Immunization

Reading: Tuckman and Serrat, Chapter 4

Asset-Liability Management

• Suppose you have liabilities or obligations consisting of a stream of fixed cash flows you must pay in the future.
  • Pension liabilities
  • Insurance liabilities
  • Bond defeasance

• How can you structure an asset portfolio to fund these liabilities?
Dedication

• The only completely riskless approach is to construct an asset portfolio with cash flows that exactly match the liability cash flows.
• This funding method is called dedication.
• This approach may be infeasible or excessively costly.
• In some situations, risk managers may want more flexibility.

Immunization

• Consider a more flexible but more risky approach, called immunization:
  • The liabilities have a certain market value.
  • That market value changes as time passes and as interest rates change.
• Construct an asset portfolio with
  • the same market value and
  • the same duration as the liabilities
  • so that the asset value tracks the liability value over time.
• Can include derivatives as well as fixed cash flows.
Typical Duration Mismatches

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From Veronesi, Fixed Income Securities, p. 97.

Duration/Market Value Matching

• Change in value \(-\)dollar duration \(\times\) change in rates

• Matching the dollar duration of assets and liabilities means matching their changes in value if all rates change by the same amount—hedges against parallel yield curve shifts.

• Matching market value means liabilities are fully funded.

• Hedging against parallel shifts is really just a first step.
Structuring an Asset Portfolio

• Suppose your liabilities have market value of $100M and duration of 6.

• You want to structure an asset portfolio with the same market value and duration.

• Construct an asset portfolio with just two securities:
  • A bond with price $110 and duration 8
  • A CMO with price $70 and duration 4

• Class Problem: What are the number of units of the bond and CMO, N1 and N2, in the immunizing asset portfolio?

Simply Dollar Duration Matching

• Suppose your liabilities have dollar duration of 100M and your assets have dollar duration 500M

• You want to leave your existing assets in place and close the gap by selling interest rate swap contracts.

• Suppose each swap contract has present value zero and dollar duration of 10M.

• Class Problem: How many contracts must you sell to give your net position zero dollar duration?
Structuring an Asset Portfolio

• Suppose your liabilities have market value of $100M and duration of 6.

• You want to structure an asset portfolio with the same market value and duration/dollar duration.

• Construct an asset portfolio with just two securities:
  • A bond with price $110 and duration 8
  • A CMO with price $70 and duration 4

• **Class Problem Solution:** What are the number of units of the bond and CMO, $N_1$ and $N_2$, in the immunizing asset portfolio?

  \[ N_1 \times 110 + N_2 \times 70 = 100M \]
  \[ N_1 \times 880 + N_2 \times 280 = 600M \]
  
  \[ \Rightarrow N_1 = 454,545, N_2 = 714,286 \]

Or, $w_1 = w_2 = 50\%$, so $N_1 = 50M/110$ and $N_2 = 50M/70$.

Simply Dollar Duration Matching

• Suppose your liabilities have dollar duration of 100M and your assets have dollar duration 500M.

• You want to leave your existing assets in place and close the gap by selling interest rate swap contracts.

• Suppose each contract has dollar duration of 10M.

• **Class Problem Solution:** How many contracts must you trade to give your net position zero dollar duration?

  \[ 500M + N \times 10M - 100M = 0 \]
  
  \[ \Rightarrow N = -400M/10M = -40. \] Sell 40 contracts.