed portion. This refresh can be mechanical: “10,000 shares total to buy at $25. Start showing 500. When that executes, immediately show another 500, until the entire amount is filled.”

This regularity, though, can render the reserve order easily detectable. 500 shares are shown; 500 shares trade; 500 more shares immediately appear, and so on. This is so obvious that it defeats the original purpose of disguising the larger amount. To control the predictability, it makes sense to randomize the refresh quantity. The NYSE Arca market offers such a feature, called a random reserve order. Some implementations of reserve orders also introduce a random delay in the refresh process.

13.5. Implementation

Is an algorithm implemented by customer, the broker, or the market center? Who actively manages it, and ensures that the instructions are correctly followed?

These questions actually arise with the primitive order types. In a floor market with bilateral trading (like a futures pit), even simple market and limit must be handled attentively. With a contract trading at a price around 80 (the units don’t matter), a broker holding a customer limit order to buy limit 75 might think that it is safe to go out for lunch. But if in his absence, the price has dropped to 70 and then gone up to 85, the broker has “missed the market”. The customer has a valid grievance. In fact, one of the early functions of the NYSE specialist was to act as agent for the book of limit orders left in his care.

Initially, pegged, discretionary and reserve orders were implemented on customers’ or brokers’ systems, and responded to data received from the market center. Over time, this functionality has moved to market centers’ computers. The BATS market, for example, offers all these types. To keep the IT architecture clean (and fast), the systems that handle the special orders are often kept separate from the system that runs the basic limit order market.

More complex algorithms may involve multiple orders and rely on broader information feeds (such as prices for other securities or markets). These algos are often implemented by bro-
kers, who make them available to customers. This reflects a broad diffusion of expertise. Algorithms that were formerly used only by sophisticated proprietary trading shops are now available at the retail level.

Summary of terms and concepts
Stop orders; trailing stops; pegged orders; discretionary orders; reserve/iceberg orders

Problems and exercises

Problem 13.1 When the last sale price of PQR is $42, I enter a stop loss order trailing by $3. In which of the following situations does my order get triggered? Circle all that apply.
   a. The trade at $42 is followed by trades at $40, $41 and $39.
   b. The trade at $42 is followed by trades at $40, $43 and $41.
   c. The trade at $42 is followed by trades at $40, $44 and $41.
   d. The trade at $42 is followed by trades at $40, $44, $43, $44, $43 and $41.

Problem 13.2
"This year's volatile stock market is driving investors to a trading technique designed to trim their losses. Yet it can also backfire on those who are in the market for the long term. The practice -- known as a "stop-loss order" -- calls for an individual stock to be automatically sold when its share price falls by a certain percentage or hits a designated price. [But] consider what happened to the Stockbusters investment club, a group of about 25 Dallas women that has been meeting and picking stocks together for two decades. Earlier this year, the club members studied and then agreed to buy two energy company stocks, XTO Energy Inc. and Chesapeake Energy Corp. [The] members put in a stop-loss order calling for the shares to be automatically sold if the prices dropped by 10%. Before the next monthly meeting, both stocks had been sold. Both then rebounded within weeks and continued to gain. "That didn't go well," says Nanci Roberts, the club's treasurer. Jeremy Siegel, finance professor at the Wharton School ... thinks long-term investors should avoid stop-losses altogether. Instead, he recommends a different tack: buying a 'put,' an option to sell the stock at a certain price on a certain date." When protecting an investment raises risks (Karen Blumenthal. Wall Street Journal. (Eastern edition). New York, N.Y.: Jun 18, 2008. pg. D.1)

What is the equivalent option strategy for a stop-buy order? What are the pros and cons of using options instead of a stop orders?
Problem 13.3 Why can't a visible limit order be pegged to or more aggressively than the same-side quote? (Consider, for example, a buy order pegged to the bid plus a penny.)

Problem 13.4 David enters an order to buy XYZ limit 50, with a discretionary price of 50.10. (That is, if the market offer drops to 50.10, David would like to buy at that price, canceling the original $50.00 limit order.) If David is willing to buy at 50.10, why doesn’t he simply post this as his visible limit price?

Problem 13.5 With ABC trading around $55, an investor enters a stop loss order: “sell 200 shares stop $52, limit $50.” This is equivalent to _____.
   a. “Sell 200 shares limit $50.”
   b. “Sell 200 shares limit $52.”
   c. “Sell 200 shares at the market.”
   d. None of the above.

Problem 13.6 Stock XYZ is 20.10 bid, offered at 20.18. At 10:00am I enter an order to buy, undisplayed, pegged to the midpoint, with a discretionary offset of $0.03. In the following sequence of quotes, when does my order first execute (if at all)?

<table>
<thead>
<tr>
<th>Time</th>
<th>Bid</th>
<th>Offer</th>
<th>Mid</th>
<th>Distance to Offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:01</td>
<td>20.10</td>
<td>20.29</td>
<td>20.195</td>
<td>0.095</td>
</tr>
<tr>
<td>10:02</td>
<td>20.18</td>
<td>20.29</td>
<td>20.235</td>
<td>0.055</td>
</tr>
<tr>
<td>10:03</td>
<td>20.19</td>
<td>20.29</td>
<td>20.240</td>
<td>0.050</td>
</tr>
<tr>
<td>10:04</td>
<td>20.20</td>
<td>20.29</td>
<td>20.245</td>
<td>0.045</td>
</tr>
<tr>
<td>10:05</td>
<td>20.21</td>
<td>20.29</td>
<td>20.250</td>
<td>0.040</td>
</tr>
<tr>
<td>10:06</td>
<td>20.22</td>
<td>20.29</td>
<td>20.255</td>
<td>0.035</td>
</tr>
<tr>
<td>10:07</td>
<td>20.23</td>
<td>20.29</td>
<td>20.260</td>
<td>0.030</td>
</tr>
<tr>
<td>10:08</td>
<td>20.24</td>
<td>20.29</td>
<td>20.265</td>
<td>0.025</td>
</tr>
<tr>
<td>10:09</td>
<td>20.25</td>
<td>20.29</td>
<td>20.270</td>
<td>0.020</td>
</tr>
<tr>
<td>10:10</td>
<td>20.26</td>
<td>20.29</td>
<td>20.275</td>
<td>0.015</td>
</tr>
</tbody>
</table>

None of these situations would lead to an execution.
Chapter 14. Transaction Cost Analysis (TCA)

Are you a trader or an investor?

In one sense, the distinction is irrelevant. Total profits are completely determined by the size and timing of the cash flows (purchases and sales). Labeling one component of the cash flows as “trading”, and the other as “investment” won’t affect the total. Still, some measureable expenses such as commissions, bid-ask spreads, and price impacts seem more closely related to how we bought or sold, with everything else being attributed to longer-run investment returns.

Even if we could separate trading costs and investment characteristics, we’d ideally want to make our trading and investment decisions jointly, taking into account any interactions. The goal would be to plan a portfolio and execution strategy by simultaneously considering the long term security attributes and the costs of actually buying and selling the securities.

But trading and investment decisions require different data and different ways of thinking. The decisions usually fall to different groups of people. The practicalities of delegation, management and evaluation seem to force trading and investment processes to be considered separately and distinctly.

The distinction is important quantitatively because trading costs can be a very large drag on portfolio performance. As a rough approximation, the one-way trading cost on a stock is about 50 bp (0.5%). This includes explicit costs (like commissions) and implicit costs (like the bid/ask spread). If we buy or sell a $100 stock, it will cost us $0.50 per share.

There are also legal considerations. Asset managers usually have a fiduciary duty to act on behalf of their investors or beneficiaries. When managers make trading decisions, the costs are ultimately borne by these investors. U.S. market centers are responsible for computing and reporting trading costs. These are posted to their web sites.
14.1. The Implementation shortfall approach for portfolios and orders.

In the portfolio implementation shortfall approach, we assume a separation between investment and trading decisions. Long term investment strategies are made by portfolio managers. They make clear decisions about what to buy, sell and hold. These decisions are implemented by a trading desk.

We compare:
- The performance of an actual portfolio (gain, loss or return) and
- The performance of an imaginary paper portfolio in which all trades are made at benchmark prices.

A benchmark price is simply a reference price that is supposed to represent the security's true value at the time the buy or sell decision is made. The performance of the paper portfolio is obviously going to depend very strongly on our choice of benchmark. We'll discuss the alternatives later in this chapter, but a common choice is the average of the bid and ask prices at the time of decision, and so for the moment let's fix on this value. This average falls halfway between the bid and ask, and so it is usually called the bid-ask midpoint (BAM).

The portfolio implementation shortfall is the difference. For example, if the return on the paper portfolio is 10% and the return on the actual portfolio is 9%, the implementation shortfall is 1%. The idea here is that with perfect markets (and a perfect trading desk), our trades would have been executed at BAMs. Any divergence between the actual and paper returns must be attributed to trading (implementation) costs.

The portfolio implementation shortfall, as originally proposed, includes both explicit and implicit costs. The key explicit costs are:
- Commissions, net of any rebates
- "Take" fees and liquidity rebates (if not included in the commission).
- Transactions taxes

In the actual portfolio, the explicit costs are paid out of a cash account, and they are usually clearly identified on the trade confirmations that the broker sends to the investor. The implicit costs are less visible. They include:
- Costs of interacting with the market (e.g., bid-ask spread or price impact costs), relative to the benchmark prices. These costs can be estimated with statistical models of the sort discussed in Chapter 14.
- Opportunity costs (the penalty associated with not completing intended trades). Examples of this include
  - The failure of a limit order to execute because the market has moved away from the limit price.
  - Failure to complete a hedging trade, which may leave the portfolio exposed to additional risk.
- Delay (failure to fill the order immediately).

The implicit costs can be very difficult to assess.

The implementation shortfall framework is generally attributed to Andre Perold (Perold (1988)). Perold originally proposed the portfolio implementation shortfall at the level of the portfolio: one number reflecting all the trades and activity.

Nowadays, however, the implementation shortfall is usually computed for individual orders that are executed in full. It is defined as:
**Implementation Shortfall**

\[
\text{Implementation Shortfall} = \begin{cases} 
\text{Trade Price} - \text{Benchmark Price}, & \text{for a buy order} \\
\text{Benchmark Price} - \text{Trade price}, & \text{for a sell order}
\end{cases}
\]  

That is, for a buy order, the implementation is how much we overpay relative to the benchmark; for a sell order, how much less we receive. In the case where a large parent order is executed over time in a sequence of smaller child orders, an average trade price is used. This is more in line with “the implicit cost of interacting with the market” in Perold’s original formulation. The Investment Technology Group (ITG) transaction-cost reports, for example, use the breakdown:

\[
\text{Total Cost} = \text{Implementation Shortfall} + \text{Commissions}
\]

Henceforth, we’ll use this narrower definition.

**14.2. Benchmark prices used in IS calculations**

The implementation shortfall calculation depends crucially on the choice of the benchmark price. The possibilities are often grouped according to where they are (before/after/during) relative to the trade. *Pre-trade benchmarks* include:

- The NBBO midpoint at the time the trading or order submission decision was made.
- The previous day’s closing price.

When a pre-trade benchmark is used, implementation shortfall is sometimes referred to as *slippage*. Examples of *post-trade benchmarks* are:

- The NBBO midpoint prior five minutes after the trade.
- The next day’s opening price

*Interval benchmarks* are also sometimes used:

- Time-weighted average price (TWAP, “Tee Wap”) over the day or duration of the order.
- Volume-weighted average price (VWAP, “Vee Wap”) over the day or duration of the order.

For individual trades, the prior NBBO midpoint and the NBBO midpoint at trade time + five minutes are popular choices.

When the prior NBBO midpoint is used as a benchmark for a parent order, the whole sequence of child orders is judged relative to the initial midpoint, typically taken when the parent order is sent (by the portfolio manager) to the portfolio manager’s trading desk, or when the parent order is sent to the broker.

An institution will typically lack the ability to directly monitor the NBBO midpoints. For these, they are dependent on their brokers’ reports. The NBBO can change significantly over the course of a few milliseconds. An institution may suspect that the broker is “gaming” the measure, choosing from a set of nearly simultaneous NBBO records the one that gives the most favorable IS. TWAP and VWAP are easier to compute. VWAP, in particular, is very widely used as a benchmark.

**14.3. Effective and realized costs**

Effective and realized costs are implementation shortfall calculations for marketable orders, that is, orders that are executed on arrival.
The effective cost uses a benchmark price equal to the midpoint of the prevailing visible bid and offer. Letting $m$ denote the bid-offer midpoint, letting $p$ denote the execution price, the effective cost is defined as:

$$\text{Effective Cost} = \begin{cases} p - m, & \text{for a marketable buy order} \\ m - p, & \text{for a marketable sell order} \end{cases}$$

The effective spread is simply twice the effective cost. If all buys executed at the NBO, and all sells at the NBB, the effective and posted spread would be the same. Accurate timing can be difficult. An institution sending the order would probably take the midpoint at the time the order was sent. If the market center is doing the calculation, the midpoint is taken as of when the order was received. Usually these times differ only by a few milliseconds, but in a volatile market even this small difference can meaningfully affect the outcome of the calculation.

As a result of a dark trade, execution against a hidden limit order, or a concession by a dealer, the execution price may lie within the posted NBBO. The amount by which the quote is bettered is called price improvement:

$$\text{Price improvement} = \begin{cases} \text{NBO} - p, & \text{for a marketable buy order} \\ p - \text{NBB}, & \text{for a marketable sell order} \end{cases}$$

The realized cost uses a post-trade benchmark. Any post-trade benchmark might be used, but for regulatory purposes, the SEC mandates the NBBO midpoint prevailing five minutes after the market receives the order (or the closing NBBO midpoint). Denoting this midpoint as $m_5$:

$$\text{Realized Cost} = \begin{cases} p - m_5, & \text{for a marketable buy order} \\ m_5 - p, & \text{for a marketable sell order} \end{cases}$$

Figure 14.1 depicts these quantities for a marketable buy order. The realized cost is on average less than the effective cost under the presumption that the price impact of the order is positive. The realized cost can be interpreted as the trading profit made by the dealer (or other trader) who acted as counterparty to the marketable order net of the impact of the trade, and assuming that they could reverse the trade at the then-prevailing quote midpoint.
The quantity Effective Cost — Realized Cost measures the movement of the quote midpoint from the trade to the five-minute mark. It is therefore an approximate measure of the price impact of the order. The correspondence is only approximate because the quote midpoint change over the interval is driven by all of the trades prior to the five-minute mark – not just the single trade in question.

Table 14.1 describes some sample calculations. The left portion of the table gives the NBBOs over the interval; the right portion contains information on three orders. The table is arranged so that the NBBO to the left of an order describes the NBBO prevailing at the order time. Note that the realized cost is in one instance negative. We would not expect this to be the case on average, because that would imply a trader generally buying before the price goes up and generally selling before a decline. For an individual trade, however, the five-minute delay used to set the benchmark price can contain substantial price variation, and almost anything is possible.
Table 14.1 Calculation of effective cost, realized cost, price improvement and VWAP for marketable orders.

<table>
<thead>
<tr>
<th>Time</th>
<th>NBB</th>
<th>NBO</th>
<th>BAM</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:14:00</td>
<td>19.81</td>
<td>19.87</td>
<td>19.840</td>
<td>0.06</td>
</tr>
<tr>
<td>10:15:00</td>
<td>19.80</td>
<td>19.85</td>
<td>19.825</td>
<td>0.05</td>
</tr>
<tr>
<td>10:16:00</td>
<td>19.78</td>
<td>19.83</td>
<td>19.805</td>
<td>0.05</td>
</tr>
<tr>
<td>10:17:00</td>
<td>19.85</td>
<td>19.90</td>
<td>19.875</td>
<td>0.05</td>
</tr>
<tr>
<td>10:18:00</td>
<td>19.92</td>
<td>19.97</td>
<td>19.945</td>
<td>0.05</td>
</tr>
<tr>
<td>10:19:00</td>
<td>19.91</td>
<td>19.97</td>
<td>19.940</td>
<td>0.06</td>
</tr>
<tr>
<td>10:20:00</td>
<td>19.84</td>
<td>19.87</td>
<td>19.855</td>
<td>0.03</td>
</tr>
<tr>
<td>10:21:00</td>
<td>19.92</td>
<td>19.96</td>
<td>19.940</td>
<td>0.04</td>
</tr>
<tr>
<td>10:22:00</td>
<td>19.84</td>
<td>19.88</td>
<td>19.860</td>
<td>0.04</td>
</tr>
<tr>
<td>10:52:00</td>
<td>19.93</td>
<td>19.97</td>
<td>19.950</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Order Time</th>
<th>Dir Vol</th>
<th>Exec Price</th>
<th>Effective Cost</th>
<th>Price Improv</th>
<th>BAM&lt;sub&gt;5&lt;/sub&gt;</th>
<th>Realized Cost</th>
<th>Vol x Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:15:09</td>
<td>B</td>
<td>300</td>
<td>19.85</td>
<td>0.025</td>
<td>19.855</td>
<td>-0.005</td>
<td>5,955</td>
</tr>
<tr>
<td>10:17:22</td>
<td>B</td>
<td>300</td>
<td>19.88</td>
<td>0.005</td>
<td>19.860</td>
<td>0.020</td>
<td>5,964</td>
</tr>
<tr>
<td>10:20:09</td>
<td>S</td>
<td>400</td>
<td>19.84</td>
<td>0.015</td>
<td>19.860</td>
<td>0.020</td>
<td>7,936</td>
</tr>
</tbody>
</table>

Total Volume: 1,000
Total (Vol x Pr): 19,855
VWAP: 19.855

Notes: BAM is the bid-ask midpoint as of the indicated quote time; BAM<sub>5</sub> is the bid-ask midpoint five minutes after the indicated order time.

14.4. Implementation shortfall and limit orders

A general principle of trading is that urgency has a cost. If you want to trade cheaply, trade patiently and passively. Why go to someone else's price? Why not wait and let them come to yours? This principle argues in favor of limit orders and generally passive strategies. The principle is a sound one, and implementation shortfall analysis can usually measure the reward to patience. In practice, though, the computations might well be biased due to neglect of failed executions.

Figure 14.2 illustrates a typical situation. The setup uses a binomial model to describe the dynamics of the stock price. The offer price starts at 10.00. At every time step (say, every minute), the offer can go up by $0.01 or down by $0.01, with equal probabilities. Over the first three minutes, then, there are 2<sup>3</sup> = 8 possible paths, all equally likely.
Suppose that we want to buy. At the start of things we can trade immediately by lifting the offer price of $10.00 with a market order. Alternatively, we could try to trade passively. Suppose that we put in a buy order, limit $9.99. If this order executes, we’ll save $0.01 over a market order. For any pre-trade benchmark, $0.01 will be difference in implementation shortfalls. It is therefore convenient to take $10.00 as the pre-trade benchmark.

If we submit a hundred limit orders in this situation, and analyze our costs only for the orders that fill, we’ll conclude that the implementation shortfall of the limit order strategy is $−0.01$, that is, that we’re trading at negative cost.

Now what is the probability of a limit order fill? This order will execute whenever a path hits 9.99. If we are willing to wait for three minutes, there are five paths on which this will occur. (Using $d$ and $u$ to denote down and up: $duu, dud, ddu, ddd, and udd$.) So the probability of execution is $5/8 = 0.625$ – better than even chances.

But what happens when the limit order doesn’t execute? If we don’t care, then we’re okay ignoring the execution failures. But if we don’t care, why were we contemplating using a market order? Why were we in the market at all?

Suppose that we’re required to purchase the stock within three minutes. This means that if the limit order hasn’t executed by the third step, we’ll have to cancel it, and lift the offer with a market order. The probability of the limit order failure is $1 − 0.625 = 0.375$.

If the limit order fails, what will we have to pay to acquire the stock? In one of the failure paths ($uuu$) the offer is 10.03; in two of the failure paths ($udu$ and $uud$) the offer is 10.01. So the expected offer conditional on a limit order failure is

\[
E[\text{offer}|\text{limit order fails}] = \frac{1}{3} \times 10.03 + \frac{2}{3} \times 10.01 = 10.0167
\]

Our overall expected cost of buying with the limit order strategy is

\[
E[\text{cost}] = 9.99 \times P(\text{LimEx}) + 10.0167 \times P(\text{LimFail})
\]
where \( \text{LimEx} \) and \( \text{LimFail} \) denote the success and failure of the limit order. So:

\[
E[\text{cost}] = 9.99 \times 0.625 + 10.0167 \times 0.375 = 10.00
\]

This is exactly the cost of using a market order. A limit order priced at $9.99 does not on average outperform a market order, and it has outcome uncertainty.

Perhaps the limit order was too aggressive. Suppose we submit an order to buy limit $9.98. If we restrict our analysis to the outcomes in which the order executes, we’ll measure an implementation shortfall of $-0.02. There are two paths on which this order will execute (\( \text{ddd} \) and \( \text{ddu} \)). So the probability of execution is \( \frac{2}{8} = 0.25 \). If

\[
E[\text{offer} | \text{limit order fails}] = \frac{1}{6} \times 10.03 + \frac{3}{6} \times 10.01 + \frac{2}{6} \times 9.99 = 10.0067
\]

and

\[
E[\text{cost}] = 9.98 \times 0.25 + 10.0067 \times 0.75 = 10.00.
\]

The practice of using a market order after a failed attempt with a limit order is called “chasing the market”. The limit order fails to execute because the price has moved in the wrong direction (up, if we’re trying to buy), and we have to “chase” a price that’s running away from us.

The strategy of first trying limit orders to achieve a passive execution, and then switching to a market order at the deadline is a fairly representative limit order strategy. In fact, the Tokyo Stock Exchange has a specific order type, the funari order, that works in exactly this fashion.

The binomial model is used in many financial settings, particularly in option pricing, where great reliance is placed on its validity. In the present case, though, if this model’s depiction of the world were accurate, no one would ever use a limit order. It is quite possible that execution would occur before the ask price moved to the limit price, as an urgent liquidity trader or other latent seller might well hit an aggressive bid. Other aspects of order-price dynamics will be discussed in Chapter 14.

It is moreover likely that the approach of imputing a market-order fill after a limit order failure overstates the opportunity cost. It is always feasible to submit such a market order, and the fact that this step is often not taken suggests that it is perceived as an unnecessary expense. There may be alternative securities or alternative hedges available that can substitute at a better price.

The main point of the analysis stands, however, which is that limit orders evaluated with no penalty for failure will always appear superior to market orders. Limit orders that are priced less aggressively will appear better yet.

### 14.5. Estimating trading costs

Most institutions compute implementation shortfalls for a sample of their trades, estimate averages, and compare these averages across brokers, algorithms, and routing destinations that the firm employs. These results then guide future order submission choices. This sort of analysis is sensible and useful. As often applied, though, the process neglects opportunity costs and the interaction with the firm’s investment and trading strategies.

The opportunity costs of failed limit orders were discussed in the last section. The point generalizes, however, to most passive strategies, including those that follow dynamic limit order
strategies. As an example, in the situation of Figure 14.2, the strategy of initially submitting an order to buy limit $9.99, but repricing it if the offer moves away. So if the offer moves to $10.01, we’d reprice our buy at $10.00. This is a pegged limit order (discussed further in section 13.2). It will have a higher execution probability than a limit order submitted once and for all at $9.99, but it will have some nonzero failure probability, which must be considered in the implementation shortfall analysis.

Interactions with the firm’s investment and trading decisions also affect the validity of the analysis. Ideally, following the protocol of drug clinical trials, we’d perform experiments on random orders in random stocks, with random quantities in random direction (buy or sell), and randomized execution strategies. In this way, we’d achieve unbiased estimates of “treatment” effects. Some firms and investors do perform experiments, but they are expensive, as they involve buying stocks that one does not really wish to own, and trading in ways that are “obviously” inefficient. The more common practice is to analyze the orders actually generated by the portfolio managers, and the executions actually achieved by the trading desk.

An example shows why this can lead to problems. Suppose that the portfolio desk generates two kinds of orders: rebalancing orders that simply seek to keep the overall portfolio close to some desired allocation weights, and momentum orders that try to profit on short-term price movements. The rebalancing orders are sent to broker R with instructions to execute gradually over time; the moment orders are sent to broker M and flagged as urgent. In a cross-broker comparison of implementation shortfall, we’d expect broker R to have the lower costs. This is not due to any special ability, but rather to the sort of orders we send.

A more subtle interaction occurs when our strategies are also being used by others and our orders are correlated with others’. The price change associated with our order, then, reflects not only our order, but also the orders originating from all other trading strategies.

Institutions do not report their trading costs directly to investors or in SEC filings. They do, however, often share data with other institutions. Some firms (e.g., ITG, Abel-Noser, TAG) produce aggregate reports of trading costs. A mutual fund might report its trade data to one of these firms. The firm then compiles summary statistics (disguising the identity of any individual fund).

14.6. Implementation shortfall decompositions

Once we settle on a benchmark price, the implementation shortfall is easily computed. The resulting number, though, doesn’t give us much information about the source of the costs. Nor does it give us guidance about how we might adjust our behavior to reduce our trading costs.

To illuminate the source of the costs, therefore, the implementation shortfall is often decomposed further. As a first step, the implementation shortfall is sometimes decomposed as:

\[
\text{Implementation shortfall} = \text{price impact} + \text{cost of delay}
\]

(14.2)

Price impact reflects the price movements induced by our executions. A portion of price impact may be temporary, as in the case where other market participants conclude that our orders are uninformed and the price reverts. The impact may also be permanent, if our orders are viewed as potentially informed. Price impact cannot be observed directly. (We traded, and the price moved, but did we cause the movement?) It is usually inferred from a statistical model of order-price dynamics.

The cost of delay is the remainder or residual. It captures the tendency of the price to move against us even if we did not in fact achieve any executions. This may occur because others are using strategies similar to ours, or responding to the same signals. It may also arise because others are detecting our intentions and trading ahead of us.
14.7. SEC Rule 605

SEC Rule 605 requires market centers (exchanges, dealers, dark pools, etc.) to provide detailed reports on the orders they receive and the outcomes. The reports cover order counts, share counts, execution rates, cancellation rates, effective spreads and realized spreads. (Effective and realized spreads are simple two times the effective and realized costs.) These statistics are broken down by stock, trade size, and pricing relative to the same-side quote. Monthly figures must be published on their websites. The SEC mandates a standardized format (U.S. Securities and Exchange Commission (2001b)).

Table 14.2 contains a portion of the Rule 605 statistics for BATS (specifically, the BATS “Z” exchange) for November 2009 for ticker EIHI (Eastern Insurance Holdings, Inc., a diversified insurance concern). In this period, BATS received only two market orders, but 363 marketable limit orders of size 100-499 shares. (This reflects the reluctance of traders and market centers to use unpriced orders.) There were 37,002 shares in these orders, of which 34,400 were cancelled, 1,122 were executed at BATS, and 1,480 were routed elsewhere. All of the executions were accomplished in under nine seconds. For these shares the average effective spread was $0.0930, and the average realized spread was $0.0685. This implies effective and realized costs of $0.0465 and $0.0343. The difference, $0.0122 is the implied price impact (see the figure above). In other words, an incoming 100 share buy order should raise the market share price by about $0.0122.

<table>
<thead>
<tr>
<th>Order type</th>
<th>Order size</th>
<th>No. orders</th>
<th>Shrs in orders</th>
<th>Shrs canc</th>
<th>Shrs exec</th>
<th>Shares exec elsewhr</th>
<th>Eff spread</th>
<th>Rlzd spread</th>
<th>Shrs rcvng price imprvmt</th>
<th>Avg price imprvmt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>1-499</td>
<td>2</td>
<td>200</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>0.050</td>
<td>0.045</td>
<td>200</td>
<td>0.0475</td>
</tr>
<tr>
<td>Mkt' limit</td>
<td>1-499</td>
<td>363</td>
<td>37,002</td>
<td>34,400</td>
<td>1,122</td>
<td>1,480</td>
<td>0.093</td>
<td>0.069</td>
<td>1228</td>
<td>0.0227</td>
</tr>
<tr>
<td>Mkt' limit</td>
<td>500-1,999</td>
<td>10</td>
<td>5000</td>
<td>0</td>
<td>0</td>
<td>5,000</td>
<td>0.198</td>
<td>0.190</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

14.8. Further reading

Because high liquidity is sometimes viewed as functionally equivalent to “low trading cost,” most liquidity measures are negatively related to most estimates of trading costs. Holden and Jacobsen (2014) provide an excellent recent survey of alternative measures and their relations.

Summary of terms and concepts

Implementation shortfall; explicit and implicit costs; effective cost; realized cost; price improvement; market impact; opportunity costs for unexecuted limit orders; delay costs; SEC Rule 605.
Problems and exercises

**Problem 14.1** Cost computations. The NBBO / Bid-ask midpoint record for XYZ is:

<table>
<thead>
<tr>
<th>Time</th>
<th>NBB</th>
<th>NBO</th>
<th>BAM</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:01:00</td>
<td>35.05</td>
<td>35.10</td>
<td>35.075</td>
<td>0.05</td>
</tr>
<tr>
<td>11:02:00</td>
<td>34.94</td>
<td>34.99</td>
<td>34.965</td>
<td>0.05</td>
</tr>
<tr>
<td>11:03:00</td>
<td>34.83</td>
<td>34.86</td>
<td>34.845</td>
<td>0.03</td>
</tr>
<tr>
<td>11:04:00</td>
<td>34.77</td>
<td>34.81</td>
<td>34.790</td>
<td>0.04</td>
</tr>
<tr>
<td>11:05:00</td>
<td>34.77</td>
<td>34.85</td>
<td>34.810</td>
<td>0.08</td>
</tr>
<tr>
<td>11:06:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:07:00</td>
<td>34.75</td>
<td>34.78</td>
<td>34.765</td>
<td>0.03</td>
</tr>
<tr>
<td>11:08:00</td>
<td>34.80</td>
<td>34.86</td>
<td>34.830</td>
<td>0.06</td>
</tr>
<tr>
<td>11:09:00</td>
<td>34.72</td>
<td>34.75</td>
<td>34.735</td>
<td>0.03</td>
</tr>
<tr>
<td>11:10:00</td>
<td>34.75</td>
<td>34.83</td>
<td>34.790</td>
<td>0.08</td>
</tr>
<tr>
<td>11:11:00</td>
<td>34.66</td>
<td>34.69</td>
<td>34.675</td>
<td>0.03</td>
</tr>
<tr>
<td>11:12:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:13:00</td>
<td>34.82</td>
<td>34.88</td>
<td>34.850</td>
<td>0.06</td>
</tr>
</tbody>
</table>

What are the effective cost, realized cost, price improvement and price impact for the following trades?

*a.* An order arriving at 11:02:44, buy 100 shares limit 35.00, was executed at 34.98.

*b.* An order arriving at 11:07:02, sell 100 shares limit 34.70, was executed at 34.73.

**Problem 14.2** The NBBO record for ABX on a given day is in part:

<table>
<thead>
<tr>
<th>Time</th>
<th>NBB</th>
<th>NBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:01</td>
<td>20.10</td>
<td>20.17</td>
</tr>
<tr>
<td>11:02</td>
<td>20.11</td>
<td>20.17</td>
</tr>
<tr>
<td>11:05</td>
<td>20.12</td>
<td>20.17</td>
</tr>
<tr>
<td>11:06</td>
<td>20.13</td>
<td>20.17</td>
</tr>
<tr>
<td>11:09</td>
<td>20.13</td>
<td>20.18</td>
</tr>
</tbody>
</table>

Market ZX receives a buy order at 11:03, which it executes at a price of 20.16.

*a.* What is the effective cost for the trade?

*b.* Using the (standard) five-minute benchmark, what is the realized cost for the trade?

**Problem 14.3** At 10am, with the NBBO at 30.20 bid, offered at 30.28, the Pegasus Fund places an order to buy 1,000 shares limit 30.15. As the market moves away (upwards), Pegasus makes the limit order more aggressive. The order gets (partial) executions of 200 shares at $30.22 and 500 shares at $30.23. The market closes at $30.39. What is the VWAP of the executed shares? Relative to the initial bid-ask midpoint, what is the implementation shortfall of the executed shares?
Problem 14.4 Non-marketable limit orders ordinarily execute at their limit prices. A limit order submitted as a funari order might execute at a price worse than the limit price. Explain how this might occur.
Chapter 15. Statistical Models of Order-Price Dynamics

This chapter develops a small number of statistical models that describe how security prices evolve and how order flow affects these dynamics. From one perspective these models are short-term predictive tools. Entire books have been written about stock price forecasting, and it is probably a safe bet that the diversity of techniques and paucity of reliable results are unmatched by any other field of scientific endeavor.

The question guiding the present analysis, though, is not, “Which way are stock prices headed?” The concern is instead, “How will my orders and trades affect prices?” The reason this second question is interesting is that many of the trading strategies that we’ll discuss in the next chapter involve sequences of trades. In a sequence, early trades may affect the prices of later ones. Clarifying this interaction and accounting for it will be an important guiding principle.

The present analysis doesn’t take a position on any prediction tool – including your current favorite. The results can peacefully coexist with most prediction models. Along the way I’ll point out how the predictions can be incorporated.

The models here tend to be descriptive and statistical, rather than theoretical and economic. The distinction is driven by practicalities. We earlier explored a simple model of informed trading that exhibited order impacts and information-dependent bid-ask spreads. It is difficult, however, to map the model directly to a real stock. (It is 9am. What will be today’s High and Low values for Microsoft?) Instead, we’ll take the intuition that orders move prices and add it to a statistical model. The statistical model, in turn, will be estimated from recent data that presumably reflect how the stock actually behaves.

The development proceeds in stages, starting with simple models, and moving to more complex ones.
15.1. The random walk.

The simplest model of security prices is the random-walk:

\[ p_t = p_{t-1} + u_t \]

where \( t = 1, 2, \ldots \) might index minutes, seconds, or some similarly brief time interval; \( p_t \) is the stock price at the close of interval \( t \), and \( u_t \) is a random perturbation, disturbance, or error. \( u_t \) reflects new information that arrives over the interval. The news might be good or bad. More formally, \( E_{t-1} u_t = 0 \): as of the end of last interval, we expected this period’s news to be neutral.

We're often concerned with the volatility of the price dynamics, characterized by the standard deviation of \( u_t \), \( \sigma_u \). If \( p_t \) is redefined, for the moment, as the logarithm of the stock price, then \( p_t - p_{t-1} \) is approximately the proportional (“percentage”) change in the stock price over the interval, and \( u \) is (approximately) a proportional disturbance. In log price form, an annual volatility of, say, \( \sigma_u \approx 40\% \) corresponds to a monthly volatility of \( \sqrt{0.40^2 / 12} \approx 11.5\% \), or a daily volatility of \( \sqrt{0.40^2 / 250} \approx 2.5\% \).

We now consider estimation. The model has one parameter, the volatility \( \sigma_u \). Suppose that we have a set of \( T \) prices \( \{p_1, p_2, \ldots, p_T\} \). Then \( u_t = p_t - p_{t-1} \) for \( t = 2, \ldots, T \) (we lose \( u_1 \) because we don’t have \( p_0 \)). The usual estimate for the variance would be

\[ \hat{\sigma}_u^2 = \frac{1}{T-1} \sum_{i=2}^{T} u_i^2. \]

This is the mean sum-of-squares. There are many ways of estimating volatility, however. In option pricing the above estimate is called the historical volatility. From the market prices of options, one can compute an implied volatility, which may be a better forecast of the volatility that will arise in the future.

Stock prices lie on a discrete grid. Bids and offers in U.S. equity markets cannot normally specify an increment finer than $0.01, that is, there can’t be more than two places to the right of the decimal. Other markets adopt similar conventions. While accepting this discreteness as a feature of the data, however, we typically don’t incorporate it as a feature of the statistical model. This simplifies estimation. It doesn’t usually cause problems, but we must remember some of our statistical intuitions implicitly invoke continuous models, and well-behaved ones at that. With a normally-distributed (Gaussian) random variable, for example, the intervals defined by the mean \( \pm \sigma, \pm 2\sigma, \pm 3\sigma \) contain on average 68.2%, 95.4%, and 99.7% of the outcomes. This generally won’t be true for most of the discrete distributions encountered in security pricing.

15.2. Drift and alpha

An obvious shortcoming of the model is that it doesn’t allow for any expected capital gains. So we introduce a drift term:

\[ \text{Note: there are about 250 trading days in a year. Most of the cumulative volatility in financial markets arises on trading days. Some people read this as “trading causes volatility.”} \]
If \( p_t \) is redefined as the logarithm of the stock price, then \( p_t - p_{t-1} \) is approximately the proportional ("percentage") change in the stock price over the month, and \( \mu \) is the expected percentage price appreciation. An annual return of 20% per year works out to about \( \mu = 0.017 \) (1.7%) per month. The implied daily expected return is about \( \mu \approx 0.0008 \) (0.08%) per day (using 250 trading days per year).

The random-walk model is important due to:

- The concept of informational efficiency. This is an economic argument that a rational market incorporates all known information into the security price, so all future movements must arise from information not previously known. This implies that price movements should be unpredictable. The random-walk model is a simple construct with unpredictable movements.
- Statistical tests generally show that the model is "difficult to reject." There aren’t solid grounds for preferring any simple alternative model.

In monthly, weekly or daily data, the model is sufficiently close to reality that it is useful for many purposes, notably option pricing. For trading (intraday) horizons, the model needs enhancement to allow for drift (alpha) and order impact. We turn now to the specifics of these modification.

When we’re using the random-walk model applied to, say, monthly log prices, the drift term \( \mu \) is the expected price change over the month. Ignoring dividends this is the expected return over the month. Expected returns over monthly and longer horizons are determined by economic fundamentals, risk, and investors’ beliefs about these things. At intraday horizons, we take a broader view of the drift.

The drift corresponds to predictable short-term price changes. The extent of the predictability depends on the information, the model, and the faith placed in the model. It may be different for different market participants.

In finance, predictable over- or under-performance (net of compensation for risk) is often called “alpha” (\( \alpha \)). In a trading context, to distinguish this predictability from effects associated with long-run fundamentals, it is sometimes called "short-term alpha". This parameter, then, incorporates the output of any supplemental prediction model. The present analysis assumes that it is constant over the horizon of the trading problem, but with a little additional effort it can be viewed as time-varying (\( \alpha_t \)).

The random walk with drift has two parameters, \( \mu \) and \( \sigma_u \). The first difference of the price is \( \Delta p_t = p_t - p_{t-1} = \mu + u_t \). So we can estimate \( \mu \) and \( \sigma_u^2 \) as the mean and variance of the \( \Delta p_t \), something that might easily be accomplished with a calculator or spreadsheet.

In practice things are bit more complicated. If the drift is attributed to the expected long-run return, its mean is likely to be quite close to zero. As a result, while useful estimates of \( \sigma_u \) can be obtained from a sample of 30 days, important changes in \( \mu \) require a few years of data to discern.

### 15.3. Permanent order impacts

The discussion of private information in Chapter 11 noted that it can lead to order price impacts. An incoming customer buy order, for example, that lifts the market offer price might have originated from an informed buyer, someone who holds positive information not publicly known. This
possibility causes other traders to revise upwards their beliefs about the security value. Buying drives the price up; selling drives the price down.

We can modify the random-walk model to incorporate short-term alpha and an order price impact. Let $S_t = \text{net buy volume} = (\# \text{ shares actively purchased} - \# \text{ shares actively sold})$, the actively bought or sold shares are understood to be those that lift the offer or hit the bid. Then:

$$p_t = p_{t-1} + \alpha + \lambda S_t + u_t$$

(15.1)

where $\lambda$ is the price impact parameter. We'd expect $\lambda > 0$. Later, when we use this model to make predictions, $S_t$ for future times will include our own trades. But for the moment, think of the model as describing what's happened in the past.

By rearranging, we obtain

$$\Delta p_t = p_t - p_{t-1} = \alpha + \lambda S_t + u_t$$

(15.2)

This looks like a simple linear regression of $\Delta p_t$ on $S_t$. The model is typically estimated over short time intervals (like one minute). By aligning the trade prices with the record of the bids and asks, we can compute $S_t$ as the number of executed shares priced at the ask less the number of shares priced at the bid. The model parameters ($\alpha$, $\lambda$, and $\sigma_u^2$) can then be estimated using a spreadsheet (or calculator) regression command. Not only is the model convenient to estimate, but it will also turn out to be convenient for solving the order splitting problem.

Of course, models chosen for convenience aren’t necessarily the most realistic. The intuition of the private information model only suggested a generally positive relation between order flow and quote changes. Why should the order flow variable be net buy volume? Why should the relation be linear?

This basic model admits many variations. The extent to which a regression fits the data is usually summarized in its coefficient of determination, $R^2$. This quantity varies between zero (no fit at all) and one (perfect fit). We can investigate alternative specifications and see which changes seem to improve the fit.

The basic specification is linear in signed order flow. This linearity implies that the price change caused by a 10,000 share purchase should be 100 times the impact of a 100 share purchase. In practice it is usually much less.

One way of lessening the impact of large trades is to simply ignore size altogether. Net buys, defined as $S'_t = (\# \text{ of active buys} - \# \text{ of active sells})$, is based on counts. A buy counts as “+1” no matter how many shares are involved.

The use of count-based variables goes against the intuition that a large order should move the price at least somewhat more than a small order. The desired property is an incremental impact that diminishes with size: the additional price impact associated with adding one share to a 10-share order should be less than if one share were added to a 10,000 share order. The declining incremental impact implies a concave relation. At present, the preferred specification involves a signed square root:

$$S''_t = \text{Sign}(S_t)\sqrt{|S_t|}.$$ 

The $\text{Sign}$ function is defined by $\text{Sign}(x) = +1$ if $x > 0$; $-1$ if $x < 0$; and $0$ if $x = 0$. For example, if $S_t = -10,000 \text{ shares}$, then $S''_t = (-1)\sqrt{10,000} = -100$. This form is suggested by supportive
evidence from various studies in different markets. It is sometimes called the “square root law” due to its apparent universality (Bouchaud, Farmer and Lillo (2008)).

Sometimes $S_t$, $S'_t$, and $S''_t$ are used together in the specification (which then becomes a multiple regression). If the goal is fitting a set of data, after all, why not use all reasonable variations? The problem is that the ultimate goal is not explaining the past, but forecasting the future. Market models are notoriously prone to a phenomenon known as “over-fitting”. (The analyst carefully crafts a statistical model that achieves an “in-sample” $R^2$ of 0.99, but then falls apart when applied to a different sample or used for out-of-sample prediction.)

15.4. Temporary order impacts

All of the action in the model (or the variants discussed) takes place within interval $t$, suggesting that the effect of an order on price is instantaneous. There are a number of reasons why we might want to allow for more flexibility in the timing.

Different market participants receive information in differing timeliness, process information at different speeds, and trade over different horizons. The business-cycle investor lies at one extreme, the high-frequency trader at the other. The existence of slow participants opens the possibility that some of the reaction to new information might involve a gradual price adjustment over time.

The market might also be subject to temporary price movements due to liquidity needs of large investors. In contrast to the lagged information adjustment effect, though, these effects might well involve “overshooting” and reversion. A large mutual fund, for example, that experiences a large unexpected investor outflow might need to trade in size quickly. As the fund’s large sell orders hit the market, the price is pushed down. Bidders initially suspect negative private information and react accordingly. But once the selling order flow abates, and it appears that there are no pending news announcements that could explain the drop, bidders will return and the price will rise.

A simple specification that allows for a temporary impact is:

$$ p_t = p_{t-1} + \lambda_0 S_t + \lambda_1 S_{t-1} + u_t $$

The total impact coefficient for an order is $\lambda_0 + \lambda_1 > 0$. If the process is dominated by lagged adjustment, then typically $\lambda_0 > \lambda_1 > 0$. If there is overshooting, then $\lambda_1 < 0$.

15.5. Estimating the trade price impact using daily data

The statistical models here are properly estimated with intraday, high-frequency data. These datasets, however, are large and complex. With some simplifying assumptions, we can form an estimate from the more common daily price and volume data.

Consider the model given in equation (15.1). On average, $\alpha \approx 0$ and $u_t \approx 0$. If we assume that these approximations hold period-by-period, we can write

$$ \Delta p_t = \lambda S_t $$

and, taking the absolute value of each side,

$$ |\Delta p_t| = \lambda |S_t| $$
Now $S_t$ is the signed order flow over the interval. If the interval is short, we can approximate its absolute value by the trading volume, $V_t \approx |S_t|$. Rearranging, the implied value of $\lambda$ is:

$$
\lambda = \frac{\Delta p_t}{V_t}
$$

This suggests estimating $\lambda$ by

$$
\hat{\lambda} = \left( \frac{\Delta p_t}{V_t} \right)
$$

where the overbar indicates an average taken over a sample of intervals. Since price and volume data are widely reported at a daily frequency, it is convenient to use a sample of daily data. This estimate is a variant of the Amihud illiquidity measure, often denoted $I$ (Amihud (2002)).

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**Summary of terms and concepts**

Three models (random-walk; random-walk plus drift; random-walk plus drift plus order impact).

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**Problems and exercises**

**Problem 15.1** Consider the pure random-walk model, $p_t = p_{t-1} + u_t$. The $u_t$ in this model captures

a. New public information.

b. Drift or price trend.

c. New private (asymmetric) information.

d. The probability of a takeover bid.

**Problem 15.2** In the random-walk-plus-drift model, $p_t = p_{t-1} + \alpha + u_t$, we’d expect a positive drift in a situation with

a. A positive earnings surprise announced last month.

b. A belief that a large investment fund was trying to sell a large position in the stock.

c. A dividend payment made yesterday.

d. An immediate positive earnings surprise.

**Problem 15.3** A trading strategy is using a random-walk-plus-drift model, $p_t = p_{t-1} + \alpha + u_t$ with parameter values $\alpha = -0.01$ and a standard deviation $SD(u) = \sigma_u = 0.02$. The current stock price is $p_0 = $20.00. Set up the recursions that define $p_3$. What are the expected value and standard deviation of the stock price three periods later, that is, $EP_3$ and $SD(p_3)$.
Chapter 16. Order Splitting

A $1 Billion mutual fund might hold 1% of its assets ($10 Million) in a single stock. At a representative share price of $50, this holding consists of 200,000 shares. The median trade size in US equity markets is about 200 shares. It is therefore almost certain that should the fund seek to sell or reallocate the holding, the shift will have to be accomplished in multiple trades. A larger order, like “sell 200,000 XYZ” will typically be divided among many smaller “child” (or “daughter”) orders that are fed to the market over time. This is called order splitting.

For each child order, we face complex problems of how to peg, how much to show, what discretionary conditions should be used, whether to make our orders visible or dark, and so on. The larger problem, though, involving the number, size and timing of the child orders is more tractable.

Order splitting results from minimizing the cost of buying (or maximizing the receipts from selling) relative to some benchmark price. (This is a special case of the implementation shortfall approach discussed in Chapter 14.) There are two important special cases. In first instance, the benchmark is an average price computed over the working period or interval of the parent order. In the second instance, we’re using a pre-trade benchmark like the bid-ask midpoint at the time the parent order is given to the trading desk. We’ll consider each in turn.

16.1. Order splitting with interval benchmarks

Among the candidates for benchmarks, we encountered the time-weighted average price (TWAP) and the volume-weighted average price (VWAP). Now if one is trying to buy as low as possible, or sell as high as possible, why should either of these, (or indeed, any benchmark) be considered as a goal? The answer is that sometimes investors are more concerned with monitoring and measuring the performance of their broker.

For example, suppose that at 9:30 on Monday, the NBBO midpoint stands at $10, the manager of the Essex fund sends a large buy order to a broker to be worked over three days, and that
Essex ends up paying an average price of $10.20 per share. The implementation shortfall cost is $0.20 per share. Essex confronts the broker with this large trading cost, but broker simply replies, “Sorry, but the market was generally tending up over those three days.” But if the volume-weighted average price over the period was $10.15, Essex can come back with, “Alright, the market trend can explain $0.15 of the implementation shortfall, but that still leaves $0.05. Other brokers working for other customers over this same period somehow managed to buy their shares $0.05 cheaper than ours.”

Now perhaps Essex should be evaluating the original instruction. Why was the horizon three days? If it had been one day, the overall cost would probably have been lower. What was the manager thinking? There will be answers to these questions, but they will be qualitative, imprecise, and probably constructed after the fact. The implementation shortfall relative to VWAP, on the other hand, is easily measured, interpreted, and assigned to a particular agent (the broker). The measurement issue also arises at higher levels of delegation. It is likely that the Essex manager is herself bearing a fiduciary responsibility to manage and report trading costs. TWAP and VWAP are relatively easy to justify, and also to (approximately) achieve.

TWAP is the simpler of the two. Because the passage of time is perfectly predictable, one simply trades at a constant rate over the working horizon. If 80,000 shares are to be purchased over four hours, then 20,000 must be purchased in each hour, 5,000 over each fifteen minutes, or 1,000 every three minutes. This does not, of course, guarantee that TWAP will be achieved, because it depends on the trades of others as well as our own. There also remains the question of how to manage the child orders. But the strategy of using a constant rate of trade is certainly the place to begin.

VWAP is more complicated because average volume per unit time changes throughout the day, and volume is not perfectly predictable. Volume (and volatility) tend to be “U”-shaped: elevated at and immediately after the open, declining and leveling off during mid-day, and rising again toward the end of regular closing hours. This deterministic (predictable) variation is easily handled by matching the intended order flow to the volume. But there is also substantial random variation in volume at the daily and intra-day level. This requires predictive time-series models, and adaptive order strategies.

TWAP and VWAP strategies simply distribute orders over time. They do not explicitly take into account the total size of the trade relative to overall market activity. If a stock has an average daily volume of 1,000 shares, then a sell order for 50,000 shares being worked over the day will roil the market if the strategy is TWAP, VWAP or just about anything else. To avoid extreme market impacts, one can impose participation constraints. For example, “sell 50,000 VWAP, but our own trades should not exceed ten percent of the total volume.”

Participation targets, though, can have unexpected consequences. In the May 6, 2010 “flash crash”, the S&P 500 index futures contract fell about 6% over the course of a few minutes. The precipitating event was a large trade. According to the joint CFTC-SEC report (U.S. Commodities Futures Trading Commission and Commission (2010)):

A large fundamental trader (a mutual fund complex) initiated a sell program to sell a total of 75,000 E-Mini contracts (valued at approximately $4.1 billion) as a hedge to an existing equity position. … This large fundamental trader chose to execute this sell program via an automated execution algorithm (“Sell Algorithm”) that was programmed to feed orders into the June 2010 E-Mini market to target an execution rate set to 9% of the trading volume calculated over the previous minute, but without regard to price or time.
An algorithm that was more aware of the impact of its orders might have not have behaved so wildly. We now turn to a discussion of algorithms that incorporate these effects.

16.2. Order splitting with pre-trade benchmarks

A typical pre-trade benchmark is the bid-ask midpoint at the time the parent order was released to the trading desk, denoted $BAM_{pre}$. The implementation shortfall for, say, a buy order is then $IS = (average\ execution\ price) - BAM_{pre}$. Since the $BAM_{pre}$ doesn’t change over the execution process, however, we can ignore it in the following sense. Any strategy that minimizes the average execution price will also minimize the implementation shortfall.

A representative trading problem in this case involves buying a given number of shares before some deadline in a way that minimizes the overall price. The problem is dynamic because trades and market conditions evolve over time. It is stochastic (that is, driven by random behavior) because we have to plan a strategy before we know the future path of prices. The solutions are achieved by applying principles of stochastic dynamic programming.

The basic approach is due to Bertsimas and Lo (1998). To see it at work, suppose that the price dynamics are given by equ. (15.1), and consider the problem of minimizing the cost of buying $S_{Total}$ shares over two periods, (period 0 is "right now"). This cost is simply

$$C = p_1 S_1 + p_2 S_2$$

We start by forecasting the prices forward using equation (15.1):

$$p_1 = \alpha + p_0 + \lambda S_1 + u_1$$
$$p_2 = \alpha + p_1 + \lambda S_2 + u_2$$
$$= 2\alpha + p_0 + \lambda S_1 + \lambda S_2 + u_1 + u_2$$

We substitute these forecasts into the cost expression

$$C = p_1 S_1 + p_2 S_2 = S_1 (\alpha + p_0 + \lambda S_1 + u_1) + S_2 (2\alpha + p_0 + \lambda S_1 + \lambda S_2 + u_1 + u_2) \quad (16.1)$$

The expected cost is obtained by setting $u_1$ and $u_2$ to zero (as, in expectation, they are):

$$EC = S_1 (\alpha + p_0 + \lambda S_1) + S_2 (2\alpha + p_0 + \lambda S_1 + \lambda S_2) \quad (16.2)$$

We want to minimize $EC$ subject to the constraint $S_1 + S_2 = S_{Total}$. We can eliminate this constraint by letting $S_2 = S_{Total} - S_1$. Substituting this into (16.2) gives

$$EC = \lambda S_1^2 - S_1 (\alpha + \lambda S_{Total}) + S_{Total} (2\alpha + p_0 + \lambda S_{Total})$$

Now the expression involves just $S_1$. The optimal value, $S_1^*$, is obtained by computing the derivative, setting it to zero, and solving:

$$\frac{dEC}{dS_1} = -\alpha + 2\lambda S_1^* - \lambda S_{Total} = 0$$

or
In the special case of $\alpha = 0$, $S_1^* = S_2^* = S_{\text{Total}}/2$. If $\alpha$ is slightly positive, $S_1^*$ will be a bit larger and $S_2^*$ will be a bit smaller. Essentially, we’re accelerating our purchases, to partially avoid the anticipated price increase. We don’t, however, try to accomplish everything in the first period. Order impact considerations still come into play. If $\alpha$ is very positive, we might initially buy more than we need ($S_1^* > S_{\text{Total}}$), and we’ll sell off what we don’t need in period 2.

Using the optimal values in (16.2), the expected cost becomes

$$EC^* = -\frac{(\alpha + \lambda S_{\text{Total}})^2}{4\lambda} + S_{\text{Total}}(2\alpha + p_0 + \lambda S_{\text{Total}})$$

Note that this is quadratic. Expected trading cost rises as the square of the total amount purchase. In the special case of $\alpha = 0$

$$EC^* = p_0 S_{\text{Total}} + \frac{3\lambda S_{\text{Total}}^2}{4}$$

Since we don’t know (at time 0) the prices at which the trades will be made, our trading plan involves risk. The total cost from equation (16.1) evaluated at the optimal quantities is:

$$C^* = \frac{1}{4} S_{\text{Total}}(4p_0 + 3\lambda S_{\text{Total}} + 4u_1 + 2u_2)$$

All the uncertainty here is due to the $u_t$. Isolating them gives

$$C^* = S_{\text{Total}}(\cdots + u_1 + \frac{1}{2}u_2)$$

From this, the variance of the trading cost is

$$Var(C^*) = S_{\text{Total}}^2 \left[ Var(u_1) + \frac{1}{4} Var(u_2) \right] = \frac{5}{4} S_{\text{Total}}^2 \sigma_u^2$$

Risk also rises as the square of the total amount traded.

The analysis extends to more than two periods. If we’re spreading the purchase over three periods:

$$S_1^* = \frac{\alpha + \lambda S_{\text{Total}}}{2\lambda}, \quad S_2^* = -\frac{\alpha - \lambda S_{\text{Total}}}{2\lambda}$$

If $\alpha = 0$, the optimal strategy over $T$ periods is

$$S_t^* = \frac{S_{\text{Total}}}{T} \text{ for } t = 1, ..., T$$
Table 16.1 summarizes the results for expected cost and risk. As $T$ goes up, expected cost goes down, but risk rises. In portfolio theory there is a fundamental trade-off between risk and return. There is a similar trade-off here. In portfolio theory the trade-off line is called the efficient portfolio frontier; here it is sometimes called the efficient trading frontier.

Table 16.1 Trading horizon, expected cost and risk

<table>
<thead>
<tr>
<th>$T$</th>
<th>$EC^*$</th>
<th>$Var(C^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_0S_{Total} + \lambda S_{Total}^2$</td>
<td>$S_{Total}^2\sigma_u^2$</td>
</tr>
<tr>
<td>2</td>
<td>$p_0S_{Total} + \frac{3\lambda S_{Total}^2}{4}$</td>
<td>$\frac{5}{4}S_{Total}^2\sigma_u^2$</td>
</tr>
<tr>
<td>3</td>
<td>$p_0S_{Total} + \frac{2\lambda S_{Total}^2}{3}$</td>
<td>( \frac{14S_{Total}^2}{9}\sigma_u^2 )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$T$</td>
<td>$p_0S_{Total} + \frac{(T + 1)\lambda S_{Total}^2}{2T}$</td>
<td>$(1 + n)(2 + 2n)S_{Total}^2\frac{\sigma_u^2}{6n}$</td>
</tr>
</tbody>
</table>

16.3. Quasi-manipulations

To this point we’ve been considering how to trade a large order, $S_{Total}$ by splitting it up into smaller orders which are accomplished over a $T$-period horizon, with $S_{Total} = S_1 + S_2 + \cdots + S_T$. The goal has been buying $S_{Total}$ as cheaply as possible, or selling $S_{Total}$ for the highest amount, or (equivalently) minimizing the total trading cost. Huberman and Stanzl (2004) suggest looking at the problem from a different perspective. Consider a strategy where some of the smaller orders are buys ($S_t > 0$), some are sells ($S_t < 0$), and the total amount purchased is zero, $S_{Total} = 0$. Huberman and Stanzl (HS) ask if it is possible to find such a strategy that gives a positive profit.

What is manipulative about this? Don’t all buyers expect to sell their holdings later at a profit? The manipulative aspect comes in because HS are concentrating on short-term strategies where the trader has no information and the profits are due entirely to order impact. For example, we’d buy in a way to maximize order impact, driving the price up. Then we’d sell in a manner that would have minimal impact, closing out our position by selling at high prices. HS ask, what sort of order impact functions would allow for such profitable trading strategies? They show that profitable strategies exist for a wide range of order impact functions.
Summary of terms and concepts
Parent and child orders; VWAP; alpha; order impact; efficient trading frontier; manipulations.

Problems and exercises

Problem 16.1 The following table gives the daily volume profile for XYZ. A fund manager wants to sell 20,000 shares between 9:30 and noon, and seeks to sell at VWAP over that period. How many shares should she aim to sell in each of the five 30-minute intervals that span 9:30 to 12:00?

<table>
<thead>
<tr>
<th>Interval</th>
<th>Avg volume</th>
<th>% of daily total</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30</td>
<td>26,000</td>
<td>13%</td>
</tr>
<tr>
<td>10:00</td>
<td>20,000</td>
<td>10%</td>
</tr>
<tr>
<td>10:30</td>
<td>18,000</td>
<td>9%</td>
</tr>
<tr>
<td>11:00</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>11:30</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>12:00</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>12:30</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>13:00</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>13:30</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>14:00</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>14:30</td>
<td>12,000</td>
<td>6%</td>
</tr>
<tr>
<td>15:00</td>
<td>16,000</td>
<td>8%</td>
</tr>
<tr>
<td>15:30</td>
<td>24,000</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>200,000</td>
<td>100%</td>
</tr>
</tbody>
</table>

Problem 16.2 In typical order splitting models of trading strategy, whenever the trading horizon lengthens, the expected execution cost ____ and the variance of the execution cost ____.

a. rises; rises  
b. falls; falls  
c. rises; falls  
d. falls; rises

Problem 16.3 In the typical order splitting models of trading strategy, an increase in short-term alpha causes optimal purchases to be made ____ and sales to be made ____.

a. Earlier; earlier  
b. Later; later  
c. Earlier; later  
d. Later; earlier

Problem 16.4 Suppose that expected stock price dynamics are described by $p_t = p_{t-1} + 0.001S_t$, where $S_t$ is the number of shares bought (or, if negative, sold), and the current price is $20. What is the implementation shortfall if 600 shares purchased in trades of 100, 200, and 300 shares? What is the total cost if the three trades are optimally sized?
Part V. Special Topics
Chapter 17. Pricing, Fees, and Rebates

Our markets and other institutions of trade are not funded by kindly donations. Exchanges and brokers incur costs in providing their services, and to survive they must recover these costs through customer fees. In this, of course, they are no different from most other firms in the economy.

The structure of these fees (and rebates), however, is complex. Some are direct; some are indirect. Sometimes they are levied on customers in a fairly transparent fashion; sometimes they are buried in other charges. A customer cannot simply focus on the “bottom line” commission because as her order passes from broker to market center, fees and rebates change hands that affect how her order is handled. Many industries, of course, have complicated pricing arrangements. In this arena, though, the opaqueness of the pricing is especially remarkable as it stands in sharp contrast with the transparency, availability and uniformity of the security's bids, offers, and reported trade prices.

This chapter focuses on two arrangements: exchange maker/taker pricing and payment for retail order flow.

17.1. Exchange pricing

It is not surprising that exchanges charge for executions. What might seem perplexing, though, is that this cost is coupled with a subsidy.

Maker/taker pricing

Under the maker/taker model, market and marketable limit orders pay a small “taker” fee, and limit orders that are added to the book and subsequently executed receive a small consideration called a “maker” payment or “liquidity rebate”. The taker fee is usually larger than the maker rebate, with the market center capturing the difference.
The terms on NASDAQ’s limit order system were recently as follows:

- An executed limit order (“maker”) generally received $0.0029 per share (that is, $0.29 per one-hundred share “standard” round lot) if the limit order was visible, and $0.0015 per share ($0.15 per hundred shares) if the limit order was hidden.
- A taker pays $0.0030 per share “liquidity removal fee”.

The practice of providing a rebate for supplying liquidity and charging a bit extra for removing it is consistent with a view that liquidity is worth rewarding. NASDAQ keeps $0.0001 per share ($0.01 on a hundred shares).

Maker/taker fees can distort posted prices. Suppose that trader A is offering $10.00 per share (visible), and trader B lifts that offer. Net of fees and rebates, A receives $10.0029, and B pays $10.0030. Effectively, except for the $0.0001 that goes to NASDAQ, it’s as if A had priced his/her offer at $10.0030.

Different market centers have different price schedules. We may be looking at posted offers from multiple exchanges that appear to be the same. But without consulting the fee/rebate schedule, we don’t know which offer is really the best.

To reward the more consistent liquidity suppliers (“de facto market makers”), maker fee schedules can have quantity premia. The BATS-X liquidity rebate starts at $0.0025 per share, but rises to $0.0029 for a member whose average daily volume is at least 1% of the consolidated average daily volume (June 1, 2012 fee schedule). Maker fees for executions against non-displayed orders are lower ($0.0017): displayed size is a better advertisement for the exchange. The taker fee is a uniform $0.0029.

**Inverted pricing (taker/maker pricing)**

Some exchanges offer inverted maker/taker fees. The BATS-Y exchange charges $0.0030 for providing liquidity, and pays $0.0020 for removing it. At first glance this defies competitive logic. But suppose that BATS-X and BATS-Y are both offering at $10. If a trader intends to lift the offer, she will certainly send her order, or at least the first part of it, to BATS-Y. (It’s better to receive $0.0020 then pay $0.029.)

Now consider another situation. BATS-X has 10,000 shares offered at $10, and the BATS-Y book is empty. We’re considering where to send a $10 limit sell order. If we send it to BATS-X, we’ll receive $0.0025. But we’ll only get this if our order is executed, and there will be 10,000 shares ahead of us. If we send our order to BATS-Y, we’ll pay $0.0030, but our order will be at the front of the book. We also know that BATS-Y is a more attractive place to send a market order.

It is not a feasible alternative to send to BATS-X a sell order priced at $10 — $0.0030 = $9.9997 because quotes on an increment finer that $0.01 aren’t permitted by the sub-penny part of Reg NMS (discussed in Section 18.4).

**Routing charges**

The SEC has not directed industry to set up a single consolidated access system. When U.S. equity markets began to go electronic, most of the newer markets (like Inet) built systems so their subscribers could send in orders directly. The SEC envisioned a network of point-to-point connections, rather than a centralized hub-and-spoke system: “private linkages approach”. This is in fact largely what has happened.

Not everyone has direct connections. If there are \( n \) nodes a point-to-point network needs \( n(n - 1)/2 \) connections. There are over 200 market centers, so there would need to be about
20,000 links. A smaller number of market centers and brokers have developed their routing capabilities (speed, intelligence, number of connections) as a means of differentiation. Rather than access all market centers directly, a trader might set up direct links to a few centers, but go through a broker’s routing system (or NASDAQ’s) to access the others. Market centers charge for routing orders out to other centers (typically around $0.0030).

17.2. Payment for (retail) order flow

Some retail investors believe that the brokers send their orders to exchanges where they interact in some central fashion with all other buyers and sellers, large and small. This may have once been true, but current practice is very different. The SEC Concept Release states, “A review of the order routing disclosures required by Rule 606 of Regulation NMS of eight broker-dealers with significant retail customer accounts reveals that nearly 100% of their customer market orders are routed to OTC market makers,” (U.S. Securities and Exchange Commission (2010)).

This arrangement is described in Section 8.3 in connection with dark trading. Typically, the broker-dealer receiving a marketable customer buy order will sell directly to the customer at the NBO; a marketable sell order will receive the NBB.

This arrangement is driven by considerations of private information. As described in Section 11.2, a dealer loses to incoming informed traders, but profits from incoming uninformed traders. Retail traders are, as a group, less informed, and are therefore more desirable counterparties. Why, though, should a broker send a retail order to a dealer? Or, if the order will be sent, which dealer should receive it?

One factor bearing on these decisions is a payment from the dealer to the broker in exchange for the order. For various reasons discussed below, this compensation, akin to a referral fee, is controversial. The SEC permits the practice, but requires disclosure.

Whereas Rule 605 is aimed at accountability by market centers (see Section 14.7), Rule 606 applies to brokers. They must report what orders they received, what they did with those orders, and other aspects of their relationships with market centers.

Charles Schwab is a full-service broker with large retail customer base. The 606 report for 2009 Q 3 documents these relationships (From Schwab website, Jan, 2010). Table 17.1 summarizes the Charles Schwab’s customer orders (“Securities Listed on the NYSE/Network A Eligible Securities”). Table 17.2 describes where the orders were sent.

<table>
<thead>
<tr>
<th>Non-directed orders as percentage (%) of total customer orders:</th>
<th>95.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Market Orders as % of total non-directed orders</td>
<td>35.2%</td>
</tr>
<tr>
<td>2. Limit Orders as % of total non-directed orders</td>
<td>57.3%</td>
</tr>
<tr>
<td>3. Other Orders as % of total non-directed orders</td>
<td>7.5%</td>
</tr>
</tbody>
</table>
The 606 reports also disclose the broker’s arrangements with market venues.

UBS is a market maker in certain NASDAQ, OTC and listed equity securities. Part of the consideration Schwab received for the sale of its capital markets business to UBS in 2004 related to execution services agreements with UBS and Schwab’s commitments to route most types of equity and listed options orders through UBS for eight years.

However, Schwab does not earn rebates or other consideration from UBS or other firms or exchanges for equity and options orders routed through UBS or routed by Schwab directly.

Other OTC Market Makers do receive payment. From E*TRADE’s 2009 Q3 606 report:

E*TRADE receives payment from its affiliate, E*TRADE Capital Markets, LLC (“ETCM”), a wholly owned subsidiary of E*TRADE Financial Corp. (“ETFC”), for directing listed equity order flow.

Payments received from ETCM averaged approximately $.0006 per share [six cents on a 100-share order].

ETCM executes on a principal basis and may have profited or lost in connection with such transactions.

Brokers are supposed to act in their clients’ interests. Do side payments provide incentives to disregard those interests? From a comment letter on an earlier concept release (AGS Specialist Partners):

When order flow is earned through better execution, customers are rewarded with tighter spreads and the efficiency of the marketplace is improved. When order flow is guaranteed through cash payments or through other means of bribery, customers suffer as spreads widen and the marketplace becomes less efficient. Prepaid order flow, if anything, gives MMs an incentive to widen their quotes as order flow is guaranteed regardless of performance.

From a Reg NMS comment letter (George A. Carroon):

As a Member of the New York Stock Exchange, and a Specialist for more than forty years, ... I also encourage the Commission to give careful consideration to the issue of payment for order flow, which, in the opinion of many, can only be considered commercial bribery.

The dispute over payment for order flow last spilled into public view during Senate hearings in 2014 (Alden (2014)):
TD Ameritrade, a brokerage firm that handles vast numbers of stock trades for average investors, promises to execute those orders on the best possible terms. But in practice, TD Ameritrade routes a large number of the customer orders to the exchanges that pay it the most, Steven Quirk, an executive at the firm, said at a Senate hearing on Tuesday.

... The issue has divided the stock exchange sector. The president of the New York Stock Exchange, Thomas W. Farley, asserted on Tuesday that a system called maker-taker payments, in which exchanges pay rebates to brokerage firms for orders, created "inherent" conflicts. But the chief executive of the BATS Global Markets exchange company, Joseph P. Ratterman, played down that concern, saying the conflicts could be managed.

But when Mr. Quirk was questioned, the talk was less hypothetical. Senator Carl Levin, Democrat of Michigan, who leads the panel, pointed to data from the fourth quarter of 2012 that showed that TD Ameritrade directed all nonmarketable customer orders -- meaning, orders that could not immediately be consummated based on the market price -- to one trading venue, Direct Edge. It so happened that Direct Edge paid the highest rebate.

"Your subjective judgment as to which market provided best execution for tens of millions of customer orders a year allowed you to route all of the orders to the market that paid you the most," Mr. Levin said. "I find that to be a frankly pretty incredible coincidence."

17.3. Further reading


Summary of terms and concepts

Taker fees (also called liquidity removal, liquidity access); rebates for adding liquidity (also called maker rebates); inverted ("taker/maker") pricing and the logic behind it; payment for order flow; [SEC] Rule 606 information.

Problems and exercises

Problem 17.1 The PAX Exchange charges a $0.003 per share access ("taker") fee and pays a ("maker") liquidity rebate of $0.002 per share. How do these fees/rebates apply to an execution in which Tae’s limit order (Sell 100 shares, limit $25.00) is lifted by Sam’s order (Buy 100 shares, limit $25.00)?

Problem 17.2 (Continuation) For each execution, the ZAP Exchange charges a per share maker fee of $0.0029, and rebates $0.0018 to the active (marketable) order. On PAX, the bid is $25.00 for 10,000 shares, and the book at ZAP is empty. Vanessa enters an order on ZAP to sell 100 shares, limit 25.00. How do ZAP’s fees/rebates apply if Sam’s buy order is directed to ZAP instead of PAX?
Problem 17.3 DirectBook advertises a liquidity rebate of $0.0025 per share and an access fee of $0.0030 per share. The top of the bid book is a limit order to buy at $10.10 (Beth’s); the top of the offer book is a limit order to sell at $10.20 (Sam’s). If Charlie submits an order to sell limit $10.10, what are the total fees/rebates; who pays/receives them? Assume all orders are for 100 shares.

Problem 17.4 DirectBook advertises a liquidity rebate of $0.0025 per share and an access fee of $0.0030 per share. The market in XYZ is $10.10 bid; offered at $10.20. Briefly explain what this means to someone who’s sending in a limit order to sell 100 shares limit $10.20.

Problem 17.5 A comment letter received by the SEC reads, “... I also encourage the Commission to give careful consideration to the issue of payment for order flow, which, in the opinion of many, can only be considered commercial bribery.” What is payment for order flow, and why does the letter writer draw the comparison?

Problem 17.6 The PX exchange uses maker/taker pricing: orders that add liquidity receive a rebate of $0.001 per share; orders that take liquidity pay $0.002 per share. Sam just entered an order to buy 100 shares limit $20. This order goes into the book. Shortly thereafter Mona enters an order to sell 100 shares, limit $19. Sam’s and Mona’s orders are matched, and a trade occurs. In this situation, which of the following is correct?

a. Sam pays $0.20; Mona receives $0.10
b. Sam receives $0.20; Mona pays $0.10
c. Sam receives $0.10; Mona pays $0.20
d. Sam pays $0.10; Mona receives $0.20
e. Sam and Mona each receive $0.20
f. Sam and Mona each pay $0.10
The SEC’s Regulation NMS heavily influences the form and operations of the US equity markets. The rules were initially proposed in 2004, adopted (after comment and revision) in 2005, and phased in over 2006. The process provided an opportunity for public debate and speculation on how the markets would and should evolve.

“NMS” stands for “National Market System”. In 1975, Congress passed the Securities Acts Amendments (of 1975), which directed the SEC to facilitate development of a “national market system”. The amendments did not define the system precisely, but they were widely interpreted as calling for one comprehensive market for US stocks. At a time when trading was dominated by floor-based exchanges, the 1975 act held out a vision of multiple trading centers that would be electronically linked, with orders flowing to the trading venues with the best prices. Three decades later, Reg NMS was positioned and promoted as a culmination and fulfillment of the 1975 Act.

The final version of Reg NMS (U.S. Securities and Exchange Commission (2005)) and the online comment letters offer a detailed discussion of the logic behind the rule.

18.1. Background

*Market and order competition*

In the 1980s and 1990s, the two principal US exchanges (NYSE and NASDAQ) found themselves beset by competition from new markets that took full advantage of modern technology. One of these, Instinet, had evolved into something close to a modern all-electronic limit order book, but participation was limited to large institutions (and NASDAQ dealers). Island, another limit order market, attracted more retail traders. Other systems included Optimark, Wunsch Auction Systems, and Delta Options. The SEC termed this process *market competition*, that is, competition
among exchanges and similar venues. By the 1990s, it was clear that market competition was bringing many benefits to investors, in the form of lower trading costs and new ways of trading.

The pursuit of market competition, though, led to fragmentation. The new environment with many competitive markets was confusing, as investors who formerly faced simple choices in routing their orders (like, NYSE vs. NASDAQ) found themselves forced to consider a multitude of possible market destinations. The problem is that when orders are resting in many places, it’s difficult to determine where the best prices are. The interplay of individual orders, described as order competition, works best when all orders go to a single destination. Enhancing order competition favors consolidation.

In designing Reg NMS the SEC sought to balance market competition and order competition. The aim is sometimes (but not in the actual text of the rule) called virtual consolidation, that is, separate competitive markets that function as one cohesive market, and present to the user one unified view of the market.

Other trade-offs involved balancing the interests of the different kinds of traders. US equities markets, for example, have an unusually large number of retail traders. Should market regulation favor retail or institutional clientele? Retail traders are more numerous, but aren’t institutions like mutual funds and pension funds representing the interests of their beneficiaries, who tend to be individuals? This debate was affected by the SEC’s regulatory experience, in the 1990s, involving “two-tier” markets in which institutions received favorable bids and asks, while retail investors faced large spreads.

Yet another apparent trade-off involved balancing the interests of short-term traders (like dealers) vs. long-term investors (presumably the dealers’ customers). The SEC’s response in this case was more one-sided. The SEC views its mandate as acting on behalf of companies trying to raise capital for long-term projects and investors with long-term horizons. Market makers, intermediaries, and day traders are not considered to have a valid interest (for regulatory purposes) except insofar as they contribute to or facilitate the long-term investors’ interests. From the final rule: “... it makes little sense to refer to someone as ‘investing’ in a company for a few seconds, minutes or hours.”

Alternative trading systems (ATSs)

The new markets posed many regulatory challenges. To begin with, what should they be called? In the set of available regulatory categories, a “market” pretty much had to be an “exchange,” that is, something that was owned and operated by its members. The new markets did not have members, at least in the traditional sense, and most could not withstand the expense of complying with all the legal and regulatory overhead that seemed to come with being labeled an exchange. The SEC initially granted them “no action” letters, provisional assurances that they could operate without fear of legal action. But as the numbers of these quasi-exchanges grew, the SEC needed a clearer and more consistent approach. In 1997, the Commission adopted Regulation ATS (Alternative Trading Systems).

Reg ATS provided several levels of regulatory requirements, depending on how much trading volume the system executed, as a proportion of the total volume across all market centers. Under the present (amended) rule, an ATS in the smallest size category (under 5% of the total volume): files an initial notice of operation and quarterly reports; keeps records (an audit trade) of its activities; and, does not call itself an exchange. At or above 5%, the ATS must also link with a registered market (like an exchange), comply with the market’s rules, and disseminate (that is,
publish) its best bids and offers on the consolidated quote system. (Orders in a dark pool that aren’t shown to anyone don’t have to be published.) Also (at or above 5%), the ATS must establish objective written standards, applied in a nondiscriminatory fashion, for admitting and denying access to its system (“fair access”).

The SEC maintains a current list of ATS on its website. As of April 2015 there were slightly under one hundred registrants. Most dark pools and crossing networks are registered as ATSs, but a market does not need to be dark. Most markets trade US equities, but some trade bonds. MTS BondsPro, for example, is a lit market for US dollar-denominated bonds.

The structure of Reg NMS

Reg NMS has four main components:
- the order protection rule
- the access rule;
- the subpenny pricing rule
- the market data rules

(The regulation also mandated a consolidation and renumbering of preexisting rules, but this was mainly a legal formality.) Each seeks to contribute to the balance of market and order competition.

18.2. The order protection rule

This is sometimes (but inaccurately) called the “no trade-through” rule. Recall that a trade-through is an execution outside of a market’s best bid or offer. This part of Reg NMS has two main provisions:
- Market centers will put in place procedures to avoid trade-throughs.
- A bid/offer is not protected against trade-through unless it gives automatic execution.

In Chapter 5 we encountered the basics of this rule. Here we consider some of the finer points.

We noted earlier that although the order protection rule is sometimes characterized as prohibiting trade-throughs, this isn’t quite correct. The rule simply says that markets need to avoid them. Nor does the rule require a broker to route a customer order to the exchange at the best visible quote (at the NBBO). It only says that an exchange shouldn’t execute the order through the NBBO. Finally, it does not guarantee that the customer always gets the best price. No reasonable rule could accomplish this. The automatic execution condition for protections was far-reaching. It made a distinction between “fast” and “slow” markets. It forced floor markets (notably the NYSE) to quickly become electronic.

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18 In the original rule, the fair access requirement did not apply until the ATS reached 20% of the total volume. The 20% threshold was dropped to 5% in 2005.
19 The SEC has recently proposed amendments to require that the operator of the ATS provide more detail about its ownership and affiliates, and also that the ATS have written “safeguards and procedures to protect subscribers’ confidential trading information,” (U.S. Securities and Exchange Commission (2016c))
**Intermarket sweep orders (ISOs)**

Sometimes a trader wants to execute a large quantity, quickly, against all of the protected quotes in all of the market centers. If uncoordinated orders are simply routed to each destination, some markets might refuse to execute their components, believing that these executions will cause trade-throughs, even though the protected bids and offers have already been executed by the same trader. In this situation, Reg NMS permits the trader to use an intermarket sweep order (ISO). Functionally, this allows the trader to “sweep” all protected bids or offers, notifying the individual markets of his intent, and relieving them of the responsibility to check for executions that would trade through protected bids or offers.

Formally, from the rule’s text:

An intermarket sweep order is ... a limit order that meets the following requirements:

1. The limit order is identified as an intermarket sweep order when routed to a trading center; and
2. Simultaneously ... one or more additional limit orders, as necessary, are routed to execute against the full displayed size of any protected bid, in the case of a limit order to sell, or the full displayed size of any protected offer, in the case of a limit order to buy, for the NMS stock with a price that is superior to the limit price of the limit order identified as an intermarket sweep order.

Here’s how it works. The ISO provision specifies that the quantity to be routed to a market must accommodate all protected bids or offers. That is, the rule specifies a minimum. The eventual outcome may result in executions that are, in the usual sense, trade-throughs, as long as they aren’t through protected quotes.

For example, consider a two-market case, with the bids given as in Table 18.1. It would satisfy the rule to submit:

- Sell 400 shares, limit $100, ISO to Exchange A, and
- Sell 100 shares, limit $100, ISO to Exchange B

The 100 shares to Exchange B suffices to execute B’s protected bid at $103. On Exchange A, 100 shares would execute against A’s protected bid of $102, and 300 shares would execute deeper in A’s book, at $100. Now this last execution trades through orders in B’s book that are priced at $102 and $101, but because these orders weren’t at the top of B’s book, they weren’t protected.

### Table 18.1 Two-market example. Bids on two exchanges.

<table>
<thead>
<tr>
<th>Bid</th>
<th>Exchange A's Book</th>
<th>Exchange B's Book</th>
</tr>
</thead>
<tbody>
<tr>
<td>$103</td>
<td></td>
<td>$100 sh</td>
</tr>
<tr>
<td>$102</td>
<td>100 sh</td>
<td>100 sh</td>
</tr>
<tr>
<td>$101</td>
<td></td>
<td>300 sh</td>
</tr>
<tr>
<td>$100</td>
<td>300 sh</td>
<td></td>
</tr>
</tbody>
</table>

The order protection rule was the most contested part of Reg NMS, and it received the strongest comment letters. In the final vote, two (of the five) SEC Commissioners voted against it. (Their dissent is available on the SEC website.) The objection was that the rule wasn’t needed. It
was asserted that brokers, in fulfilling their fiduciary duty of best execution, would ensure that their customers would always get the best price, and so trade-throughs wouldn’t occur. If brokers didn’t do this, the customer could always get a new broker. In any event, the problem should remain a matter between the broker and the customer, not something that needs to be addressed by federal statute. The counterargument was that investors needed protection. They lacked the means to monitor what their brokers were doing. Furthermore, the brokers themselves might not know at any given instant which particular market had the best quote.

18.3. Access rule
For a market to have its quotes protected under the order protection rule, anyone must be able to execute against those quotes quickly and *inexpensively*. The market must give everyone equal access. No discrimination in favor of subscribers. The rule also states that any “access fee” can’t be higher than $0.0030 per share (see Section 17.1).

18.4. The sub-penny rule
Until the end of the 20th century, trading in US equity markets was conducted in eighths ($0.125). Congress’s Common Cents Pricing Act of 1997 established the penny as the price increment in US stock market. As a transitional step, exchanges went first to 1/16ths, “steenths”. (Quickly now: if you’re a buyer, would you rather have five steenths or three eighths? If a seller, thirteen steenths or three quarters?) In 2001 the transition to penny pricing was completed.20

It is sometimes said that the tick size is the price of time priority. Suppose the bid is $10.00 for 20,000 shares. If I put in a bid at $10.00 for 100 shares, I’m at the end of a 20,000 share line (due to time priority). If I put in at $10.01, I move to the front of the line. If the tick size were $0.000001, I could jump by the queue by paying an extra $0.000001 x 100 shares = $0.0001. The effectiveness of the sub-penny rule has been partially undone by sub-penny maker/taker fees (see Section 17.1).

The sub-penny rule applies to bid and ask quotes, not trades. If the NBBO is $25.00 bid, offered at $25.01, a dark pool trade that is priced at the midpoint will executed at $25.005. This is permissible.

The US Congress usually delegates rule-making to the SEC. The 1997 act was an exception, but it was not the last time Congress involved itself with tick-size. The Small Cap Liquidity Reform Act of 2014 directed the SEC to study the desirability of *raising* the pricing increment for some stocks to $0.05.

18.5. Market data rules
There are two main sources of exchange revenue: listing fees and data fees. Historically, they were about equal. Nowadays, data fees dominate.

The Consolidated Tape Association runs CTS and CQS and shares its revenues with the data providers (exchanges and non-exchange SIPs, securities information processors) under complex formulas. Prior to Reg NMS, exchanges were a fixed fee for reporting a trade (no matter how

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20 The title of the act is highly referential: Cents vs. “sense”; “common sense” used to mean practical knowledge; “Common Sense” was also a pamphlet published during the American Revolutionary War.
This led to “tape shredding” breaking up large trades into small 100-share trades to maximize the data revenue. Reg NMS removed this incentive. Also, prior to Reg NMS, an exchange was paid for posting a quote, no matter how aggressive it was. If the NBB was $10.00 for 10,000 shares, an exchange would get paid for posting a bid of $1 for 100 shares. Reg NMS revised the formula, so that revenue is based on size and time at NBBO.

What sort of data can the exchanges charge for? The basic principles were laid out in 1999 SEC report (“Seligman Commission”). The BBO and trade reports should be provided at low cost. All other data can be priced at what the market will bear, e.g., order book data, historical data, short-sale data, and so on.

When Reg NMS was being debated, the modern era of high-frequency trading was just beginning. It passed notice that an exchange might want to discriminate among its market data customers on the basis of speed. Is it presently permissible for an exchange to intentionally slow down market data transmission speed for one class of customers? Probably not. Can exchanges charge extra for high-speed transmission channels that outrun the consolidated trade and quote systems? This is certainly okay.

18.6. An appraisal

For the most part Reg NMS has functioned well. It has survived ten years with no substantial modifications. Market center competition has been robust. An innovative new exchange, IEX, was approved in 2016. Order competition is also vigorous. Trading volumes are large, and bid-ask spreads are low. O’Hara and Ye (2011) make case that market quality has not been impaired by fragmentation. Other commentators are more critical (see, for example, Hatheway, Kwan and Zheng (2013)). They claim that while the decline in bid-ask spreads has been good for small retail traders, institutional traders are finding large trades more costly. Some people believe that the market fragmentation that has occurred under the rule has encouraged high frequency trading (discussed in the next chapter). Access fees remain problematic, and have proliferated in variety.

A fragmented market is a complicated place. The virtual consolidation vision of Reg NMS has certainly not resulted in virtual simplicity. For this reason, some commentators believe that while market competition may have been necessary during a period of technological innovation, that phase is now largely over, and it is time to encourage consolidation, perhaps even moving to one consolidated limit order market. That market, though, would be a monopoly, wielding enormous economic and political power. This might be counterbalanced by aggressive regulation, but experience is not encouraging. Throughout much of the last century, the SEC struggled to open up US exchanges to more competition (Seligman (1985); (1995)).

In providing a framework for market competition, Reg NMS has been influential. Shortly afterwards, the European Union adopted the Markets in Financial Instruments Directive (“MiFinD”). As a result, the older exchanges (e.g., the Paris and Amsterdam Bourses) have been placed in competition with new entrants (e.g., BATS Europe, Chi-X, etc.).

Not all markets have followed the Reg NMS lead. US futures exchanges have remained consolidated. Their regulator (the CFTC) and key market constituents have not shown strong interest in reshaping the exchanges along Reg NMS lines. The differences in regulatory and market design philosophies that split US equity and futures markets are striking given the strong resemblances in what they trade. If we want to invest in the S&P 500, we can buy the SPY ETF on an equities exchange, or we can go long an S&P stock index contract on a futures exchange. The profit/loss outcomes will be very similar, but the trading environments are very different.
Summary of terms and concepts

Market competition; order competition; alternative trading system (ATS); electronic communications network (ECN); order handling rule; Reg NMS; order protection rule; access rule; sub-penny pricing rule; protected quotes; intermarket sweep orders (ISOs); tick size; The Common Cents Pricing Act of 1997; Small Cap Liquidity Reform Act of 2014.

Problems and Exercises

**Problem 18.1** For stock XYZ markets AX and BX are the only two market centers bidding. Their bid books look like this:

<table>
<thead>
<tr>
<th>Price</th>
<th>AX</th>
<th>BX</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.10</td>
<td>100 sh</td>
<td>Hidden</td>
</tr>
<tr>
<td>$10.09</td>
<td>200</td>
<td>Visible, 100 sh Visible</td>
</tr>
<tr>
<td>$10.08</td>
<td></td>
<td>300 Visible</td>
</tr>
<tr>
<td>$10.07</td>
<td>300</td>
<td>Visible, 500 Visible</td>
</tr>
</tbody>
</table>

a. What is the National Best Bid?

b. Market BX receives an order to sell 400 shares limit 10.08 (not marked “sweep”). It executes 100 shares at 10.09 and 300 more shares at 10.08. Does this constitute a trade-through from the perspective of Reg NMS? Explain.

c. Suppose the seller divides her orders:
   - Sell 200 shares limit 10.08 (marked sweep) to AX
   - Sell 200 shares limit 10.08 (marked sweep) to BX.
   Is this consistent with the Reg NMS trade-through policy? Explain. What executions will result?

**Problem 18.2** The bid sides of the books at three market centers are as follows. There are no hidden orders.

<table>
<thead>
<tr>
<th>Price</th>
<th>Exchange A shares</th>
<th>Exchange B shares</th>
<th>Exchange C shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.49</td>
<td>300</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>50.48</td>
<td>100</td>
<td>800</td>
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</tr>
<tr>
<td>50.47</td>
<td>100</td>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>50.46</td>
<td></td>
<td>900</td>
<td>600</td>
</tr>
<tr>
<td>50.45</td>
<td>2,000</td>
<td>2,000</td>
<td>2,000</td>
</tr>
</tbody>
</table>

a. For each exchange, what are the protected bids (prices and quantities)?

b. A trader wants to sell 1,000 shares limit 50.45 using ISOs (intermarket sweep orders). What is the minimum quantity that must be submitted to A? to B? to C?

c. A trader wants to sell 6,000 shares limit 10.48 using ISOs. What is the minimum quantity that must be submitted to A? to B? to C?

d. If the trader submits sell 2,000 shares limit 10.45 to each exchange using ISOs, are there any trade-throughs of displayed bids that aren’t protected.
Problem 18.3  The bid sides of the books at three market centers are:

<table>
<thead>
<tr>
<th>Price</th>
<th>Exchange A</th>
<th></th>
<th>Exchange B</th>
<th></th>
<th>Exchange C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shares (Displayed)</td>
<td>Shares (Hidden)</td>
<td>Shares (Displayed)</td>
<td>Shares (Hidden)</td>
<td>Shares (Displayed)</td>
</tr>
<tr>
<td>10.49</td>
<td>100</td>
<td>900</td>
<td>800</td>
<td>2,000</td>
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<tr>
<td>10.48</td>
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<td>10.45</td>
<td>800</td>
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<td>200</td>
<td></td>
<td>2,000</td>
</tr>
</tbody>
</table>

a. For each exchange, what are the protected bids (prices and quantities)?

b. A trader simultaneously sends the following sell orders, all with limit prices of 10.46, all marked ISO: sell 800 to A, sell 2,000 to B, sell 500 to C. What executions occur?
Chapter 19. High Frequency Trading (HFT)

The technology that exchanges presently use to manage continuous limit order books and run opening or closing auctions is far superior to what was available near the end of the last century. In fragmented markets, this is also true of the technology used to link market centers. Present day information, access and routing systems are more advanced, possessing superior speed, capacity and reliability.

At the start of transition to electronic markets, it was believed that technology would weaken exchanges’ floor constituencies, that electronic access would reduce the advantage of being physically present at the center of trading. Although we’ll see that this prediction eventually proved false in some interesting respects, it was initially correct: the floor trading crowd was largely left behind, unable to keep up with automatic execution.

More broadly, though, electronic markets were initially viewed as fostering equality among participants. Morris Mendelson and Junius Peake, two early advocates, wrote that, ““Bids and offers [will] compete instantly, equally and fairly regardless of their origin since, once entered into the system, they [will] be instantly reflected in the summary displays and capable of being executed. ... All brokers, dealers and investors [will] have equal, simultaneous access to all data,” (Mendelson and Peake (1979)). In this vision, everyone from retail customer to hedge fund trader sits at a computer terminal, equal but for their inborn quickness of mind.

As matters unfolded, however, equality did not come to pass. The reach of technology did not stop at improvements within the market centers, it extended to the markets' users, that is, the brokers, investors, and traders. We might distinguish, therefore, between market technology (inside the market center) and trading technology (outside). Advances in trading technology altered the relative economic power and standing of key players. The changes have not been neutral. Technological shifts rarely are.

The effects of trading technology have been most profound with respect to speed. Simply put, in the most of the trading mechanisms and strategies discussed throughout this book, speed
confers an advantage. The first trader to hit a bid or lift an offer, the first to cancel a stale bid or offer, whether in a floor crowd or electronic limit order book, generally wins. Even in an auction, the bidder with the fastest technology can submit or cancel his order after everyone else.

Traders whose strategies depend on speed are generally lumped together as “high frequency traders.” The term is widely used, but carries some misleading connotations. “High frequency” implies that they trade often. They might, but it’s probably more accurate to say that when they decide to trade, they trade quickly. For this reason, Hasbrouck and Saar (2013) suggest “low latency,” that is, “with minimal delay.”

Since technology plays a pervasive role in the subject, it also serves as a good starting point.

**19.1. Technology**

What does a trader need to win, place, or at least finish with the field in the speed race?

The first step in building a fast system is to eliminate (at least for trading purposes) the person entering orders on the screen and keyboard. The human blink reflex, in response to a loud noise (for example), requires about 200 milliseconds (that is, 0.2 seconds). Add in a bit more time to press the “send” key (assuming that the order screen is already correctly populated) and we might well be facing a response time of one second or more. Present markets are generally accessible through their application program interfaces (APIs) that allow a program written in a familiar language (like Java or C) to directly access a market’s information and order entry ports.

Once we have optimized the speed of our programs, we might think about how to make the hardware faster. The off-the-shelf Intel and AMD chips that power our personal computers are too slow, even when multiple processors are used in parallel, so programmers turn to advanced customizable logic chips. The curious might start with “A Low-Latency Library in FPGA Hardware for High-Frequency Trading (HFT),” (Lockwood, Gupte, Mehta, Blott, English and Vissers (2012)). By using APIs and bit of clever programming, we might be able to generate an order in a few ms. But now the order must be communicated to the market center. At this point we run up against a physical limit – the speed of light ($3 \times 10^8 \text{ m/sec}$). If our offices are in Chicago and the market is in New York (a separation of about 1,200 km), one-way transmission time is at least

$$\frac{1.2 \times 10^6 \text{ meters}}{3 \times 10^8 \text{ meters/second}} = 0.004 \text{ sec} = 4 \text{ ms}; \text{ Shanghai to New York is about 12,000 km (about 40 ms).}$$

Now since we can’t circumvent the speed of light, the next best alternative is to position our computer closer to the market. How close do we need to be? Is the same city good enough? The same street? The same building?

Modern market data centers are composed of blade servers: racks filled with boxes that are filled with slots. The blades that fit into these slots are circuit boards that hold processors and memory. One blade constitutes “the market”. Ideally, we’ll place our blade (the one holding our trading logic) in the same server in a slot adjacent to the market blade. Our computer is now said to be collocated (with the market). The practice is generally termed “collocation”. Through collocation we can achieve fast two-way communication with the market where we’re collocated. This facilitates single-market strategies. We can lift a new offer, for example, immediately after it is posted.

Our collocation does not protect us, however, from delays involving information produced in other places, such as other markets. Suppose that we’re collocated with market $X$ and we’d like to hit $X$’s bid quickly when market $Y$’s bid (in the same stock) is hit. In learning about market $Y$ we are again faced with transmission latencies. We could collocate a second computer with market $Y$, but this won’t help us communicate between the two markets.
Faced with the intermarket communication problem and the light-speed barrier, we can at most achieve marginal improvements in speed. Laughlin, Aguirre and Grundfest (2013) note:

Using relativistically correct millisecond-resolution tick data, we document a 3-millisecond decrease in one-way communication time between the Chicago and New York areas that has occurred from April 27th, 2010 to August 17th, 2012. We attribute the first segment of this decline to the introduction of a latency-optimized fiber optic connection in late 2010. A second phase of latency decrease can be attributed to line-of-sight microwave networks, operating primarily in the 6-11 GHz region of the spectrum, licensed during 2011 and 2012. Using publicly available information, we estimate these networks’ latencies and bandwidths. We estimate the total infrastructure and 5-year operations costs associated with these latency improvements to exceed $500 million.

Or consider the following press release (Hibernia Networks (2015)):

DUBLIN, IRELAND – September 24, 2015 - Hibernia Networks, a leading provider of high speed global telecommunications services, announces that its new Hibernia Express transatlantic cable has an actual tested latency of better than 58.95ms (milliseconds) from New York to London, which is faster by more than half a millisecond off the original projected speed. The new 4,600km ultra low latency submarine cable is the first transatlantic cable build in over a decade.

The cost of the cable was generally thought to be in excess of $300 Million.

Clearly our pursuit of speed carries a high price tag. In what senses, if any, are these expenditures justifiable?

19.2. The private and public value of speed

At long horizons, faster information is undoubtedly better information. We are learning about the future sooner. This can produce real benefits. The US government publishes Weekly Weather and Crop Bulletins that summarize recent weather conditions and their impact on key crops. It would be wonderful if some improvement in forecasting technology allowed us to predict in January the content of the coming August report. If we discovered and owned the forecasting technology we could make large profits in the grain futures markets. We’d establish our positions, publicly release our forecasts, and reverse our trades. The private gains (our profits) would be substantial. But the public (social) gains arising from better planting and harvesting decisions would probably be even larger.

Someone making the case for allowing us to trade on our information might appeal to the principle of market efficiency. It would be argued that the possibility of trade gives us the incentive to produce the information, and that our profits capture only a small portion of the overall benefit. Such statements have frequently arisen in the public debate on high frequency trading, claiming that a faster market is a more efficient market. (We are invoking here the simplest view of private informational efficiency, ignoring the complications discussed in Chapter 11.)

Given equal accuracy, a July forecast of August conditions is less valuable than a January forecast. Farmers might still be able to make better harvesting decisions, but July is probably a little late to change plantings. Here too, though, if we could trade on the information, we’d realize sizable profits.

Now consider a “forecast” of the crop report that is accessible (privately, to us) ten seconds before the public release. Our trading profits would still be substantial, but the social gains would
be virtually nonexistent. It is difficult to imagine a ten-second time difference affecting any farmer’s decision.\textsuperscript{21}

The social value of information derives from its importance in affecting real production and consumption activities. Production and consumption decisions, though, are made at time horizons of days, weeks and months. The private value of information in trading decisions is much more time sensitive. Financial markets reward those who move “first”, not necessarily those who are “fast” in any absolute sense. If it takes everyone else one minute to hit a bid, I can be first with a response time of fifty-nine seconds; if the normal response time is one ms, though, being first requires reaction within microseconds.\textsuperscript{22}

\subsection*{19.3. The high frequency traders}

So who are the high-frequency traders? It depends on how broadly we define the category. Our mutual funds and pension funds routinely use computerized algorithms to implement their trading decisions. Many of these algorithms are available to retail investors. Retail investors, even if they are communicating with their brokers by telephone, would find that the best handling of their orders required them to be passed by sophisticated routing technology. (This is sometimes called agency algorithmic trading because the brokers are deploying it in service to and as an agent for their customers.) In a broad sense, by the standards of the late twentieth century, we are all high frequency traders.

Public discussions of high frequency trading, though, generally refer to a smaller set of players, proprietary trading firms that rely on the fastest technology. Although most of them were born in the current century, some have already become large market participants. In 2013, several of them (Global Trading Systems, Hudson River Trading, Quantlab Financial, and Tower Research Capital) formed a trade group (modernmarketsinitiative.org). Their most important role is “de facto” market making, having displaced traditional market makers (like the NYSE specialists). While many observers feel that they have functioned well overall in this capacity, concerns persist (see Section 7.4).

One parallel is especially striking. On the old floor exchanges, members enjoyed better access and information that off-floor customers. This was viewed as an inherent limitation of a market that was convened in a small physical space. Although the new electronic markets held the promise of equal access, old ways hung on. The new “floor” is a blade server, and traders who are on the floor (that is, collocated) still have advantages relative to off-floor customers.

\textsuperscript{21} The plot of the comedy “Trading Places” turns on illicit advance knowledge of an orange juice crop report.

\textsuperscript{22} It has always been thus. “For some years prior to [the introduction of the telegraph in 1846], William C. Bridges, a stock broker... maintained a unique private ‘telegraph’ system between Philadelphia and New York. By the ingenious device of establishing stations on high points across New Jersey on which signals were given by semaphore [flags] in the daytime and by light flashes at night, discerned with the aid of telescopes, information on lottery numbers, stock prices, etc., was conveyed in as short a time as ten minutes between the two cities. Some of the mysterious movements in the stock markets were ascribed to this pioneer financial news bureau,” (Barnes (1911), italics mine). Such was the practice of high-frequency trading in the nineteenth century. The reader may have elsewhere encountered the story that Nathan Rothschild profited by quickly trading on the news of Wellington’s victory at Waterloo (1815), conveyed to him by carrier pigeon. Were it true this anecdote would be very on point, but Ferguson (1998) refutes it.
There are, of course, some important differences. The old floor had formidable barriers to entry. If you lacked the good fortune to have inherited a membership, you would have to purchase one. In the latter twentieth century memberships (“seats”) on the NYSE sold for several million dollars. Current collocation prices appear to be much lower (see NYSE (2016)). When barriers to entry are lower, an economist would expect more competition, and lower profits for HFT firms. Many commentators, however, remain critical of high-frequency trading. Some of the accusations are similar to those that have leveled at human brokers and market-makers, notably that they front-run (that is, trade ahead of) customers’ orders, and destabilize markets (see recurrent blog postings at themistrading.com and nanex.net).

19.4. THOR and the start of IEX

As the speed advantages of the fastest traders become ever shorter, the claims to enhanced market efficiency become less persuasive. Even when the information is of a fundamental nature, like the Michigan Consumer Sentiment Index, the social gains from release a few seconds early to select traders is doubtful (see Section 9.1). We are even more skeptical when the information advantage involves market data.

The tension simmered in industry symposia and regulatory hearings for years, but burst into public awareness with the 2014 publication of Michael Lewis’ *Flash Boys* (Lewis (2014)). The book was a sweeping indictment of many financial industry practices (including payment for order flow). The book pushed into prominence a firm (IEX), which went on to play a major role in the HFT regulatory debate.

IEX was started in 2012 by Brad Katsuyama, head of electronic trading at Royal Bank of Canada’s (RBC’s) Capital Markets Group. Initially IEX was a dark pool (an alternative trading system, or ATS). In 2014 it filed an application to become a full-fledged national securities exchange. The application was approved in June, 2016.

In *Flash Boys*, Katsuyama recounts a demonstration from 2007, while he was at RBC:

I’d say, ‘Watch closely. I am about to buy one hundred thousand shares of AMD. I am willing to pay forty-eight dollars a share. There are currently one hundred thousand shares of AMD being offered at forty-eight dollars a share—ten thousand on BATS, thirty-five thousand on the New York Stock Exchange, thirty thousand on Nasdaq, and twenty-five thousand on Direct Edge.’ You could see it all on the screens. We’d all sit there and stare at the screen and I’d have my finger over the Enter button. I’d count out loud to five . . .

‘One . . .
‘Two . . . See, nothing’s happened.
‘Three . . . Offers are still there at forty-eight . . .
‘Four . . . Still no movement.
‘Five.’ Then I’d hit the Enter button and—boom!—all hell would break loose. The offerings would all disappear, and the stock would pop higher.”

At which point he [Katsuyama] turned to the guys standing behind him and said, “You see, I’m the event. I am the news.”

The fact that a lifted offer conveys information is, of course, unsurprising (to us, and certainly to Katsuyama as well). But there is more to the story. Why would the offerings all disappear? It was not because Katsuyama’s orders were getting filled. It turned out that the only market center
where the orders were consistently executed was BATS. This suggested that the BATS execution was the key event in the causal chain that ended with offers on other exchanges vanishing.\footnote{To comply with Reg NMS, the order would have been entered as an intermarket sweep (ISO), with the components sent to all exchanges “simultaneously.” With an ISO, of course, nothing is guaranteed, and prudence would dictate that the components be flagged IOC (immediate or cancel) to prevent unintended display of unexecuted portions.}

If the offers on other exchanges were simply being cancelled, there might be little cause for concern. A market maker might simply place offers on all exchanges, intending to cancel the others after one of them executed. If I am selling a boat by posting “for sale” messages on five different bulletin boards in my building, nobody would presume that I have five boats for sale. In situations where this needs to be formally stated (as for auto dealerships), the ad will generally include a statement that the one item is being offered “subject to prior sale.”

If the offers on other exchanges were being executed, however, we’d be more concerned. The purpose of Reg NMS was to promote virtual consolidation. In a single limit order book, if we submit a large order, it is impossible for another trader to jump in front of us as soon as we have executed a portion of our order. If this becomes possible in a fragmented market, it suggests that the linkage mechanisms are falling short.

To illustrate, suppose exchanges A and B are each posting offers of 10,000 shares (of anything) at $50.10, and that Mega Mutual Fund wants to sweep the top of two markets’ ask books (Figure 19.1). But now suppose that transmission speed to A is fast, but transmission to B is slow. The difference in latencies (delays) opens a window of opportunity for a faster predatory trader “Clarence”. As soon as there’s an execution on A, Clarence buys the shares on exchange B. By the time that Mega’s order arrives at B, the shares it wanted have been purchased by Clarence (Figure 19.2).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure19_1.png}
\caption{Figure 19.1}
\end{figure}

The market has seen the offer lifted twice (at each exchange), and this will tend to drive the ask higher. Perhaps Clarence will offer the shares a cent higher (at 20.11). If Mega buys them at this higher price, Clarence will make $100.

Clarence’s strategy is one form of latency arbitrage. Strictly speaking it is not an arbitrage because it involves some risk: the price might drop while Clarence is holding the shares. Given two big trades at the ask, though, this isn’t very likely, at least in the short run. Whether or not
we call it an arbitrage, it is clear that the profit is deriving not from production of any fundamental information about the stock, but rather from Clarence’s anticipation of A’s order.

*Figure 19.2*

1. Mega’s order arrives and executes at Exchange A
2. HFT/PTF “Clarence” sees execution of sweep order at A, and immediately lifts offer on Exchange B.
3. When Mega’s order arrives at Exchange B, the 10,000 shares aren’t there anymore.

The RBC remedy was to delay transmission of the order to A so that it arrived at the same time as the order to B (Figure 19.3). The delay synchronizes the order arrivals so that Clarence cannot preemptively purchase the shares.

*Figure 19.3*

Although RBC could have introduced the delay by inserting a timer in the software, it decided instead to employ a length of fiber optic cable long enough to ensure that an order submitted at
one end would emerge from the other end 350 microseconds later. The cable was compactly coiled in a way that resembled a spool of fishing line. Pictures of coil (once widely available) gave RBC’s solution visibility and concreteness, attributes usually absent from the HFT debate. RBC called its system the Tactical Hybrid Order Router (THOR, after the figure from Norse mythology).

THOR was based on selective delay, the idea that by slowing down one part of the market, things could effectively be reordered in time to eliminate the possibility of latency arbitrage, that is, to recreate a level playing field. It was an effective response to the latency arbitrage directed at sweep orders, but it would not have been effective against similar strategies. In the situation described above, for example, if exchange A’s executions generally occurred before exchange B’s, Clarence could use A’s trades as a signal, to pick off Exchange B’s stale bids and offers.

IEX was formed with the intent of applying the delay principle more broadly, to construct a market center where latency arbitrage would not be possible, even when the market center was embedded in the complicated Reg NMS framework of competing exchanges and trade-through prevention. We’ll return to the IEX story, but since delay plays such a prominent role, we need to step back and look at how other delays have been introduced and how they are viewed.

19.5. Delay

To explore the role of delay, let’s start with a simple limit order market (Figure 19.4). The figure explicitly maps the inbound and outbound message queues, the book, and the processor that links them and makes the decisions. The inbound messages are orders and cancellations; the outbound messages convey the state of the book and report executions. The book itself is depicted as a separate data structure, distinct from the queues and processor. We can imagine the processor functioning as a loop, taking a message from the inbound queue, processing it (executing, canceling from, or adding to the book). The queues are FIFO (first-in, first-out), which implies that the outcomes of this market are determined by the order of messages in the inbound queue.

In any given trade, the buyer and seller would generally prefer that publication of the execution be delayed as long as possible. A trade is often a piece of a broader plan, such as when a large order is split into smaller orders. Delaying publication of a trade makes it more difficult for others to see the big picture.

Disclosure is nevertheless usually viewed as an important aspect of transparency. In one floor market the rule states, “The buyer and seller in a pit transaction must report immediately to the pit observer any change in the last sales price or last quotation and it shall be their duty to
make certain that such change in quotation is properly posted," (Chicago Mercantile Exchange, Rule 528). In US securities markets, FINRA rules also require prompt publication.

There are nevertheless some interesting exceptions. In the 1990’s, NASDAQ reportedly delayed trade publication to the east coast of the U.S., to offset transmission delays to the west coast. The London Stock Exchange allowed large trades to be published the following day. The intent here was to give dealers an opportunity to work off large positions that they acquired when they accommodated a customer.

There have also been some accidental delays. When a trade occurs, it is usually disseminated to the outside world via broadcast on a market information system. But there are also reports (called confirmations) that are sent back to buyer and seller directly, informing them that the execution has occurred, and alerting them that clearing and settlement procedures will follow. The CME was embarrassed when it came to notice that confirmations were being sent out before public dissemination of the execution (Patterson, Strasburg and Pleven (2013)). The NYSE was fined $5 Million by the SEC because some subscribers received quote updates in advance of transmission to the consolidated feed (U.S. Securities and Exchange Commission (2012b)). In both cases, the timing advantages would have been on the order of a few milliseconds, but they were nevertheless perceived as sufficient to support latency arbitrage. The differentials also arose in complex computer systems, and so are difficult to detect.

Intentional inbound delays are rarer. THOR used a selective delay on orders sent to BATS, but the intent was not to delay processing on BATS, but instead to delay other traders’ awareness of the BATS execution.

The processing of special orders or conditional orders, such as stopped, pegged, discretionary and so forth, introduces additional complications because that the processor has to monitor the output messages. A stop order is elected in response to an execution; a pegged order is repriced in response to a quote change; a discretionary order may be repriced to be marketable. If there are many of these orders, they may impose a substantial workload. Sometimes they are handled in a second system or processor, as shown in Figure 19.5. NASDAQ's system, RASH (for routing and special handling) was separate from the INET system that managed the book, for example.

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**Figure 19.5**

![Diagram of special orders processing](image)

The processing of special orders may also introduce latencies that can be exploited. If a proprietary trader can insert himself advantageously, he might be able to act before all of the special
and conditional orders have been updated. For example, if there are many offers pegged to the NBO, repegging all of them when the NBO rises might be so time-consuming that the prop trader might be able to lift some before they can be repriced (Figure 19.6).

Figure 19.6

One approach is to move the special order processing within the inbound/outbound message queues. This moves the prop trader farther away from the order updating processors (Figure 19.7).

Figure 19.7
19.6. The IEX Exchange Application

In its exchange application, IEX proposed the imposition of delays on inbound and outbound messages. Were this a consolidated market and IEX the only exchange, the proposal would have seemed straightforward.

Compatibility with Reg NMS posed difficulties. IEX would certainly want its visible bids and offers to be protected, but in order to be protected, they had to be immediately and electronically accessible. Was this consistent with deliberate delay? The SEC decided that in this case, the delay was "de minimis", that is, so trivial or minor as to be consistent with the intent of Reg NMS, particularly given that the avowed purpose of the delay was to make markets more fair. Furthermore, for purposes of determining and disseminating the NBBO, IEX's links with other markets would not be subject to the delay.

A summary of IEX's proposal, summaries of comment letters, and the SEC's analysis are given in order approving the exchange application (U.S. Securities and Exchange Commission (2016a)). The following discussion draws extensively on this document.

What emerged is diagrammed in Figure 19.8. For clarity, the diagram has all orders (including special orders) going through one processor. IEX's delays on inbound and outbound message queues are actually 350 microseconds. The prop trader (in fact, all customers submitting orders) experiences the delay. Market information, though, at least that conveyed through other market centers, is not delayed in either direction. That is, IEX receives unimpeded updates from other exchanges, and it promptly updates other exchanges. The SEC decided that IEX's bid and offer quotes are protected on the same terms as those of other exchanges.

Figure 19.8

Pegged orders played a prominent role in the SEC's decision: “[The] advantage IEX provides to pegged orders is ... designed to ensure that pegged orders on IEX operate as designed ... To accomplish this, IEX slows down incoming order messages by 350 microseconds to allow it to
update resting pegged orders when the NBBO changes, so that the resting pegged orders are accurately pegged to current market prices. Without this protection, pegged orders resting on IEX have the potential to be subject to ‘latency arbitrage’…” In the SEC’s view, the repegging of orders should have priority over incoming attempts to execute against them. Protecting customer orders against latency arbitrage arguably just as valuable as protecting them against trade-throughs, but the view represents a new direction in regulation.

IEX’s exchange application included one other novel feature, a new order type, called the discretionary pegged (“D-Peg”) order. It is basically a nondisplayed (hidden) limit order, pegged to the NBBO midpoint, that (like other discretionary orders) can actively take the opposing quote. The discretionary orders discussed in section 13.3, however, followed a pre-set rule. For example, a buy limit order is pegged to the NBBO midpoint, but if the NBO ever comes within $0.02 of the midpoint, take it. With the IEX D-Peg order, the buy order will become a “taker” only if IEX determines that the market is stable, that is, not in the middle of a downward movement, a condition described as a crumbling [bid] quote.

Of course, ascertaining whether a price trend will continue is usually a judgement subject to a high margin of error. (“IBM has closed higher in the last five trading sessions. What are the changes that it will also close higher today?”) Why would we expect IEX to be in any position to make an accurate prediction?

IEX’s advantage comes from the delays on inbound and, to a lesser extent, outbound messages. Suppose that I have a computer screen that is showing the current NBBO. If you are looking at a screen where the NBBO is delayed by a minute, then from your perspective, I will appear to be clairvoyant, capable of foreseeing bids and offers one minute “into the future”. The delay on the outbound side hurts anyone trying to discern the current state of the market; the delay on the inbound side penalizes anyone trying to react to a perceived market condition.

In the SEC’s analysis, the delays are supported by two lines of argument. The first is primarily operational, the allowance of a grace period to update pegged orders so that they can be repriced while removed from the threat of execution. The second is informational, allowing the exchange’s order management software a look-ahead to protect a particular order type.

The SEC’s approval was controversial. The order type being favored is nondisplayed, that is, dark. Some commentators therefore believed that the decision will tilt order strategies toward dark mechanisms. Others have suggested that in competitive response all exchanges will offer similar order types. This would lessen IEX’s advantage, but would within each exchange shift the balance of orders toward dark pegged orders, a step that would further stress lit markets and displayed limit orders.24

### 19.7. Further reading

The SEC order approving IEX’s exchange application, taken together with the comment letters, provide a thorough discussion of current issues. Both are posted to the SEC’s website.

Practitioner sentiments on HFT are strong and divided. The pro-HFT view is articulated by the trade organization Modern Markets Initiative. Critical views are more likely to be found on the blogs of Themis Trading (www.themistrading.com) and Nanex Research (www.nanex.net/NxResearch/).

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24 Nasdaq has announced that it plans to offer a limit order type that would be subject to a short no-cancel window. In exchange for this commitment to persistence, the order would have priority over a same-price order that could be cancelled at any time (Michaels (2016)).
HFT has proven to be a fertile subject for research. Jones (2013) provides an excellent review article; also see the introduction to the Journal of Financial Market’s special issue (Chordia, Goyal, Lehmann and Saar (2013)). Many of the earlier papers drew favorable conclusions about HFT, generally finding a positive association between HFT and market quality. It was more difficult to establish causality. (Were the HFT’s simply better market makers, or were they becoming more active in markets that were already improving for unrelated reasons?) Later papers, pointing to practices like advance knowledge of public news announcements and latency arbitrage, were more skeptical of the benefits. (See, for example, Biais, Foucault and Moinas (2015); Clark-Joseph (2012); Foucault, Hombert and Rosu (2016); Jarrow and Protter (2012); Weller (2016).) Angel and McCabe (2013) discuss fairness issues related to HFT. Baruch and Glosten (2013) suggest that fast technology encourages traders to used mixed (randomized) strategies in placing their limit orders and to frequently revise their bids and offers.

Gode and Sunder (2000) point out that geographically dispersed traders can’t access a market on an equal basis, and suggest moving to call markets. Budish, Cramton and Shim (2015) suggest that periodic batch auctions without time priority would eliminate overinvestment in speed (also see Section 6.6 in these notes).
Part VI. Answers to end-of-chapter problems.
Answers for Chapter 1.

Answers for Chapter 2.

Answers for Chapter 3.

Answers for Chapter 4.

*Answer to Problem 4.1*

<table>
<thead>
<tr>
<th>Priority (1=first, top)</th>
<th>Trader</th>
<th>Arrival Sequence</th>
<th>Quantity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Chad</td>
<td>3</td>
<td>200</td>
<td>20.07</td>
</tr>
<tr>
<td>2</td>
<td>Emily</td>
<td>5</td>
<td>300</td>
<td>20.07</td>
</tr>
<tr>
<td>3</td>
<td>Dana</td>
<td>4</td>
<td>100</td>
<td>20.06</td>
</tr>
<tr>
<td>4</td>
<td>Brian</td>
<td>2</td>
<td>400</td>
<td>20.05</td>
</tr>
<tr>
<td>5</td>
<td>Frank</td>
<td>6</td>
<td>300</td>
<td>20.05</td>
</tr>
<tr>
<td>6</td>
<td>Amy</td>
<td>1</td>
<td>100</td>
<td>20.02</td>
</tr>
</tbody>
</table>

*Answer to Problem 4.2*

<table>
<thead>
<tr>
<th>Priority (1=first, top)</th>
<th>Trader</th>
<th>Sequence</th>
<th>Quantity</th>
<th>Price</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gina</td>
<td>7</td>
<td>200</td>
<td>10.00</td>
<td>N</td>
</tr>
<tr>
<td>2</td>
<td>Emily</td>
<td>5</td>
<td>300</td>
<td>10.01</td>
<td>Y</td>
</tr>
<tr>
<td>3</td>
<td>Chad</td>
<td>3</td>
<td>200</td>
<td>10.01</td>
<td>N</td>
</tr>
<tr>
<td>4</td>
<td>Brian</td>
<td>2</td>
<td>400</td>
<td>10.02</td>
<td>Y</td>
</tr>
<tr>
<td>5</td>
<td>Dana</td>
<td>4</td>
<td>100</td>
<td>10.02</td>
<td>Y</td>
</tr>
<tr>
<td>6</td>
<td>Amy</td>
<td>1</td>
<td>100</td>
<td>10.10</td>
<td>Y</td>
</tr>
<tr>
<td>7</td>
<td>Frank</td>
<td>6</td>
<td>300</td>
<td>10.10</td>
<td>N</td>
</tr>
</tbody>
</table>

Note: The offer side of the book is often displayed on a screen in reverse order: the “top” of the book is displayed on the bottom.
Answer to Problem 4.3 Prior to the receipt of any new orders, the market is 15.01 bid for 300 shares (Hans), 200 shares offered at 15.05 (Alice). On the bid side, although Bruce’s bid is higher, the order is not visible.

a. Kathy buys 200 from Alice @ 15.05; Kathy buys 100 from Dmitri @ 15.05; the market is 15.01 bid for 300 shares (Hans), 100 offered at 15.07 (Esteban). Esteban’s order sets the best offer because Dmitri’s offer is hidden.

b. Lane buys 200 from Alice @ 15.05; Lane buys 300 from Dmitri @15.05; Lane buys 100 from Esteban @ 15.07; the market is 15.01 bid for 300 (Hans), 500 offered at 15.07 (Gina).

c. Maureen buys 200 from Alice @ 15.05; Maureen buys 300 from Dmitri @ 15.05; Maureen has 100 shares left to buy, so the market is 15.05 bid for 100 (Maureen), 100 offered at 15.07 (Esteban).

d. Ollie sells 400 to Bruce @ 15.03; Ollie sells 100 to Hans @ 15.01; the market is 15.01 bid for 200 (Hans), 200 offered at 15.05 (Alice)

e. Petra sells 400 to Bruce @ 15.0; her remaining 100 shares are added to the book on the sell side; the market is 15.01 bid for 300 (Hans), 100 offered at 15.02 (Petra).

f. Rama sells 400 to Bruce @ 15.03. Rama’s remaining 100 shares can’t be executed, and (because of the IOC) they are canceled; the market is 15.01 bid for 300 (Hans), 200 offered at 15.05 (Alice)

g. Only 700 shares are available at 15.01 or better, so Sydney’s entire order is cancelled. The market is (still) 15.01 bid for 300 (Hans), 200 offered at 15.05 (Alice).

Answer to Problem 4.4

<table>
<thead>
<tr>
<th></th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Buy</td>
<td>$10.20</td>
<td>11:08</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td>Price</td>
</tr>
<tr>
<td>2</td>
<td>Buy</td>
<td>$10.10</td>
<td>11:08</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td>Price</td>
</tr>
<tr>
<td>3</td>
<td>Buy</td>
<td>$10.05</td>
<td>11:10</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td>Visibility</td>
</tr>
<tr>
<td>4</td>
<td>Buy</td>
<td>$10.05</td>
<td>11:02</td>
<td>1,000</td>
<td>H</td>
<td>Beth</td>
<td>Price</td>
</tr>
<tr>
<td>5</td>
<td>Buy</td>
<td>$10.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
<td></td>
</tr>
</tbody>
</table>

Why? = "Why is this order ranked higher than the order below it? (Price, time, or visibility)" The question didn’t ask for this, but I’ve included it to give a fuller explanation.

Answer to Problem 4.5

<table>
<thead>
<tr>
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<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Buy</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td>Price</td>
</tr>
<tr>
<td>2</td>
<td>Buy</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td>Price</td>
</tr>
<tr>
<td>3</td>
<td>Buy</td>
<td>$20.40</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td>Price</td>
</tr>
<tr>
<td>4</td>
<td>Buy</td>
<td>$20.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
<td>Time</td>
</tr>
<tr>
<td>5</td>
<td>Buy</td>
<td>$20.00</td>
<td>11:02</td>
<td>100</td>
<td>D</td>
<td>Beth</td>
<td></td>
</tr>
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### Answer to Problem 4.6

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<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sell</td>
<td>$20.00</td>
<td>11:01</td>
<td>200</td>
<td>D</td>
<td>Andy</td>
<td>Time</td>
</tr>
<tr>
<td>2</td>
<td>Sell</td>
<td>$20.00</td>
<td>11:02</td>
<td>100</td>
<td>D</td>
<td>Beth</td>
<td>Price</td>
</tr>
<tr>
<td>3</td>
<td>Sell</td>
<td>$20.45</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td>Time</td>
</tr>
<tr>
<td>4</td>
<td>Sell</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td>Price</td>
</tr>
<tr>
<td>5</td>
<td>Sell</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td></td>
</tr>
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</table>

### Answer to Problem 4.7

<table>
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<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:02</td>
<td>1,000</td>
<td>D</td>
<td>Beth</td>
<td>Time</td>
</tr>
<tr>
<td>2</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td>Time</td>
</tr>
<tr>
<td>3</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:15</td>
<td>100</td>
<td>D</td>
<td>Emma</td>
<td>Visibility</td>
</tr>
<tr>
<td>4</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:01</td>
<td>200</td>
<td>H</td>
<td>Andy</td>
<td>Time</td>
</tr>
<tr>
<td>5</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:08</td>
<td>300</td>
<td>H</td>
<td>Cathy</td>
<td></td>
</tr>
</tbody>
</table>

### Answer to Problem 4.8

<table>
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<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
<th>Why?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:02</td>
<td>1,000</td>
<td>D</td>
<td>Beth</td>
<td>Visibility</td>
</tr>
<tr>
<td>2</td>
<td>Sell</td>
<td>$20.30</td>
<td>11:01</td>
<td>200</td>
<td>H</td>
<td>Andy</td>
<td>Price</td>
</tr>
<tr>
<td>3</td>
<td>Sell</td>
<td>$20.45</td>
<td>11:09</td>
<td>500</td>
<td>D</td>
<td>David</td>
<td>Visibility</td>
</tr>
<tr>
<td>4</td>
<td>Sell</td>
<td>$20.45</td>
<td>11:15</td>
<td>100</td>
<td>H</td>
<td>Emma</td>
<td>Price</td>
</tr>
<tr>
<td>5</td>
<td>Sell</td>
<td>$20.50</td>
<td>11:08</td>
<td>300</td>
<td>D</td>
<td>Cathy</td>
<td></td>
</tr>
</tbody>
</table>

### Answer to Problem 4.9

Wendy buys 400 shares from Petra, and 100 shares from Oliver. The market is 10.01 bid, offered at 10.06.

<table>
<thead>
<tr>
<th></th>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>2</td>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>100</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td></td>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td></td>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td></td>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>
Answer to Problem 4.10 Yuri buys 400 shares from Petra; Yuri’s 100 remaining shares are added to the book on the bid side:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.05</td>
<td>10:00</td>
<td>100</td>
<td>D</td>
<td>Yuri</td>
</tr>
<tr>
<td>Buy</td>
<td>$10.01</td>
<td>9:44</td>
<td>100</td>
<td>D</td>
<td>Rob</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

Answer to Problem 4.11 100 shares execute against Rob; 500 shares execute against Sandy. The new book is:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

Answer to Problem 4.12 100 shares execute against Rob; Arturo’s remaining 500 shares are cancelled. The new book is:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Price</th>
<th>Time</th>
<th>Quantity</th>
<th>Visibility</th>
<th>Trader</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell</td>
<td>$10.10</td>
<td>9:50</td>
<td>300</td>
<td>D</td>
<td>Maura</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.06</td>
<td>9:51</td>
<td>200</td>
<td>D</td>
<td>Oliver</td>
</tr>
<tr>
<td>Sell</td>
<td>$10.05</td>
<td>9:52</td>
<td>400</td>
<td>D</td>
<td>Petra</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.98</td>
<td>9:45</td>
<td>500</td>
<td>D</td>
<td>Sandy</td>
</tr>
<tr>
<td>Buy</td>
<td>$9.90</td>
<td>9:48</td>
<td>200</td>
<td>D</td>
<td>Trevor</td>
</tr>
</tbody>
</table>

The entire order can’t be executed at 10.01 or better, so it is immediately cancelled.
Answers for Chapter 5.

Answer to Problem 5.1 The NBB is the minimum across this set:

<table>
<thead>
<tr>
<th>Time</th>
<th>Exchange</th>
<th>Bid</th>
<th>NBB</th>
<th>Atlantic</th>
<th>Pacific</th>
<th>Gulf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00:01</td>
<td>Atlantic</td>
<td>23.33</td>
<td>23.33</td>
<td>23.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10:00:02</td>
<td>Pacific</td>
<td>23.32</td>
<td>23.33</td>
<td>23.33</td>
<td>23.32</td>
<td></td>
</tr>
<tr>
<td>10:00:03</td>
<td>Gulf</td>
<td>23.34</td>
<td>23.34</td>
<td>23.33</td>
<td>23.32</td>
<td>23.34</td>
</tr>
<tr>
<td>10:00:04</td>
<td>Pacific</td>
<td>23.40</td>
<td>23.40</td>
<td>23.33</td>
<td>23.40</td>
<td>23.34</td>
</tr>
<tr>
<td>10:00:05</td>
<td>Gulf</td>
<td>23.45</td>
<td>23.45</td>
<td>23.33</td>
<td>23.40</td>
<td>23.45</td>
</tr>
<tr>
<td>10:00:06</td>
<td>Pacific</td>
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<td>23.45</td>
<td>23.33</td>
<td>23.44</td>
<td>23.45</td>
</tr>
<tr>
<td>10:00:07</td>
<td>Atlantic</td>
<td>23.43</td>
<td>23.45</td>
<td>23.43</td>
<td>23.44</td>
<td>23.45</td>
</tr>
<tr>
<td>10:00:08</td>
<td>Gulf</td>
<td>23.40</td>
<td>23.44</td>
<td>23.43</td>
<td>23.44</td>
<td>23.40</td>
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</table>

Answer to Problem 5.2

<table>
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<th>Price</th>
<th>Exchange A</th>
<th>Exchange B</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.10</td>
<td>100 sh, hidden</td>
<td></td>
</tr>
<tr>
<td>50.09</td>
<td><strong>300 sh</strong></td>
<td></td>
</tr>
<tr>
<td>50.08</td>
<td></td>
<td><strong>200 sh</strong></td>
</tr>
<tr>
<td>50.07</td>
<td>400 sh</td>
<td>500 sh</td>
</tr>
</tbody>
</table>

a. The protected orders are indicated above in bold. The top of each market’s visible book. Hidden orders are not protected.

b. It can execute 400 shares without trading through B’s bid. It can send the remaining 500 shares to B for execution or it can return them, unexecuted.

c. It cannot execute anything without trading through A’s protected bid. So it can return the entire order to the customer, unexecuted. Or it can send 300 shares to A, and execute 200 shares against its own protected order.

Interestingly, in this case, there will be an apparent trade-through. On Exchange A, there will be an execution of 100 sh at 50.10, and 200 sh at 50.09; 100 sh will remain at 50.09. Exchange B executes 200 sh at 50.08. This does not constitute a trade-through, however, because B is not responsible for A’s hidden orders. B simply had to send enough to satisfy A’s protected (visible) bids.
**Answers for Chapter 6.**

*Answer to Problem 6.1*

*a.* The cumulative demand and supply schedules constructed from these orders are:

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Price</th>
<th>Quantity</th>
<th>Cum</th>
<th>Buyer</th>
<th>Price</th>
<th>Quantity</th>
<th>Cum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>$20.10</td>
<td>200</td>
<td>200</td>
<td>Ira</td>
<td>$19.80</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Amy</td>
<td>$20.00</td>
<td>200</td>
<td>400</td>
<td>Gabe</td>
<td>$19.90</td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>Cathy</td>
<td>$19.95</td>
<td>100</td>
<td>500</td>
<td>Haley</td>
<td>$19.95</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>Brian</td>
<td>$19.90</td>
<td>100</td>
<td>200</td>
<td>Fiona</td>
<td>$20.10</td>
<td>100</td>
<td>700</td>
</tr>
</tbody>
</table>

They are graphed as:

![Graph showing the cumulative demand and supply schedules]

At an indicative price of 20.00: Dave and Amy are buyers (for 400 shares); Ira, Gabe and Haley are sellers (for 600 shares). The matched volume is 400, and there’s a net sell imbalance of 200.

*a.* The clearing price is 19.95. At that price, matched volume is 500 shares, and there’s a net sell imbalance of 100 shares. Dave, Amy and Cathy buy; Ira, Gabe and Haley sell, but Haley can only sell 200 shares.

*b.* Each block in the graph is worth 100 $sh \times$ $0.05 per $sh = $5. So the profits are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>$30</td>
</tr>
<tr>
<td>Amy</td>
<td>$10</td>
</tr>
<tr>
<td>Cathy</td>
<td>$0</td>
</tr>
<tr>
<td>Ira</td>
<td>$15</td>
</tr>
<tr>
<td>Gabe</td>
<td>$10</td>
</tr>
<tr>
<td>Haley</td>
<td>$0</td>
</tr>
</tbody>
</table>

The total sellers’ profits are $25; the total buyers’ profits are $40; the overall total is $65.
d. With Brian and Gabe out, the supply and demand curves constructed from the remaining orders are:

The new clearing price is 20.00 (Dave and Amy are buyers; Ira and Haley are sellers.) At 20.00, the matched volume is 400 shares; the net imbalance is zero. (At 19.95, the matched volume would also be 400 shares, but there would be a buy imbalance from Cathy.)

The profits are:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dave</td>
<td>$20</td>
</tr>
<tr>
<td>Amy</td>
<td>$0</td>
</tr>
<tr>
<td>Ira</td>
<td>$20</td>
</tr>
<tr>
<td>Haley</td>
<td>$15</td>
</tr>
</tbody>
</table>

For a total of $55. Relative to the original outcome, this represents efficiency of \( \frac{55}{65} = 85\% \).

**Answer to Problem 6.2**

The market-clearing price is the price at which the supply and demand schedules cross: 9.80. At that price, we have buyer (demand) for 12 units and seller (supply) of 9 units. The matched volume is the minimum of the two: 9 units. There is a buy imbalance of 3 units. At a reference price of 9.70, the matched volume is 5 (the minimum of 5 and 15) and there's a buy imbalance of 10 units.

**Answer to Problem 6.3**

a. Beth’s limit order is buy two contracts, limit $26; Sara’s order is sell three contracts, limit $21.

b. At an indicative price of $27, the matched volume would have been 7 contracts and there would have been a sell imbalance of 6 contracts.

c. The exchange plans to maximize matched volume, and if two prices have the same matched volume, the clearing price will minimize the net imbalance.
   
i. Under this rule, the clearing price would have been $25 with a matched volume of 10 contracts and a buy imbalance of 3 contracts.
   
ii. The total buyers’ profits are 23; the total sellers’ profits are 22.

d. All of the remaining buyers have limit prices greater than Sara’s sell limit price (21). Any of them could trade with Sara.
The buyer’s profits are 2 (Dave) + 3 (Dara) + 2 (Brian) = 7.
The seller’s profits are 16 (Seth) + 6 (Sasha) + 0 (Sam) = 22.
The total profits are 29, for an efficiency of 29/(22+23) = 29/45=64%

---

Answers for Chapter 7.

Answer to Problem 7.1
a. False (Dealer markets don’t generally have trade reporting.)
b. False (There may be limit order books in the interdealer market, but there are no venues where customer orders are displayed.)
c. True
d. False (Dealers are the counterparty to all customer trades: selling to buyers and buying from sellers.)
e. False (Interdealer markets are closed to customers.)

Answer to Problem 7.2
a. False
b. False
c. True (This is the essence of public priority: the specialist would have to yield to public interest at the same price.)
d. False (If the specialist were to start a price trend by successively lifting customer offers, this could be considered to be trading in a destabilizing fashion. This would be against the rules, but it is not “public priority”.)

Answer to Problem 7.3
a. True (The specialist might be entering a bid/offer on behalf of someone else, such as an order in the book. But if there were no orders on the book, the specialist would have to bid/offer on his own account.)
b. False
c. False
d. False (The specialist was trading with his own money; he was not an employee of the exchange.)

Answer to Problem 7.4
a. True
b. False (Dealers are the counterparties to almost all customer orders.)
c. False
d. False (Interdealer markets are not usually open or visible to customers.)
e. False. Trades generally aren’t reported in dealer markets (although the US corporate bond market is an exception).
Answers for Chapter 8. Dark liquidity and dark mechanisms

*Answer to Problem 8.1*

*a*: Not dark. The order is executed by someone whose visible quote is 10.10.

*b*: Dark. The MM is matching a price that he himself is not posting. (Note that this would have been considered a dark trade even if the MM sold to the customer at a price slightly better than NBO, like $10.0999. In fact, MMs sometimes do this to demonstrate that they are offering their customers price improvement. Here, the price improvement is a trivial $0.0001.)

*c*: Dark

*d*: Dark. There are no visible quotes are the execution price.

Answers for Chapter 9. Public information

*Answer to Problem 9.1*  
*c*. There is evidence that the news may already have been leaked. (If the news is leaking, there is a danger of increased information asymmetries.)

*Answer to Problem 9.2*  
*b*. Widens; drops (Over the period of the announcement, volatility is higher.)

*Answer to Problem 9.3*  
*a*. Set price limits for individual stocks. These limits can trigger trading halts.

*Answer to Problem 9.4*  
*b* (only on declines)

Answers for Chapter 10. Informational efficiency and securities class action lawsuits

*Answer to Problem 10.1*  
*b*. This is one of the drawbacks of the process: it is essentially a transfer from one group of shareholders to another group of shareholders (plus the lawyers and consultants).

*Answer to Problem 10.2*  
*d*. Under market efficiency, the reliance is indirect, via the market price.

*Answer to Problem 10.3*  
*c*.

*Answer to Problem 10.4*  
*a*. The event period would probably include the date of the announcement, but the market reaction might have started before or continued after that date.

*Answer to Problem 10.5*  
*a*. If there is no leakage, the impact of the announcement shouldn’t appear before the announcement.
**Answer to Problem 10.6** We usually try to rule out other explanations for the drop. We can eliminate market-wide explanations by focusing on the error in a market model linear regression. We can rule out alternative industry-based explanations if the model also includes the return on an industry index.

**Answer to Problem 10.7** The corrective disclosures did not occur before the market close on Monday, so Monday’s return is not relevant. APX declined on Tuesday and Wednesday, but so did the market. To isolate the part of the return that might be due to company-specific news, we focus on the residuals in the regression (the company-specific components of the return).

<table>
<thead>
<tr>
<th>Date</th>
<th>$r_{APX}$</th>
<th>$r_M$</th>
<th>$e = r_{APX} - \beta_{APX} \times r_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday, March 3</td>
<td>-2.0%</td>
<td>-4.0%</td>
<td>...</td>
</tr>
<tr>
<td>Tuesday, March 4</td>
<td>-18.0%</td>
<td>-5.0%</td>
<td>$-18 - 1.3 \times (-5) = -11.5%$</td>
</tr>
<tr>
<td>Wednesday, March 5</td>
<td>-8.0%</td>
<td>-1.0%</td>
<td>$-8 - 1.3 \times (-1) = -6.7%$</td>
</tr>
<tr>
<td>Thursday, March 6</td>
<td>+3.0%</td>
<td>-1.0%</td>
<td>$3 - 1.3 \times (-1) = 4.3%$</td>
</tr>
<tr>
<td>Friday, March 7</td>
<td>6.0%</td>
<td>5.0%</td>
<td>...</td>
</tr>
</tbody>
</table>

If the company-specific returns on Tuesday and Wednesday are attributed to the corrective disclosures, the damages would be based on return of $-11.5\% - 6.7\% = -18.2\%$. But the following day (Thursday), the stock recovered on a day when the market was down. So the expert might also argue that there were over-reactions on Tuesday and Wednesday. Including Thursday’s gain, the damages would be calculated on a return of $-13.9\% (= -18.2 + 4.3)$.

**Answer to Problem 10.8** The period over which the earnings accrued (in an accounting sense) isn’t relevant here. What matters is the information that management reported (or mis-reported) to the market. Since they first reported 2013 IV earnings on January 29, what matters is the net number of shares that were purchased during the period covered by the mis-information, that is January 29 to March 6. This is 2,000 shares.

---

**Answers for Chapter 11. Private information**

**Answer to Problem 11.1 a.** Retail traders are less likely to be informed (in possession of non-public information). Market makers lose to informed traders. (Informed traders always trade in the direction of their information, buying if they have positive non-public news; selling on negative non-public news.) Market makers profit (on average) from uninformed traders. An uninformed trader is just as likely to buy as to sell; the dealer earns the spread from each pair of uninformed traders who come by. Informed traders never come by in pairs; they are always on one side of the market. Answer b is irrelevant; answer c states something that is incorrect (retail traders are less likely to know or care where their orders were sent. As for answer d, the statement is true, but it does not describe why a market maker would prefer a retail order.
Answer to Problem 11.2 The market considers the following possibilities (the full table is given, but only the numbers in bold are actually needed):

<table>
<thead>
<tr>
<th>End-of-day value</th>
<th>Trader type</th>
<th>Trader ...</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uninformed</td>
<td>Buys</td>
<td>$0.8 \times 0.9 \times \frac{1}{2} = 0.36$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>same</td>
</tr>
<tr>
<td>100</td>
<td>Informed</td>
<td>Buys</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>$0.8 \times 0.1 = 0.08$</td>
</tr>
<tr>
<td>110</td>
<td>Uninformed</td>
<td>Buys</td>
<td>$0.2 \times 0.9 \times \frac{1}{2} = 0.09$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>same</td>
</tr>
<tr>
<td></td>
<td>Informed</td>
<td>Buys</td>
<td>$0.2 \times 0.1 = 0.02$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>0</td>
</tr>
</tbody>
</table>

The probability of a buy order from the first incoming trader is

\[ P(buy) = 0.36 + 0 + 0.09 + 0.02 = 0.47 \]

The probability of a buy order and a low end-of-day value is

\[ P(buy, 100) = 0.36 \]

The probability of a low end-of-day value given a buy order is

\[ P(100|buy) = \frac{P(buy, 100)}{P(buy)} = \frac{0.36}{0.47} = 0.766 \]

The expected end-of-day value given a buy order is $0.766 \times 100 + 0.234 \times 110 = 102.34$. This is the break-even ask price.

Answer to Problem 11.3 The new probabilities are:

<table>
<thead>
<tr>
<th>End-of-day value</th>
<th>Trader type</th>
<th>Trader ...</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uninformed</td>
<td>Buys</td>
<td>$0.8 \times 0.9 \times 0.3 = 0.216$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>$0.8 \times 0.9 \times 0.7 = 0.504$</td>
</tr>
<tr>
<td>100</td>
<td>Informed</td>
<td>Buys</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>$0.8 \times 0.1 = 0.08$</td>
</tr>
<tr>
<td>110</td>
<td>Uninformed</td>
<td>Buys</td>
<td>$0.2 \times 0.9 \times 0.3 = 0.054$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>$0.2 \times 0.9 \times 0.7 = 0.126$</td>
</tr>
<tr>
<td></td>
<td>Informed</td>
<td>Buys</td>
<td>$0.2 \times 0.1 = 0.02$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sells</td>
<td>0</td>
</tr>
</tbody>
</table>

Then:

\[ P(buy) = 0.216 + 0 + 0.054 + 0.02 = 0.290 \]

\[ P(buy, 100) = 0.216 \]

\[ P(100|buy) = \frac{P(buy, 100)}{P(buy)} = \frac{0.216}{0.290} = 0.745 \]

The expected end-of-day value given a buy order is $0.745 \times 100 + 0.265 \times 110 = 102.55$. This is the break-even ask price. (A buy order is more informative than in the last case because it is more surprising.)
Answer to Problem 11.4 The evidence could well be correct: Orders move security prices due to the markets’ beliefs in the likelihood of insider trading – not the actuality. All orders move prices in the same way because the market can’t tell who is informed and who isn’t. The construction the defense puts on this claim, however, is false. Just because the market couldn’t tell, at the time of his trades, whether or not Clarence had information, it doesn’t rule out the possibility that he actually did.

Answers for Chapter 12. Insider trading

Answer to Problem 12.1 Managers might actually have an incentive to delay, so they can make their trades.

Answer to Problem 12.2 b. Found innocent; he had no fiduciary responsibility to the target companies. Note: it might have been argued that he had misappropriated information from his employer (the printing company), as in the O'Hagan case. But this argument was not originally presented to the jury.

Answer to Problem 12.3 d.

Answer to Problem 12.4 b.

Answer to Problem 12.5 b, c.

Answer to Problem 12.6 By strict disclose or abstain, Mike can't trade (he has material non-public information); it doesn’t matter that it’s not about his company. He has no fiduciary duty to SSL, but he does have a fiduciary duty to ICE. ICE produced the information that he’s planning to trade on the basis of; the information properly belongs to ICE shareholders. In using the info to trade, he is misappropriating the info from the shareholders.

Answers for Chapter 13. Conditional and complex orders

Answer to Problem 13.1

a. The trade at $42 is followed by trades at $40, $41 and $39.
   The stop price is initially set at $42 - 3 = 39, and since the stock is falling, the stop price doesn’t change. The stop is triggered (elected) at the last trade, 39.

b. The trade at $42 is followed by trades at $40, $43 and $41.
   As in part a, the stop price is initially 39. But when the price rises to 43, the stop price goes to $43 - 3 = 40. Since the trade price doesn’t drop to 40, the stop is never triggered.

c. The trade at $42 is followed by trades at $40, $44 and $41.
   The stop prices corresponding to these trades are 39, 39, and 41, Triggered at 41.
d. The trade at $42 is followed by trades at $40, $44, $43, $44, $43 and $41. The stop prices are 39, 39, 41, 41, 41, Triggered at 41.

Answer to Problem 13.2 A stock buy order is used to purchase stock when the price has increased to a certain level. This could also be accomplished by the purchase of a call option. Puts and calls are better than stop orders because they avoid the problem of regret (a stock that rebounds after triggering a stop-loss sale, or a stock that falls back after triggering a stop-buy purchase). Puts and calls are, however, costly. They also expire: if you’re planning on using one to provide on-going insurance, new options must be purchased to replace expiring ones.

Answer to Problem 13.3 Once the pegged buy order becomes the visible bid, it tries to better itself. This is unstable, and ends only when the buy limit price reaches the offer (and achieves execution).

Answer to Problem 13.4 Posting a visible bid of $50.10 may encourage other buyers to match that price. These buyers may end up purchasing while David’s order goes unexecuted. Even worse, a showing of visible interest at $50.10 might induce sellers to reprice their orders higher.

Answer to Problem 13.5 d.

Answer to Problem 13.6 At 10:07 the offer is within $0.03 of the midpoint price, and the pegged limit order is replaced with a marketable order (buy limit $20.29).
Answers for Chapter 14. Transaction cost analysis

Answer to Problem 14.1

a. The quote prevailing when the order arrived was set at 11:02:00; five minutes subsequent to arrival is 11:07:44, and the prevailing quote at that point was set at 11:07:00. These quotes are highlighted below:

<table>
<thead>
<tr>
<th>Time</th>
<th>NBB</th>
<th>NBO</th>
<th>BAM</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:01:00</td>
<td>35.05</td>
<td>35.10</td>
<td>35.075</td>
<td>0.05</td>
</tr>
<tr>
<td>11:02:00</td>
<td>34.94</td>
<td>34.99</td>
<td>34.965</td>
<td>0.05</td>
</tr>
<tr>
<td>11:03:00</td>
<td>34.83</td>
<td>34.86</td>
<td>34.845</td>
<td>0.03</td>
</tr>
<tr>
<td>11:04:00</td>
<td>34.77</td>
<td>34.81</td>
<td>34.790</td>
<td>0.04</td>
</tr>
<tr>
<td>11:05:00</td>
<td>34.77</td>
<td>34.85</td>
<td>34.810</td>
<td>0.08</td>
</tr>
<tr>
<td>11:06:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:07:00</td>
<td>34.75</td>
<td>34.78</td>
<td>34.765</td>
<td>0.03</td>
</tr>
<tr>
<td>11:08:00</td>
<td>34.80</td>
<td>34.86</td>
<td>34.830</td>
<td>0.06</td>
</tr>
<tr>
<td>11:09:00</td>
<td>34.72</td>
<td>34.75</td>
<td>34.735</td>
<td>0.03</td>
</tr>
<tr>
<td>11:10:00</td>
<td>34.75</td>
<td>34.83</td>
<td>34.790</td>
<td>0.08</td>
</tr>
<tr>
<td>11:11:00</td>
<td>34.66</td>
<td>34.69</td>
<td>34.675</td>
<td>0.03</td>
</tr>
<tr>
<td>11:12:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:13:00</td>
<td>34.82</td>
<td>34.88</td>
<td>34.850</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The effective cost is $34.98 - 34.965 = 0.015$; the realized cost is $34.98 - 34.765 = 0.215$. The price improvement is $34.99 - 34.98 = 0.01$; price impact is $34.765 - 34.965 = -0.200$.

b. The quote prevailing when the order arrived was set at 11:07:00; five minutes subsequent to arrival is 11:12:02, and the prevailing quote at that point was set at 11:12:00. These quotes are highlighted below:

<table>
<thead>
<tr>
<th>Time</th>
<th>NBB</th>
<th>NBO</th>
<th>BAM</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:01:00</td>
<td>35.05</td>
<td>35.10</td>
<td>35.075</td>
<td>0.05</td>
</tr>
<tr>
<td>11:02:00</td>
<td>34.94</td>
<td>34.99</td>
<td>34.965</td>
<td>0.05</td>
</tr>
<tr>
<td>11:03:00</td>
<td>34.83</td>
<td>34.86</td>
<td>34.845</td>
<td>0.03</td>
</tr>
<tr>
<td>11:04:00</td>
<td>34.77</td>
<td>34.81</td>
<td>34.790</td>
<td>0.04</td>
</tr>
<tr>
<td>11:05:00</td>
<td>34.77</td>
<td>34.85</td>
<td>34.810</td>
<td>0.08</td>
</tr>
<tr>
<td>11:06:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:07:00</td>
<td>34.75</td>
<td>34.78</td>
<td>34.765</td>
<td>0.03</td>
</tr>
<tr>
<td>11:08:00</td>
<td>34.80</td>
<td>34.86</td>
<td>34.830</td>
<td>0.06</td>
</tr>
<tr>
<td>11:09:00</td>
<td>34.72</td>
<td>34.75</td>
<td>34.735</td>
<td>0.03</td>
</tr>
<tr>
<td>11:10:00</td>
<td>34.75</td>
<td>34.83</td>
<td>34.790</td>
<td>0.08</td>
</tr>
<tr>
<td>11:11:00</td>
<td>34.66</td>
<td>34.69</td>
<td>34.675</td>
<td>0.03</td>
</tr>
<tr>
<td>11:12:00</td>
<td>34.74</td>
<td>34.78</td>
<td>34.760</td>
<td>0.04</td>
</tr>
<tr>
<td>11:13:00</td>
<td>34.82</td>
<td>34.88</td>
<td>34.850</td>
<td>0.06</td>
</tr>
</tbody>
</table>
The effective cost is $34.765 - 34.73 = 0.035$; the realized cost is $34.760 - 34.73 = 0.030$; the price improvement is $34.73 - 34.75 = -0.02$.

The negative price improvement is a “dis-improvement.” The person who sent the order probably expected to hit the bid, thereby selling at 34.75. They actually sold at an inferior price.

If the 34.75 bid had been available at the time the order was executed, this would have been considered a trade-through. A trade-through shouldn’t have occurred, but it might have if, for example, other sell orders had executed momentarily before, leaving the best bid in the book at 34.73. In this case, though, there should have been a new quote record generated that showed the lower bid.

The price impact is $34.765 - 34.760 = 0.005$.

**Answer to Problem 14.2**

a. The effective cost uses the NBBO prevailing at the time of the trade. For a trade at 11:03, the quotes were set at 11:02. The bid-ask midpoint is $BAM = 20.14$. For a buy order the effective cost is $price - BAM = 20.16 - 20.14 = 0.02$.

b. The realized cost uses the NBBO prevailing at the time of the trade plus five minutes. In this case, we’d use the NBBO as of 11:08, which is the NBBO that was actually set at 11:06. The BAM is 20.15, and realized cost $= 20.16 - 20.15 = 0.01$.

**Answer to Problem 14.3**

The VWAP of the executed trades is: $(200 \times 30.22 + 500 \times 30.23)/(200 + 500) = 30.227$. The initial bid-ask midpoint is $\frac{30.20 + 30.28}{2} = 30.24$. The shares are actually purchased below the midpoint: the implementation shortfall is $30.227 - 30.240 = -0.013$. With the imputed execution at the close, the VWAP is $(200 \times 30.22 + 500 \times 30.23 + 300 \times 30.39)/(200 + 500 + 300) = 30.276$. The implementation shortfall is $30.276 - 30.240 = 0.036$.

**Answer to Problem 14.4**

If market prices move away from the limit price, the closing price might lie far above the limit price. For example, at 10am, with the bid and offer $11.00 and $11.05, a buyer submits a funari order to buy limit $11.00. The price moves away, and near the close the bid and offer are $12.00 and $12.05. The funari order becomes a market on close order, and stock is purchased near $12.05 (about a dollar worse than the initial limit price).

**Answers for Chapter 15. Statistical models**

**Answer to Problem 15.1 a.**

**Answer to Problem 15.2 d.**
Answer to Problem 15.3

\[ p_1 = 20 - 0.01 + u_1 \]
\[ p_2 = p_1 - 0.01 + u_2 = 20 - 2\times0.01 + u_1 + u_2 \]
\[ p_3 = p_2 - 0.01 + u_3 = 20 - 3\times0.01 + u_1 + u_2 + u_3 \]

The expectation of each \( u \) is zero, so \( E p_3 = 20 - 3\times0.01 = 19.97 \).
The variance of \( p_3 \) is \( Var(p_3) = 3\times\sigma_u^2 = 3\times0.02^2 \);
The standard deviation is \( SD(p_3) = \sqrt{Var(p_3)} = \sqrt{3\times0.02} \)

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Answers for Chapter 16. Order splitting

Answer to Problem 16.1 We construct a volume profile for the intervals over which we’ll be trading:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Average</th>
<th>30% of 20,000=</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:30</td>
<td>26,000</td>
<td>5,909</td>
</tr>
<tr>
<td>10:00</td>
<td>20,000</td>
<td>4,545</td>
</tr>
<tr>
<td>10:30</td>
<td>18,000</td>
<td>4,091</td>
</tr>
<tr>
<td>11:00</td>
<td>12,000</td>
<td>2,727</td>
</tr>
<tr>
<td>11:30</td>
<td>12,000</td>
<td>2,727</td>
</tr>
<tr>
<td>88,000</td>
<td>100%</td>
<td>20,000</td>
</tr>
</tbody>
</table>

The numbers in bold are the trading targets.

Answer to Problem 16.2 d. (This is the risk-return trade-off in order placement decisions.)

Answer to Problem 16.3 c.

Answer to Problem 16.4 The first 100 shares are purchased at \( \$20 + 0.001\times100 = \$20.10 \);
the next 200 shares are purchased at \( 20.10 + 0.001\times200 = \$20.30 \);
the final 300 shares are purchased at \( 20.30 + 0.001\times300 = \$20.60 \).
The total cost is \( 100\times20.10 + 200\times20.30 + 300\times20.60 = 12,250 \).
With a linear price impact, the cost is minimized by doing the trades evenly: 3 trades of 200 shares.
Under this schedule:
The first 200 shares are purchased at \( \$20 + 0.001\times200 = \$20.20 \);
the next 200 shares are purchased at \( 20.20 + 0.001\times200 = \$20.40 \);
the final 300 shares are purchased at \( 20.40 + 0.001\times200 = \$20.60 \).
The total cost is \( 200\times20.20 + 200\times20.40 + 300\times20.60 = 12,240 \).
Answers for Chapter 17. Fees, rebates and other inducements

*Answer to Problem 17.1* Tae receives the rebate, \(100 \times 0.002 = 0.20\), which brings the amount she receives from the sale to $2,500.20. Sam pays the access fee, bringing his net payment to $2,500.30. PAX keeps the $0.10.

*Answer to Problem 17.2* Since Vanessa’s order is the first in an empty book, she’s at the front of ZAP’s offer queue. Sam's order executes against Vanessa’s. Vanessa pays maker fee, so the net amount she receives from the sale is \(2,500 - 0.29 = 2,499.71\); Sam pays \(2,500 - 0.18 = 2,499.82\). ZAP keeps $0.11.

*Answer to Problem 17.3* The resting bid is Beth’s. She receives a rebate of $0.25.; The taker is Charlie; he pays $0.30.

*Answer to Problem 17.4* The order is at the offer, so it might not execute, but if it does, they will receive (in addition to the $10.20 per share) a liquidity rebate of 0.0025 per share (an extra $0.25)

Note: The *maker* is the passive side of the trade, the *resting* limit order, the bid that is hit or the offer that is lifted. The taker is the person who is actively executing against the maker (“taking liquidity”).

- In *Answer to Problem 17.3*, the resting order is Beth’s: she is sitting at the bid. Charlie’s order is priced to be marketable: when his order arrives, there is an execution. So he is the taker.

- In *Answer to Problem 17.4*, the incoming sell order goes on to the book behind the order that is already there. the arrival of the new incoming sell order does not trigger an immediate trade. It will be passive, resting in the book until some marketable incoming buy order causes it to execute. It is therefore a "maker".

*Answer to Problem 17.5* Payment for order flow is money paid by a dealer or market center to a broker to encourage the broker to send retail customer orders. Since the broker is supposed to be the agent for the customer, this gives the broker an incentive to act in a way that might not be in the customer’s interest.

*Answer to Problem 17.6* Sam receives $0.10; Mona pays $0.20

Answers for Chapter 18. Reg NMS

*Answer to Problem 18.1*

1. 10.09

2. This is a trade-through. AX is posting a visible quantity of 200 shares; BX trades at an inferior price without executing AX’s shares.
c. At AX, 100 sh trade at 10.10; 100 sh trade at 10.09. At BX, 100 sh trade at 10.09, and 100 more trade at 10.08. Although AX still has 100 shares left at 10.09, this does not constitute a trade-through because the trader’s order was sufficient to execute all of AX’s protected quantity. AX’s hidden quantities are not protected.

Answer to Problem 18.2

a. A’s protected bid is 300 sh @ 50.49; B’s is 200 sh @ 50.49; C’s is 400 @ 50.47. It is not necessary for a protected quote to match the market-wide best bid or offer.

b. The limit price of 50.45 is below all of the protected bids, so the trader must send 300 to A, 200 to B, and 400 to C.

c. Only A’s and B’s protected bids are better than the limit price, so the trader must send at least 300 to A and 200 to B. There is no need to send anything to C.

d. On exchanges A and C, the final executions will occur at 50.45. But B will still have 200 shares left at 50.46 (a better price).

Answer to Problem 18.3

a. Only the top of the visible book is protected. A’s protected quote is 10.49 bid for 100; B’s is 10.49 bid for 800; C’s is 10.47 bid for 200.

b. On exchange A, 800 shares @ 10.49; on B, 2,000 shares @ 10.49; on C, 500 shares @ 10.47. Based on what was displayed, the trader would have expected that the orders would take out all of the displayed interest on A and B above 10.46. In fact, due to the hidden executions, everything executed at each markets best bid. In retrospect, the executions on exchange C could have been filled at better prices on the other exchanges.

In this situation you might wonder why all 500 shares get sold on market C at 10.47? Wouldn’t the only protected orders be the 200 visible orders? And if that is the case then the ISO wouldn’t require the remaining 300 shares to be sold from the hidden quantity at 10.47?

And wouldn’t it be the case that all orders that are priced better than the ISO order must be executed in a market before they can move to the next market?

The answers to these questions arise in the distinction between routing and execution. The trade-through rule only dictates what quantities must be routed (sent) to a market. The executions those orders actually receive depends on the state of the order books at those markets. Sweep orders are sent simultaneously, not sequentially. Each receiving market handles its own portion of the ISO independently of what other markets are doing.

Relative to what is on the books at A and B, the entire execution of 500 shares at 10.47 at C could be judged a trade-through. That is, both A and B are holding buy orders at prices better than 10.47. But these orders aren’t protected unless they’re visible and at the top of the book. C will execute the incoming 500 share sell order instantly: once C’s front end
sees that the order is marked “ISO”, C is relieved of the responsibility of checking other markets protect bids.
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