American Options and Callable Bonds

**Concepts and Buzzwords**
- American Options
- Valuing an American Call on a Coupon Bond
- Valuing a Callable Bond
- Interest Rate Sensitivity of a Callable Bond
- exercise policy
- value-maximization
- redeem
- negative convexity
- option-adjusted spread

**Reading**
- Veronesi, Chapter 12
- Tuckman, Chapter 19
**American Options**

- Most corporate bonds, and virtually all mortgages, contain an embedded option giving the borrower the option to call, or prepay, the loan at a pre-specified price on a date of the borrower’s choosing.
- Valuing these securities amounts to valuing the embedded American option.
- With an American option, the holder has a choice of when to exercise.
- Thus, valuing and assessing the risk of an American option involves determining the holder’s optimal exercise policy.

**No Early Exercise of Calls on Assets that Have No Payments Prior to Expiration:**

- Prior to expiration, an American call on a “non-paying” asset, such as a zero, is worth more than its exercise value:
  
  \[
  \text{American call value} \geq \text{European call value} = \text{European put value} + V - d_1K > V - d_1K > V - K = \text{exercise value}.
  \]
- Therefore, better to sell the call than to exercise it (assuming the strike price stays constant over time).
Early Exercise of American Calls on Assets with Intervening Payments

▪ If the underlying asset makes payments prior to option expiration—“intervening payments” such as coupons or dividends, than early exercise can be optimal when the call gets deep in the money and the payment is high:

American call value ≥ European call value

= European put value + \[ V^{excluding-intervening-payments} - d_1K \]

> ?= ?< ? \[ V^{including-intervening-payments} - K \]

▪ Exercise or wait?
  ▪ Exercise: get PV of intervening payment
  ▪ Wait: keep the interest on the strike price
  ▪ Wait: save the “put value”

American Call on the Coupon Bond

▪ Consider an American call option on
  ▪ $100 par of the 2-year, 5.5%-coupon bond.
  ▪ The strike price of the call is $100.
  ▪ The call expires at time 1.5.
  ▪ The call is exercisable on any coupon date, ex-coupon, i.e., immediately after that date’s coupon is paid.
Underlying 5.5%-Coupon Bond

- Each node in the tree below lists
  - the current ex-coupon price of the 5.5%-coupon bond, and
  - the current price of a zero with 6 months to maturity.

- At each node, the bond is priced as a package of zeroes.

Class Problems: What are the time 0 and time 0.5 ex-coupon prices of the bond?

Exercise Policy for the American Call

- To value the call, we need to know its future cash flows at each time and state.

- At each time and state the option holder must decide whether or not to exercise the option

- The future cash flows of the option depend on the option holder's exercise policy.
Value-Maximizing Exercise Policy

- We assume that the option holder will choose, at each time and state, to exercise or not depending on which action maximizes the option's market value.
- This assumption is reasonable as long as the option holder can freely sell the option.
- Then, if ever the option holder wants to get out of his position when the option is worth more than its exercise value, the option holder just sells the option instead of exercising it.

Class Problem: American Call

- Working backwards in time, decide what the call holder would do if he got to a given point with the call still alive—exercise, or wait?
- At time 1.5, the last call date, if the option were still alive, the holder would just exercise it if in the money or else let it expire worthless.
- At earlier dates, the holder can either exercise, or else wait one period, pay the coupon, and proceed optimally from there, whichever gives the option greater value.
Callable Bond

- Consider a firm that issues a 2-year, 5.5%-coupon bond that is callable at par.
- This means that the issuer has the option to call (or ‘redeem’, or buy back) the bond for par at any time.
- By selling a callable bond to investors, the issuer has effectively
  - sold a noncallable bond to the investors and
  - simultaneously bought a call option on the bond from those investors.

Valuing the Callable Bond

- Callable Bond = Noncallable Bond - Call Option
  - Issuer (borrower): short the bond, long the option
  - Investor (lender): long the bond, short the option
- To value the callable bond, assume that the issuer follows a strategy for exercising the call that
  - maximizes the value of the call, or equivalently,
  - minimizes the cost of the callable bond.
- Since we have already valued the noncallable bond and the option, we know the value of the callable bond at each point.
- The next slides calculate the callable bond value from scratch and verify that Callable = Noncallable – Call always holds.
Class Problem: Callable Bond

- Working backwards, decide what the issuer would do to minimize the cost (pv) of debt if he got to a given point with the bond still outstanding.
- The issuer’s choices are to either
  - call the bond and pay par, or
  - leave it outstanding for another period, pay another coupon, and then proceed with optimal debt service from there.

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
<th>Time 1</th>
<th>Time 1.5</th>
</tr>
</thead>
</table>

Another Way to See Callable = Noncallable – Option: See Refunding Profit as Option Exercise Value

- Suppose the way the firm finances the call is by selling new noncallable bonds with the same coupon and maturity as the old debt—a “refunding.”
- The profit from refunding
  = proceeds of sale of new debt - call price
  = value of the noncallable -100
  = option exercise value
- With this plan in place, a firm that has issued a callable bond pays the noncallable cash flows until maturity but gets to do a refunding along the way.
- The firms’ net position = short a callable bond
  = short a noncallable bond, long an option.
Class Problems

What are the SR dollar durations and SR durations of

1) the noncallable bond?

2) the call?

3) the callable bond?

Negative Convexity of Callable Bonds

- The value of the embedded call option is a highly convex function of interest rates.
- So the short call position in the callable bond not only reduces its duration, it also gives it “negative convexity.”

![Graph showing bond price vs. short rate for noncallable and callable bonds](image)
Call Price Schedules and Call Protection

- In the preceding example, the callable bond was callable at par at any time.
- In practice, a bond may have an initial period of call protection, during which it is not callable, and then it becomes callable.
- In addition, the contractual call price, i.e., the strike price of the embedded call option, may be greater than par, and may decline over time according to a published schedule.
- The preceding analysis still applies, except that the borrower can call only on certain dates, at the scheduled contractual call price (strike price).
- Bonds may also be called between coupon dates, at the call price plus accrued interest.

Callable Bond with Call Protection and Declining Call Price

- Suppose the 5.5%-coupon, 2-year bond is only callable at time 1 at 101 and at time 1.5 at 100.50.
- Now what is the value of the callable bond?

\[
\begin{align*}
\text{Time 0} & \\
w: 99.9286 &= 0.973047 \times [2.75 + 0.5 \times (99.0155+100.8777)] \\
\text{Time 0.5} & \\
w: 99.0155 &= 0.976941 \times [2.75 + 0.5 \times (100.0207+100.9967)] \\
\text{Time 1} & \\
w: 98.6063 &= 0.979071 \times [2.75 + 0.5 \times (100.3113+100.5)] \\
\text{Time 1.5} & \\
w: 99.6684 &= 0.969150 \times [2.75 + 0.5 \times (100.9967+100.9967)] \\
c: 100.50
\end{align*}
\]
Option-Adjusted Spread

- Corporate bonds pay less than promised in the event of default.
- This credit risk makes them worth less than a nondefaultable bond with same coupon and maturity.
- This price difference results in a higher yield, that is, a credit spread over the yield of a similar Treasury.
- If the bond is callable, the price is lower because of the embedded call, which also results in a higher yield to maturity.
- In fact, the call and default risks interact in a complicated way.
- In practice, people try to calculate the component of the spread of a callable defaultable bond over a noncallable Treasury that is due to just to credit risk, and not call risk.
- This is the so-called option-adjusted spread.

OAS

- First consider that in the absence of the call option, the credit spread of the bond is the amount by which the yield curve must be shifted up to correctly price the bond if it were nondefaultable.
- In the same spirit, the OAS is the amount that the interest rates in the model must be shifted upward to make the modeled callable bond price match the market price.
- The idea is that the model adjusts for the option, and then the OAS captures the additional price discount attributable to credit risk.
- In fact, OAS is calculated and quoted for all kinds of bonds: illiquid Treasuries, mortgage-backed securities, etc.