New York University
Stern School of Business

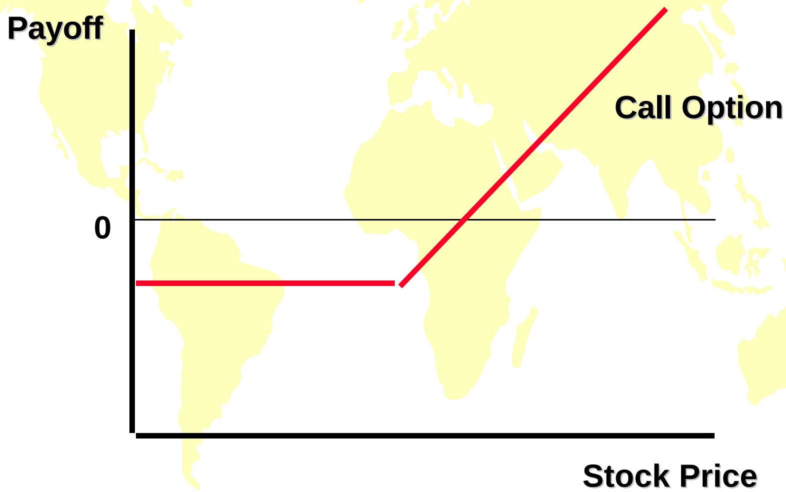
Options

Prof. Ian Giddy
New York University

Options

- Puts and Calls
- Put-Call Parity
- Combinations and Trading Strategies
- Valuation
- Hedging

Options



Payoffs and Profits on Options at Expiration - Calls

<u>Notation</u>

Stock Price = S_T Exercise Price = X

<u>Payoff to Call Holder</u>

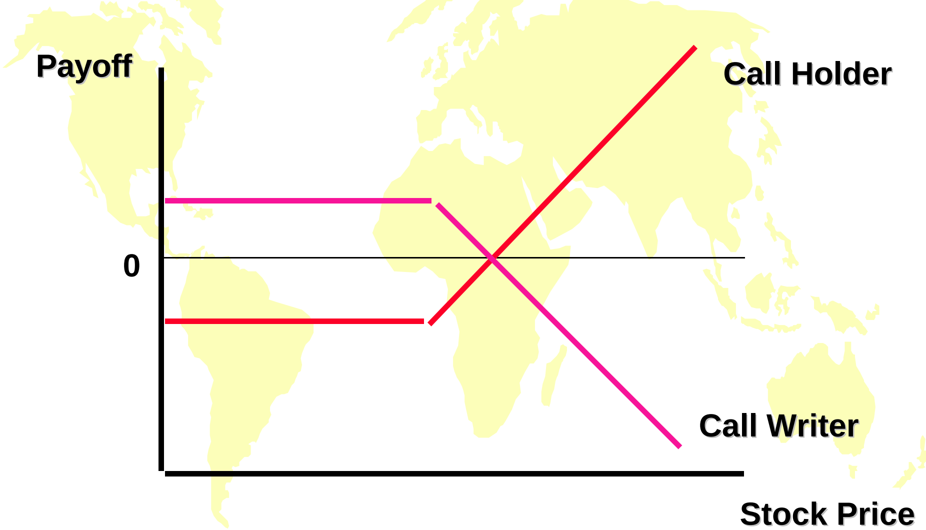
(S_T - X) if S_T >X

0 if $S_T \leq$ X

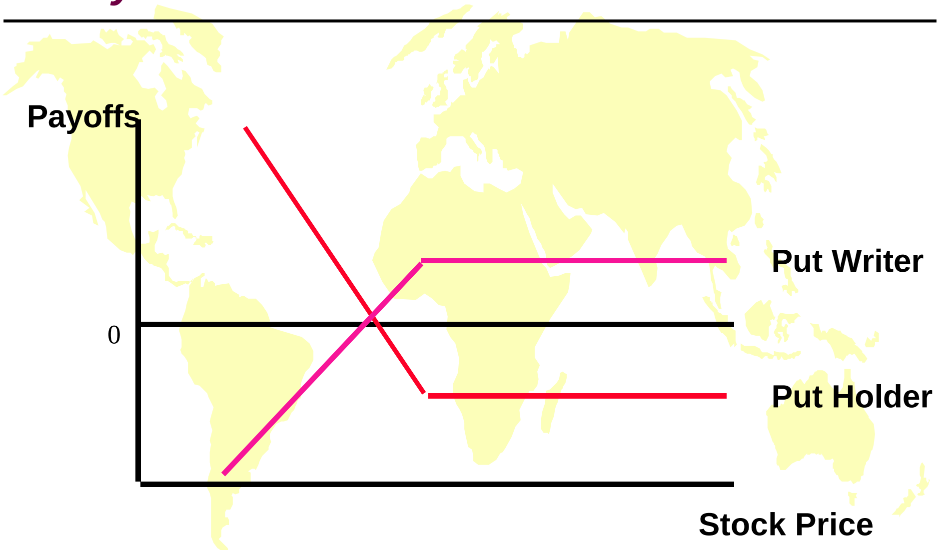
<u>Profit to Call Holder</u>

Payoff - Purchase Price

Payoff Profiles for Calls



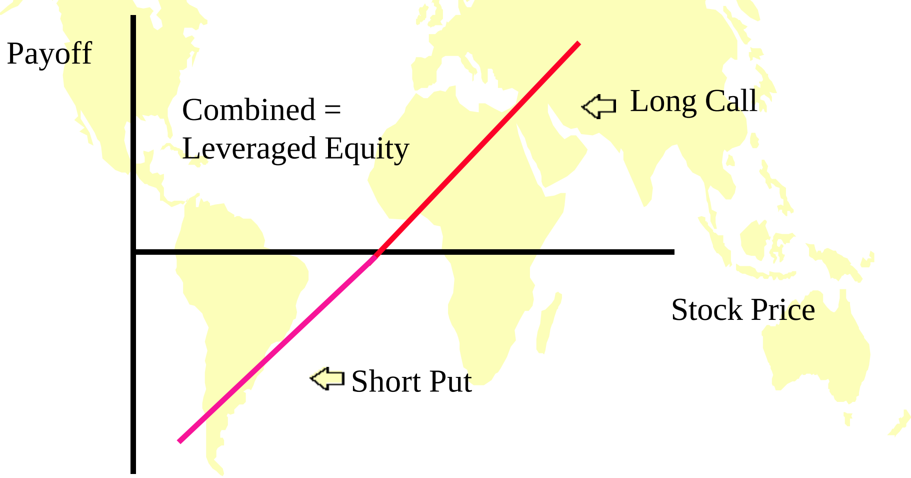
Payoff Profiles for Puts



Put-Call Parity Relationship

	$S_T \leq X$	$S_T > X$
Payoff for Call Owned	**0**	$S_T - X$
Payoff for Put Written	**-(X -ST)**	**0**
Total Payoff	$S_T - X$	$S_T - X$

Payoff of Long Call & Short Put



Arbitrage & Put Call Parity

Since the payoff on a combination of a long call and a short put are equivalent to leveraged equity, the prices must be equal.

$$C - P = S_0 - X / (1 + r_f)^T$$

If the prices are not equal arbitrage will be possible

Cost of buying stock by borrowing $X

Cost of buying a call and selling a put

Put Call Parity - Disequilibrium Example

Stock Price = 110   Call Price = 17

Put Price = 5   Risk Free rate = 10.25%

Maturity = .5 yr   Strike price X = 105

$$C - P > S_0 - X / (1 + r_f)^T$$

$$17 - 5 > 110 - (105/1.05)$$

$$12 > 10$$

Since the leveraged equity is less expensive, acquire the low cost alternative and sell the high cost alternative

Put-Call Parity Arbitrage

Position	Immediate Cashflow	Cashflow in Six Months $S_T<105$	Cashflow in Six Months $S_T \geq 105$
Buy Stock	**-110**	S_T	S_T
Borrow $X/(1+r)^T = 100$	**+100**	**-105**	**-105**
Sell Call	**+17**	**0**	$-(S_T-105)$
Buy Put	**-5**	$105-S_T$	**0**
Total	**2**	**0**	**0**

Option Strategies

<u>Protective Put</u>

- Long Stock
- Long Put

<u>Covered Call</u>

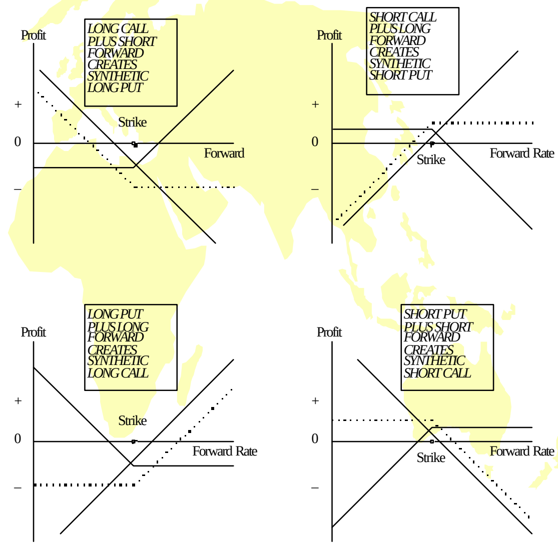
- Long Stock
- Short Call

<u>Straddle (Same Exercise Price)</u>

- Long Call
- Long Put

Option Combinations

- Four classic ways of combining an option with a futures to create the opposite option

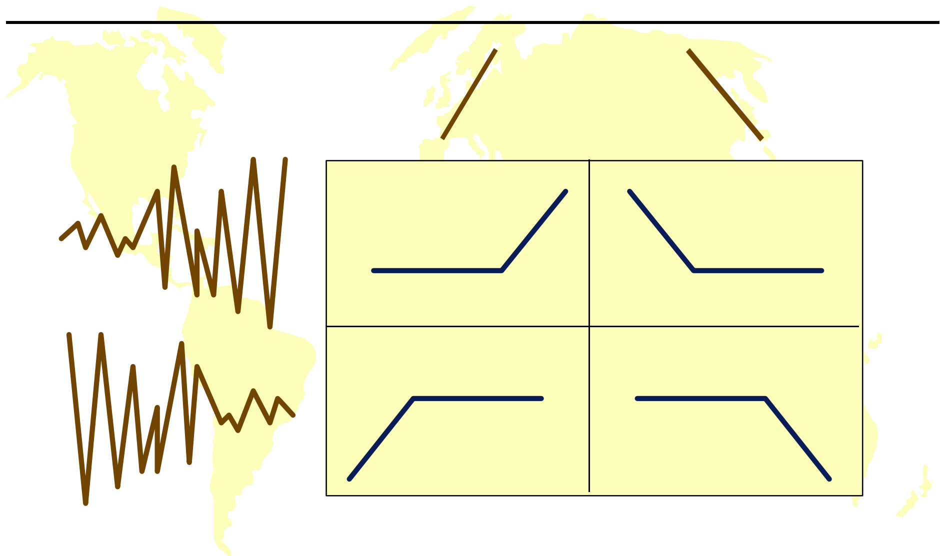


When Use Options to Take a View?

- View on *direction*
- View on *volatility*

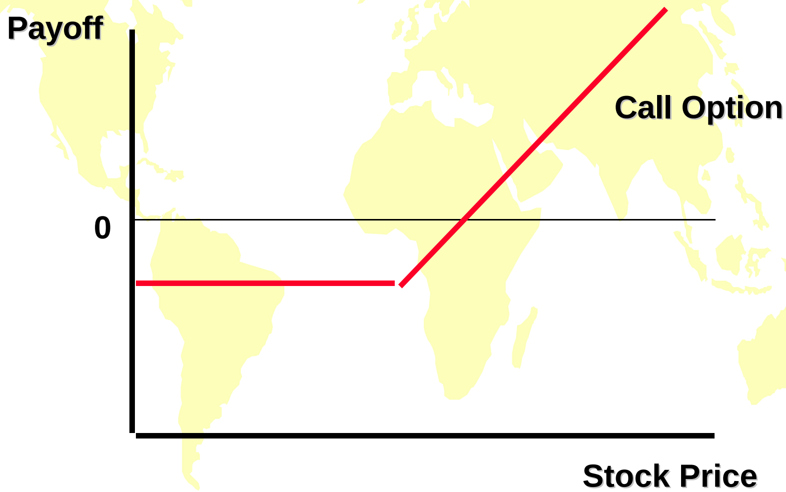
Direction: / Volatility	**Security rising**	**Security falling**	**No trend**
Volatility increasing	*Buy call*	*Buy put*	*Buy straddle*
Volatility falling	*Sell put*	*Sell call*	*Sell straddle*
No trend in volatility	*Buy forward*	*Sell forward*	*Arbitrage*

View on Direction, Volatility or Both?



Option Valuation

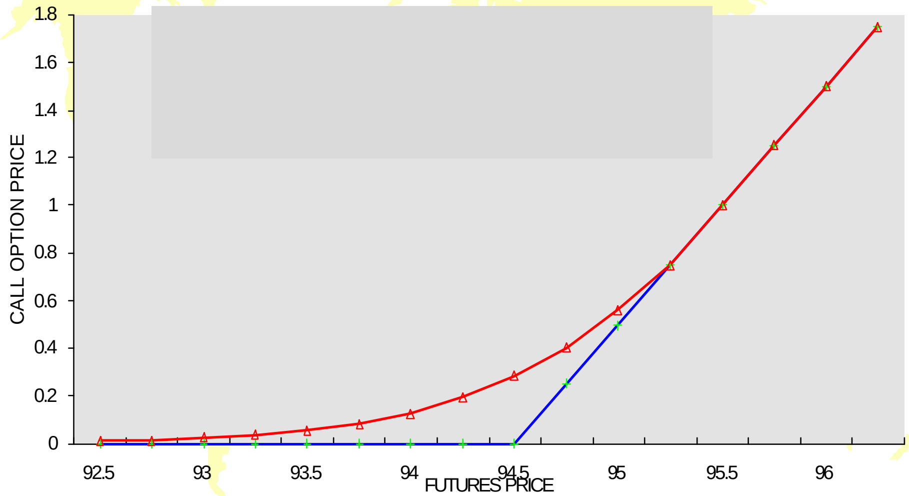
Options



Option Values

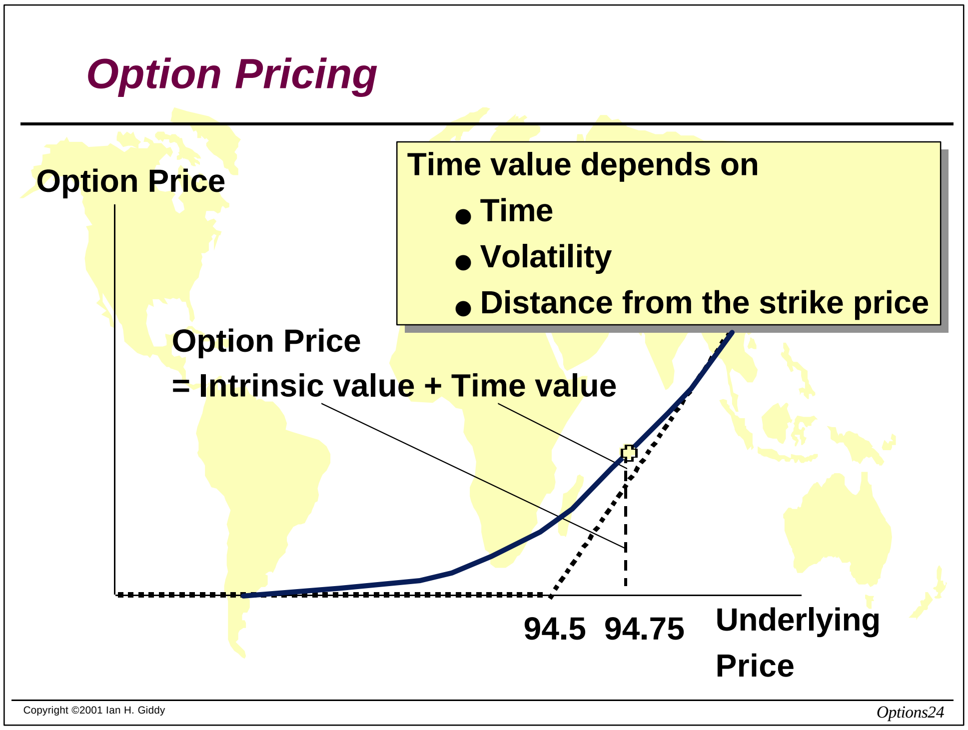
- Intrinsic value - profit that could be made if the option was immediately exercised
  - ◆Call: stock price - exercise price
  - ◆Put: exercise price - stock price
- Time value - the difference between the option price and the intrinsic value

Option Pricing Model



Factors Influencing Option Values: Calls

Factor	Effect on value
Stock price	increases
Exercise price	decreases
Volatility of stock price	increases
Time to expiration	increases
Interest rate	increases
Dividend Rate	decreases


Option Pricing
Option Price
Time value depends on
● Time
● Volatility
● Distance from the strike price
Option Price
= Intrinsic value + Time value
94.5 94.75
Underlying Price
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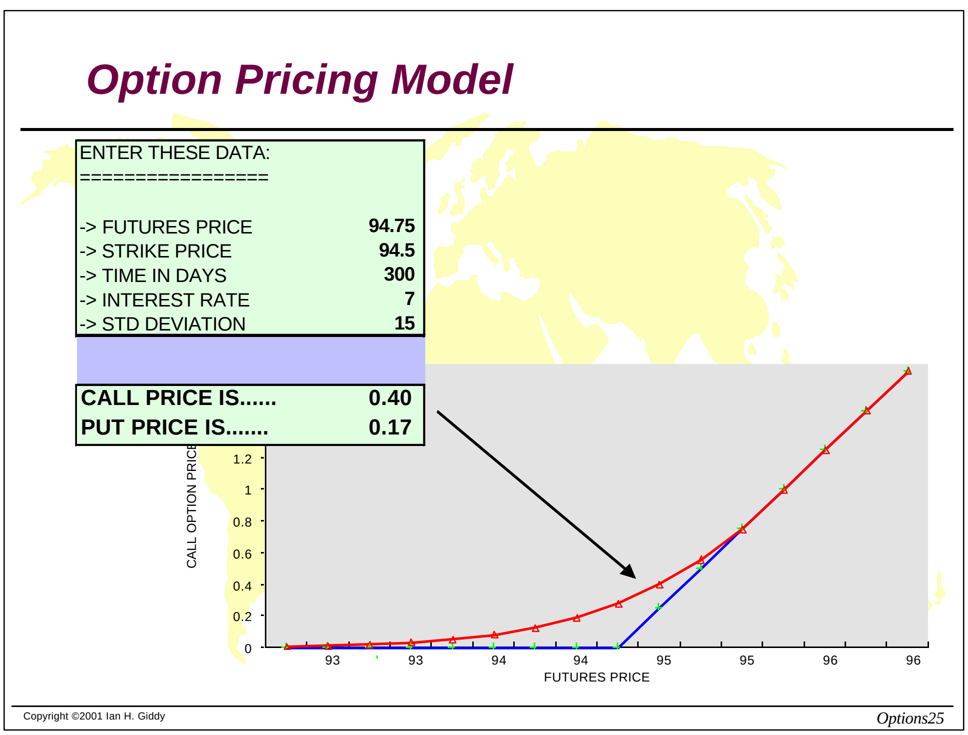
Options24


Option Pricing Model
ENTER THESE DATA:
================
-> FUTURES PRICE 94.75
-> STRIKE PRICE 94.5
-> TIME IN DAYS 300
-> INTEREST RATE 7
-> STD DEVIATION 15
CALL PRICE IS...... 0.40
PUT PRICE IS....... 0.17
CALL OPTION PRICE
1.2
1
0.8
0.6
0.4
0.2
0
93 93 94 94 95 95 96 96
FUTURES PRICE
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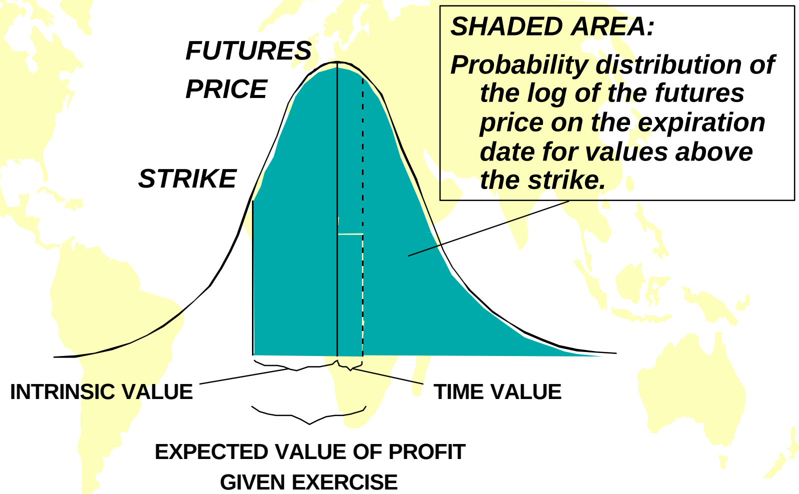
Options25

Value of Call Option



Black-Scholes Option Valuation

Call value = $S_o N(d_1) - Xe^{-rT} N(d_2)$

$d_1 = [\ln(S_o/X) + (r + \sigma^2/2)T] / (\sigma\ T^{1/2})$

$d_2 = d_1 - (\sigma\ T^{1/2})$

where

S_o = Current stock price

X = Strike price, T = time, r = interest rate

N(d) = probability that a random draw from a normal distribution will be less than d.

Black-Scholes Option Valuation

X = Exercise price.

e = 2.71828, the base of the natural log.

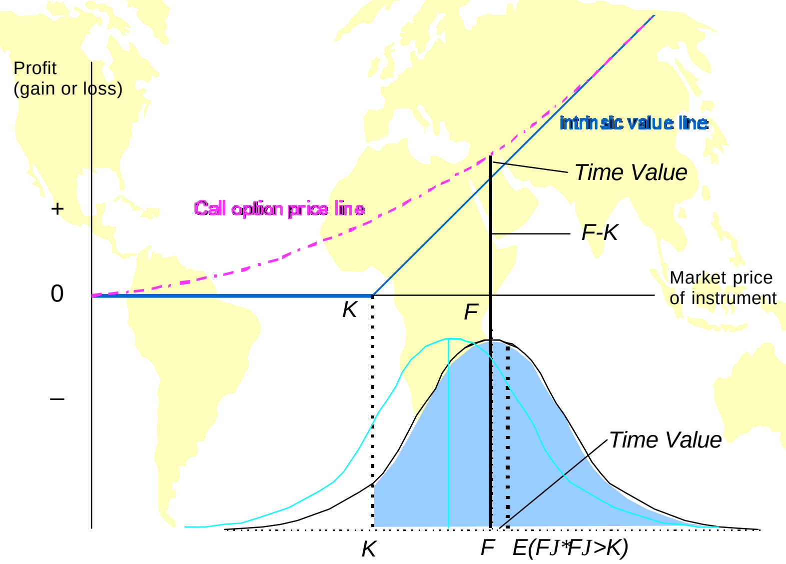
r = Risk-free interest rate (annualizes continuously compounded with the same maturity as the option.

T = time to maturity of the option in years.

ln = Natural log function

σ = Standard deviation of annualized cont. compounded rate of return on the stock

How a Change in the Futures Price Changes the Option's Price



Call Option Example

$S_o = 100$ $X = 95$

$r = .10$ $T = .25$ (quarter)

$\sigma = .50$

$d_1 = [\ln(100/95) + (.10+(.5^{\,2}/2))] / (.5\ .25^{1/2})$

$= .43$

$d_2 = .43 - ((.5)(\ .25^{1/2})$

$= .18$

Probabilities from Normal Dist.

N (.43) = .6664

d	N(d)	
.42	.6628	
.43	.6664	Interpolation
.44	.6700	

Probabilities from Normal Dist.

N (.18) = .5714

d	N(d)
.16	.5636
.18	.5714
.20	.5793

Call Option Value

$C_o = S_oN(d_1) - Xe^{-rT}N(d_2)$

$C_o = 100 \text{ X } .6664 - 95 \text{ e}^{-.10 \text{ X } .25} \text{ X } .5714$

$C_o = 13.70$

Implied Volatility

Using Black-Scholes and the actual price of the option, solve for volatility.

Is the implied volatility consistent with the stock?

Put Option Valuation: Using Put-Call Parity

$P = C + PV(X) - S_o$

$= C + Xe^{-rT} - S_o$

Using the example data

$C = 13.70$ $X = 95$ $S = 100$

$r = .10$ $T = .25$

$P = 13.70 + 95\,e^{-.10 \times .25} - 100$

$P = 8.35$

Using the Black-Scholes Formula

Hedging: Hedge ratio or delta

The number of stocks required to hedge against the price risk of holding one option

Call = $N(d_1)$

Put = $N(d_1) - 1$

Option Elasticity

Percentage change in the option's value given a 1% change in the value of the underlying stock

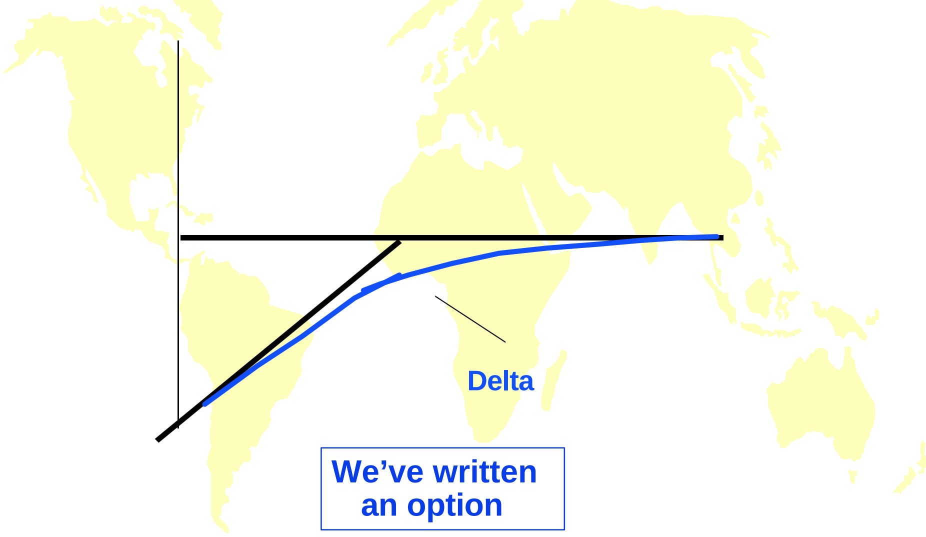
Portfolio Insurance - Protecting Against Declines in Stock Value

- Buying Puts - results in downside protection with unlimited upside potential
- Limitations
  - Tracking errors if indexes are used for the puts
  - Maturity of puts may be too short
  - Hedge ratios or deltas change as stock values change

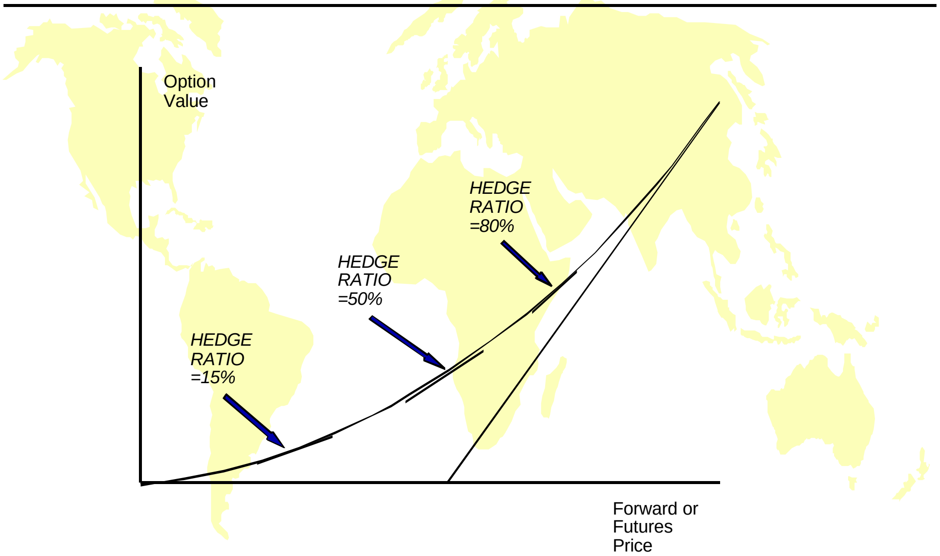
Trading Options: Risk Management Concepts

- Hedger's view: "An option is an insurance premium--you can gain, but you cannot lose more than the premium you pay."
- Trader's view: "An option is just another security--I can buy it or sell it, take a long or short position in puts or calls. Its price fluctuates with market variables. For any position, the gain or loss is the change in price of the position."

Trading Options: Delta Hedging



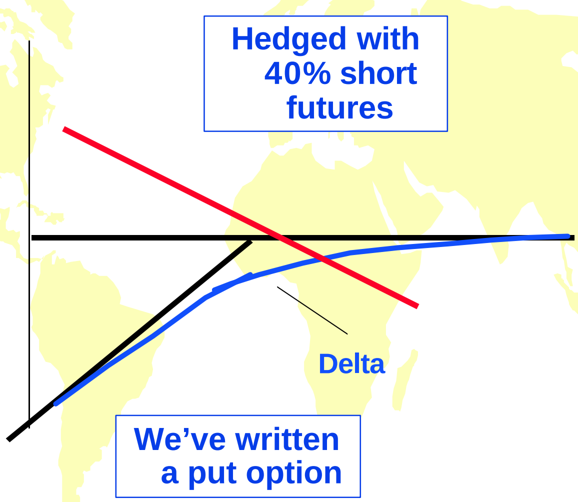
Delta Hedging



Trading Options: Delta Hedging with Futures

- Delta or hedge ratio is the change in the option price for a given change in the price of the underlying.
- Delta is also the probability of exercise.
- Delta is also the hedge ratio that tells one how many futures you need to hedge a given options position.

Trading Options: Delta Hedging



Options Trading: Delta Hedging and the Gamma

- Delta is not constant: it's low for out of the money options, high for in the money options.
- Change in delta makes it tough to know exactly how many futures to use.
- Change in delta is the *gamma*. Positive gamma is nice. Writing options produces negative gamma, which is nasty.

Goal: Understand Options' Sensitivity

An option trader has a portfolio of options with different deltas, gammas, etc. The goal is to discover the sensitivities of the portfolio to changes in rates, time, volatility, etc, and to neutralize them.

	Greek	Measures
Δ	Delta	Sensitivity of portfolio value to change in price of the underlying asset
Γ	Gamma	Sensitivity of delta to change in price of underlying asset
θ	Theta	Sensitivity of portfolio value to change in time
Λ	Lambda (Vega)	Sensitivity of portfolio value to change in volatility
Ρ	Rho	Sensitivity of portfolio to change in interest rate

Options: Summary

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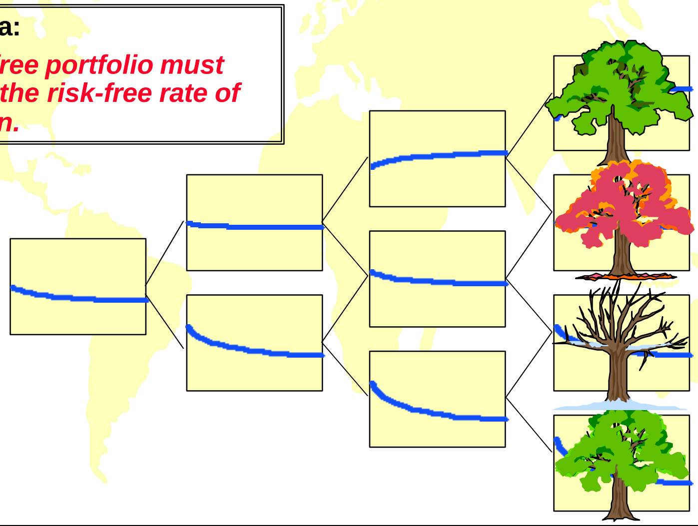
Appendix:
Option Pricing: Trees

The Idea:

A risk-free portfolio must earn the risk-free rate of return.



Binomial Tree

Consider a call option on a Treasury Bill

- The strike price is set at $21.
- The underlying bill could go to $22 or $18 in 1 month.

UP

- Bill=$22
- Call=$1

- Bill=$20
- Call=?

- Bill=$18
- Call=$0

DOWN

A risk-free portfolio must earn the risk-free rate of return.

Binomial Tree

Consider a call option on a Treasury Bill

- The strike price is set at $21.
- The underlying bill could go to $22 or $18 in 1 month.

UP

- Bill=$22
- Call=$1

- Bill=$20
- Call=?

- Bill=$18
- Call=$0

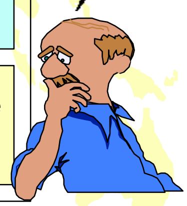
DOWN

PORTFOLIO:
Long Δ bills
Short 1 Call

Portfolio value = $22\,\Delta - 1$

Portfolio value = $18\,\Delta$

What's the delta that will make the portfolio risk free?



A risk-free portfolio must earn the risk-free rate of return.

Binomial Tree

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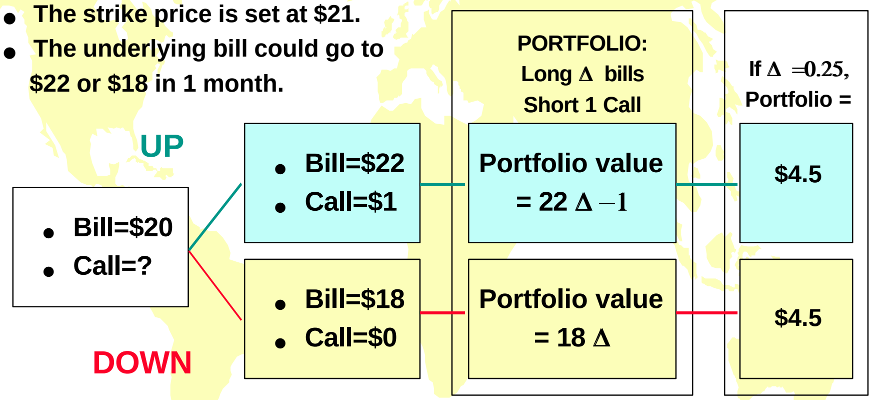
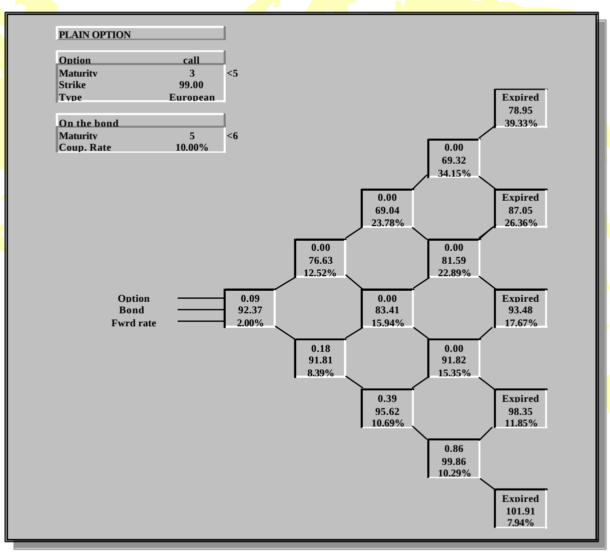


A risk-free portfolio must earn the risk-free rate of return.

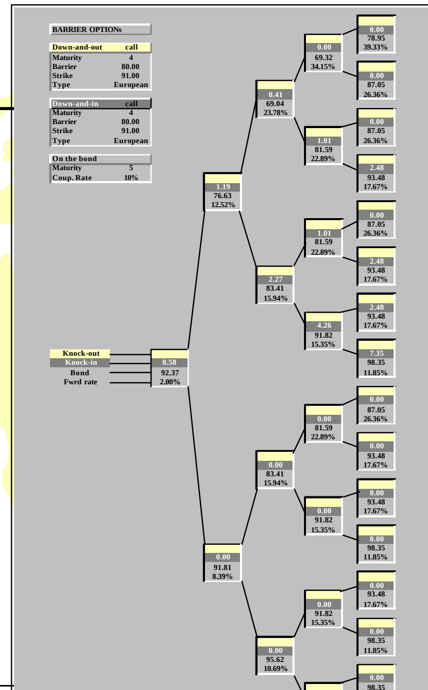
Discounting the *certain outcome* of $4.5 by the risk-free rate (10%) and subtracting cost of Δ bills leads to the call price: *PV(4.5)-5, ie $0.91*.

Plain Vanilla



Barrier Options

This *Down-and-In option* is an example of path-dependent options, where *how you get there* matters.



Options

- Puts and Calls
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- Combinations and Trading Strategies
- Valuation
- Hedging
- Valuation of Exotics

Option Valuation: Applications

- Warrant bonds
- Convertibles
- Callable bonds
- *Corporate valuation*

What's a Company Worth?

IBM

- Required returns
- Types of Models
  - Balance sheet models
  - Dividend discount models
  - Corporate cash flow models
  - Price/Earnings ratios
- Estimating Growth Rates
- Application

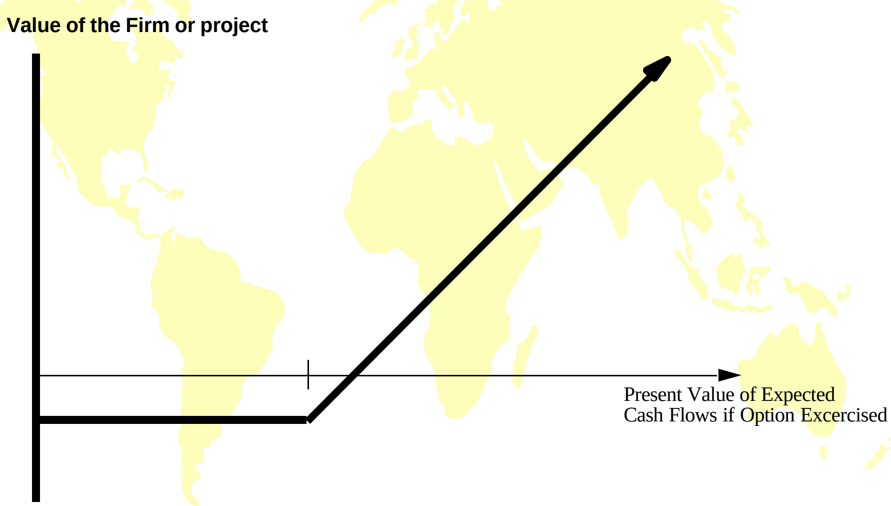
What's a Company Worth? Alternative Models

- The options approach
  - Option to expand
  - Option to abandon
- Creation of key resources that another company would pay for
  - Patents or trademarks
  - Teams of employees
  - Customers
- *Examples?*

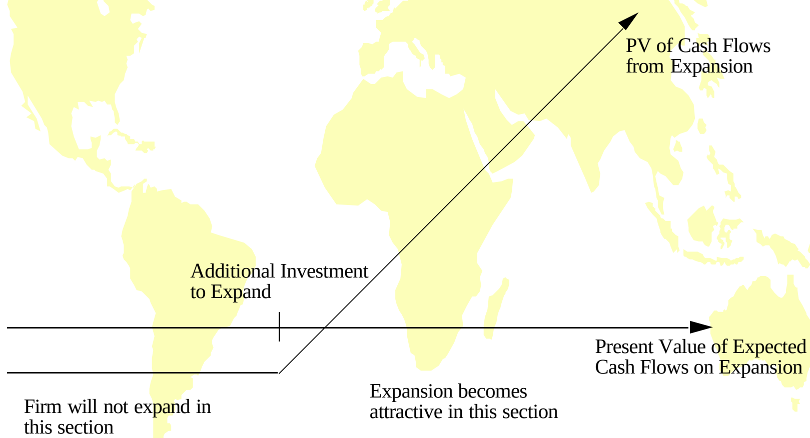
Lycos

Messageclick.com

What's a Company Worth? The Options Approach



The Option to Expand



An Example of a Corporate Option

- J&J is considering investing $110 million to purchase an internet distribution company to serve the growing on-line market.
- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to J&J will be only $80 million. Thus, by itself, the corporate venture has a **negative NPV of $ 30 million**.
- If the on-line market turns out to be more lucrative than currently anticipated, J&J **could expand** its reach a global on-line market with **an additional investment of $ 150 million** any time over the next 2 years. While the current expectation is that the cash flows from having a worldwide on-line distribution channel is only $100 million, there is considerable uncertainty about both the potential for such an channel and the shape of the market itself, leading to significant variance in this estimate.
- ***This uncertainty is what makes the corporate venture valuable!***

Valuing the Corporate Venture Option

- Value of the underlying asset (S) = PV of cash flows from purchase of on-line selling venture, if done now =$100 Million
- Strike Price (K) = cost of expansion into global on-line selling = $150 Million
- We estimate the variance in the estimate of the project value by using the annualized variance in firm value of publicly traded on-line marketing firms in the global markets, which is approximately 20%.
  - Variance in Underlying Asset's Value = 0.20
- Time to expiration = Period for which "venture option" applies = 2 years
- 2-year interest rate: 6.5%

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$d_1 = [\ln(S_o/X) + (r + \sigma^2/2)T] / (\sigma T^{1/2})$

$d_2 = d_1 - (\sigma T^{1/2})$

where

S_o = Current stock price

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N(d) = probability that a random draw from a normal distribution will be less than d.

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- We estimate the variance in the estimate of the project value by using the annualized variance in firm value of publicly traded on-line marketing firms in the global markets, which is approximately 20%.
  - Variance in Underlying Asset's Value = 0.20
- Time to expiration = Period for which "venture option" applies = 2 years
- 2-year interest rate: 6.5%

Call Value= 100 (0.7915) -150 (exp(-0.065)(2) (0.3400)

= $ 52.5 Million

Summary

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- Hedging
- Valuation of Exotics
- Applications to corporate securities and corporate valuation