Floating Rate Notes

Concepts and Buzzwords

- Floating Rate Notes
  - Cash flows
  - Valuation
  - Interest Rate Sensitivity

- floater, FRN, ARN, VRN, benchmark interest rate, index

Reading

- Veronesi, Chapter 1
- Tuckman, Chapter 18
Introduction to Floating-Rate Notes

- A floating rate note is a bond with a coupon that is indexed to a benchmark interest rate.
- Possible benchmark rates include US Treasury rates, LIBOR, prime rate, municipal and mortgage interest rate indexes.
- Examples of floating-rate notes
  - Corporate (especially financial institutions)
  - Adjustable-rate mortgages (ARMs)
  - Governments (inflation-indexed notes)

Floating Rate Jargon

- Other terms used for floating-rate notes include
  - FRNs
  - Floaters and Inverse Floaters
  - Variable-rate notes (VRNs)
  - Adjustable-rate notes
- FRN usually refers to an instrument whose coupon is based on a short term rate (3-month T-bill, 6-month LIBOR)
- VRNs are based on longer-term rates
  - (1-year T-bill, 5-year T-bond)
Cash Flow Rule for Plain Vanilla Semi-Annual Floater

- The basic semi-annual coupon floating rate note has the coupon indexed to the 6-month interest rate.
- Each coupon date, the coupon is equal to the par value of the note times one-half the 6-month rate quoted 6 months earlier, at the beginning of the coupon period. In other words, the time \( t \) coupon payment as percent of par is \( r_{t-0.5} \).
- The note pays par value at maturity.

Floating Rate Note Cash Flows

- Each coupon is based on the previous 0.5-year rate.
- Only the next coupon is known at the current date. The later ones are random.
### Example: Two-Year Semi-Annual Floater

What are the cash flows from $100 par of the note in this scenario?

- The first coupon on the bond is \(100 \times 0.0554/2=2.77\).
- Later coupons set by the future 6-month interest rates.
- For example, suppose the future 6-month interest rates turn out as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.54%</td>
</tr>
<tr>
<td>0.5</td>
<td>6.00%</td>
</tr>
<tr>
<td>1</td>
<td>5.44%</td>
</tr>
<tr>
<td>1.5</td>
<td>6.18%</td>
</tr>
</tbody>
</table>

Floater Cash Flows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.77</td>
</tr>
<tr>
<td>0.5</td>
<td>3.00</td>
</tr>
<tr>
<td>1</td>
<td>2.72</td>
</tr>
<tr>
<td>2</td>
<td>103.09</td>
</tr>
</tbody>
</table>

### Replicating a T-Year Floater with 0.5-Year Par Bonds

Consider the following trading strategy:

- At time 0, buy a 0.5-year par bond: pay $1.
- At time 0.5, buy another 0.5-year par bond: collect $1 + \(r_{0.5}/2\), pay $1 = collect \(r_{0.5}/2\).
- At time 1, buy another 0.5-year par bond: collect $1 + \(0.5r_{1}/2\), pay $1 = collect \(0.5r_{1}/2\).
- and so on, every six months until floater maturity date \(T\).
- At time \(T\): collect $1 + \(T \cdot 0.5r_{T}/2\).
A Semi-Annual-Coupon Floater is Equivalent to a 0.5-Year Par Bond

- A dynamic strategy of rolling six-month par bonds until floater maturity, collecting the coupons along the way, replicates the cash flows of a floater.
- So as semi-annual coupon floater is equivalent to the six-month par bond in its replicating trading strategy.
- Like its replicating trading strategy, a floater is always worth par on the next coupon date with certainty.
- Its coupon is set to make it worth par today.
- The duration of the floater is therefore equal to the duration of a six-month par bond.
- Their convexities are the same too.

<table>
<thead>
<tr>
<th>Time</th>
<th>Strategy</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Buy 0.5-yr par bond</td>
<td>$-1 \cdot 1 + (1+0.5r_{0.5}/2)$</td>
</tr>
<tr>
<td>0.5</td>
<td>Buy 0.5-yr par bond</td>
<td>$-1 \cdot 1 + (1+0.5r_{0.5}/2)$</td>
</tr>
<tr>
<td>1</td>
<td>Buy 0.5-yr par bond</td>
<td>$-1 \cdot 1 + (1+r_{1.5}/2)$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>T</td>
<td>Buy 0.5-yr par bond</td>
<td>$-1 \cdot 1 + (1+r_{T-0.5}r_T/2)$</td>
</tr>
</tbody>
</table>

Net: $-1 \cdot 0r_{0.5}/2 - 0.5r_{0.5}/2 - 1r_{1.5}/2 - ... - 1 + r_{T-0.5}r_T/2$

0 0.5 1 1.5 ... T
Class Problems

Assume the 0.5-year rate is 5.54%.

1) What is the duration of a semi-annual paying floating-rate note?

2) What is the dollar duration of $100 par of this note?

3) What is the convexity of this floater?

4) What is the dollar convexity?