

# Chapter 1

## Incomplete Contracts

### 1.1 Introduction

- **Complete Contracts:** Arrow-Debreu contracts. Contracts that condition on every possible state of the world. Need not be optimal contracts. Second-best contracts under moral hazard and adverse selection are not complete since not based on agent's type  $\theta$  or action  $a$ .
- **Comprehensive Contracts:** Contracts that make optimal use of all commonly observable information. Second-best contracts under moral hazard and adverse selection are comprehensive. Optimality (i.e. contract ensues from (principal's) optimization problem) guarantees that contract need not be renegotiated.
- **Remark:** If information is commonly observable but not verifiable, comprehensive contract would be based on outcome of mechanism.
- **Remark:** Complete contracts need not be comprehensive. Example: State-contingent sales contract that defines that buyer receives 5 widgets in every possible state of the world. But 5 widgets may not always be optimal  $\Rightarrow$  contract will be renegotiated.
- **Incomplete Contracts:** Do not optimally use all commonly observable information. Will be renegotiated at some point in time (bargaining).
- Reasons for contractual incompleteness:
  1. **Bounded rationality:** Parties cannot write long-term state-contingent contracts. Cannot foresee all possible future states.

2. **Transaction cost:** Conditioning long-term contract on all possible future states prohibitively costly.
  3. **Nonverifiability:** Some information observable but not verifiable vis-a-vis court and mechanism cannot be used (for instance, because information concerns quality attribute that is hard to measure).
- Typical assumptions:
    - Long-term state-contingent contracts not possible. After state of the world is realized, spot contracts possible with no transaction cost. Spot contracts result from renegotiation (bargaining) process.
    - But possible to write ex ante contracts regarding **governance** or **control structure:** Rules that determine how conflicts should be resolved, who has bargaining power, how renegotiation process should be designed, etc.

## 1.2 Theory of the Firm

- Central question: What determines the **boundaries of the firm**? Why do some firms merge but others don't? Why are there many firms and not just one large firm?

### Neoclassical Approach

- Firm as a black box (production function). Optimal "firm" size  $q^*$  determined by minimum of average cost curve (under perfect competition). Average cost rise after some point due to fixed factor: managerial talent. As firm becomes larger, manager's productivity decreases.
- More a theory of optimal plant size than of optimal firm size. Suppose two firms (plants) merge and become divisions of large firm. Nothing would be changed. Each division produces at  $q^*$  but large firm produces at  $2q^*$ . Neoclassical theory consistent with one big firm that comprises all firms as subdivisions.
- No disadvantage of merger, but possible benefits: Large firm employs CEO who controls subdivisions. Can let subdivisions produce independently  $\Rightarrow$  welfare same as if subdivisions were independent firms. But possible to **selectively intervene** and coordinate production if

Pareto-improvement. Hence large firm never worse off but sometimes better off than independent firms (**Williamson puzzle**). So why isn't there just one big firm?

### Principal-Agent Approach

- Neoclassical view ignores incentive problems and internal structure of firms (black box). Suppose incentive problems (moral hazard) between buyer and supplier. Better if firms merge? If incentive contract possible, then doesn't matter for efficiency whether buyer and supplier are independent firms or divisions of same firm. **Comprehensive contracts cannot explain boundaries of the firm.**
- Caveat: Assumed that after merger, principal's (i.e. buyer's) ability to monitor agent's effort remains unchanged. But if monitoring employee is easier than monitoring independent contractor, mergers do matter!

### Transaction Cost Approach

- **Coase (1937)**: Transactions within a firm and between firms governed by different mechanisms:
  - Within a firm: authority, commands, etc.
  - Between firms: price mechanism.

Market mechanism entails coordination cost. Coase therefore argues that with small number of transactions, authority better. As firm gets bigger, costs of authority rise exponentially: Managers make mistakes, bureaucracy costs, etc. Optimal firm size minimizes **transaction cost**.

- Objections:
  1. Firms can simulate market mechanism: Transactions between divisions can take place through price mechanism (transfer pricing). So why is there not just one firm where each division has optimal (transaction cost-minimizing) size? Williamson puzzle not solved!
  2. **Alchian and Demsetz (1972)**: Distinction between command system and market system artificial: If boss threatens to fire worker, same as if buyer threatens to terminate relationship with seller.

- **Williamson (1975, 1985), Klein, Crawford and Alchian (1978):** Particular source of transaction cost originates with **hold-up problem**: One party exploits fact that other party has sunk relationship-specific investment. Consider buyer-seller relationship. Seller can produce good at zero cost.
  - At date 0, buyer can make relationship-specific investment of \$ 0 or 60. Investment is nonverifiable (e.g. investment in human capital).
  - If buyer invests 0, utility from transaction with seller is 0. If buyer invests 60, utility is 100.
  - Conclusion: Investment is efficient ( $100 - 60 = 40 > 0$ ) and should be carried out.
  - Buyer and seller cannot write long-term contract regarding price of investment.
  - Question: Will buyer invest? Suppose buyer invests.
  - At date 1, investment is sunk and buyer's utility from transaction is 100. Parties will bargain over price. Nash bargaining yields 50 : 50 division of surplus: Buyer makes loss of  $50 - 60 = -10 \Rightarrow$  buyer will not invest.
- Ingredients of hold-up problem:
  1. **Nonverifiable investment:** If investment was verifiable, seller could pay buyer 20 conditional upon investment. Buyer would get  $50 - 60 + 20 = 10$  and invest.
  2. **Relationship-specific investment:** If investment was not specific, buyer could threaten to trade with other seller unless he sells for 20. Buyer would get  $100 - 20 - 60 = 20$  and invest.
  3. **Sunk investment:** If investment was not sunk, buyer could threaten to undo investment unless seller sells for 20.
  4. **Impossibility to write long-term contract:** If long-term contract was possible, parties could set price at 20.
- Solution to hold-up problem: Buyer and seller merge (**vertical integration**). At date 0, buyer buys seller's firm for 20. At date 1, transaction takes place (at price of 0) and buyer gets  $100 - 60 - 20 = 20 \Rightarrow$  buyer invests.

- Does transaction cost theory solve the Williamson puzzle? Not really:
  1. Does not explain why buyer's bargaining power increases after vertical integration (above assumed that seller delivers at price of 0, i.e. buyer has full bargaining power). Seller could threaten to quit  $\Rightarrow$  parties bargain (within firm)  $\Rightarrow$  same problem as under non-integration. Mechanism that determines bargaining power as consequence of integration must be spelled out more clearly!
  2. Does not explain disadvantages of integration. So why isn't there just one fully integrated firm? (Williamson names bureaucracy cost as cost of integration. But argued earlier that this doesn't solve puzzle).

### Property Rights Approach

- **Grossman and Hart (1986), Hart and Moore (1990):** Also use hold-up problem as main explanation for integration, but provide more satisfactory answers to questions 1 & 2.
- **Definition 1 (Ownership):** Ownership of (physical) asset is **residual right of control** over asset, i.e. right to determine use of asset in contingencies not governed by explicit contract. In particular, right to exclude others from use of asset.
- **Remarks:**
  1. Before Grossman-Hart-Moore (GHM), ownership mainly regarded as right to residual income.
  2. Residual control rights matter only if contracts are incomplete. When comprehensive contracts possible, optimal use of asset can be specified in all contingencies.
  3. Control over physical assets can lead indirectly to control over human assets if physical assets necessary for person to be productive.
- **Definition 2 (Firm):** A firm consists of its physical assets.
- Why do GHM give more satisfactory answers to questions 1 & 2?
  1. If buyer and seller are independent, seller can threaten to withhold productive assets and personal service. If buyer buys seller's firm, he possesses seller's assets. Seller can now only threaten to quit

(i.e. withhold personal service). If assets can also be operated by other agents (i.e. seller is **dispensable**), seller's threat to quit is less consequential  $\Rightarrow$  buyer has strong bargaining power. If seller is **indispensable**, seller's bargaining power is strong, even though buyer owns assets. GHM explain factors that determine ex post bargaining power.

2. Suppose seller can also make relationship-specific ex ante investment. If buyer owns seller's assets, buyer's ex post bargaining power is strong  $\Rightarrow$  buyer may make investment, but seller may not invest for fear of expropriation ex post. Hence, **benefits of integration** are better ex ante incentives for buyer. **Costs of integration** are reduced ex ante incentives for seller. Reverse is true if seller owns buyer. Who should own whom depends on parameters of model: Whose investment decision is more important, who is dispensable, etc. May even be optimal for firms to stay independent. GHM view costs and benefits of integration as two sides of same coin, not like Williamson who evokes bureaucracy costs to explain costs of integration.

### 1.3 Hart-Moore (1990)

- Hart, O.D., and Moore J.H. (1990), "Property Rights and the Nature of the Firm," *Journal of Political Economy* **98**, 1119-1158.
- General model of **asset ownership** (not only vertical integration).
  - Many agents (not just buyer and seller). Physical assets already in place. Question is who should own them.
  - Some agents invest at date 0 in human capital. Investment is asset-specific, i.e. yields surplus only if at date 1 agents have access to particular assets.
  - Investments nonverifiable, long-term contracts not possible.
  - At date 1, bargaining over surplus. Agent's bargaining position depends on ability to generate surplus and hence on assets he has access to (doesn't mean that he has to own them). Implies that if agent does not own particular asset, agent's bargaining power depends on *who* owns this asset (example A: Suppose cook does not own yacht. If tycoon owns yacht, cook's bargaining position is stronger (must only bargain with tycoon) than if skipper owns yacht (then cook must bargain with tycoon and skipper)).

**The Model**

1. Set  $\underline{S} = \{1, \dots, I\}$  of risk neutral agents.
2. Set  $\underline{A} = \{a_1, \dots, a_N\}$  of assets in place.
3. Date 0: Relationship-specific investments in human capital  $x_i \in [0, \bar{x}_i]$ , where  $\bar{x}_i > 0$ . Investments observable by all parties but not verifiable vis-a-vis courts.
4. Agents incur investment cost  $C_i(x_i) \geq 0$ , where  $C_i(\cdot)$  is increasing, strictly convex, and twice differentiable with  $\lim_{x_i \rightarrow 0} C'_i(\cdot) = 0$  and  $\lim_{x_i \rightarrow \bar{x}_i} C'_i(\cdot) = \infty$ .
5. Date 0: Parties can define (and contractually commit to) **control structure**  $\alpha(S)$  that specifies subset of assets  $A \subseteq \underline{A}$  controlled by coalition  $S \subseteq \underline{S}$ . If coalition controls asset, any member in coalition has access to it. Any asset can be controlled by at most one coalition. Example: Agent  $i$  owns asset  $a_n : a_n \in \alpha(S)$  if and only if  $i \in S$ . If assets can be jointly owned, then asset is controlled by coalition that includes majority owner(s).
6. Date 0: Parties cannot write contract specifying trade at date 1  $\Rightarrow$  outcome from trade at date 1 determined by bargaining. Since date 0-investments observable, bargaining under symmetric information.
7. Date 1: Coalitions can form and trade (bargain) with each other. Let  $v(S, A|x) \geq 0$  denote potential gains from trade (surplus, value) of coalition  $S$  if coalition controls assets  $A$  and investments  $x = (x_1, \dots, x_N)$  were made at date 0.
8.  $v(\cdot)$  is twice differentiable and concave in  $x$  with  $v(\emptyset, A|x) = 0$ .
9.  $\frac{\partial v(S, A|x)}{\partial x_i} \equiv v^i(S, A|x) > 0$  if  $i \in S$  and  $v^i(S, A|x) = 0$  if  $i \notin S$ , i.e. additional investment by agent  $i$  increases value of coalition if and only if agent  $i$  is member of coalition ( $\Rightarrow$  investment in human capital; coalition can only benefit from investment by including agent  $i$ ).
10.  $\frac{\partial}{\partial x_j} v^i(S, A|x) \geq 0 \forall j \neq i$ , i.e. additional investment by agent  $i$  increases marginal productivity of investments by other agents in coalition ( $\Rightarrow$  investments complementary at the margin).
11. Superadditivity:  $v(S, A|x) \geq v(S', A'|x) + v(S \setminus S', A \setminus A'|x) \forall S' \subseteq S$  and  $A' \subseteq A$ . Intuition: If not satisfied, coalition  $S$  could create greater

value by splitting up into coalitions  $S'$  and  $S \setminus S'$ . Implies that total surplus (value) is maximized in grand coalition  $\underline{S}$ .

12.  $v^i(S, A|x) \geq v^i(S', A'|x) \forall S' \subseteq S$  and  $A' \subseteq A$ . Implies that marginal productivity of agents in coalition cannot fall if additional agents and/or assets are included in coalition. Together, assumptions 11 and 12 imply that total and marginal returns go in same direction (i.e. are positively correlated) when number of agents or assets in coalition increases. Without this assumption, can get overinvestment in addition to underinvestment.

13. Agent  $i$ 's expected return from date 1-bargaining for given control structure  $\alpha$  and given vector of investments  $x$  is denoted by  $B_i(\alpha|x)$ .  $B_i(\alpha|x)$  is defined as agent  $i$ 's **Shapley value**

$$B_i(\alpha|x) \equiv \sum_{S|i \in S} p(S) [v(S, \alpha(S)|x) - v(S \setminus \{i\}, \alpha(S \setminus \{i\})|x)],$$

where

$$p(S) = \frac{(|S| - 1)! (I - |S|)!}{I!}$$

and where  $|S|$  is number of agents in coalition  $S$ .

14. Shapley value gives agent  $i$ 's expected contribution to a coalition ("a" refers to coalition in general, not to specific one). Computed as agent  $i$ 's (marginal) contribution to specific coalition  $S$ , weighted with probability  $p(S)$  that coalition  $S$  will form, summed over all coalitions where agent  $i$  is part of.

15. Simple way to determine Shapley value for agent  $i$  :

- Write down all permutations of  $\underline{S}$ .
- In each permutation,  $S \setminus \{i\}$  is group of agents that are placed before (to left) of  $i$ .
- Compute  $v(S, \alpha(S)|x) - v(S \setminus \{i\}, \alpha(S \setminus \{i\})|x)$  for every permutation ( $S$  is coalition of agents left to  $i$  plus agent  $i$ ).
- Multiply  $v(S, \alpha(S)|x) - v(S \setminus \{i\}, \alpha(S \setminus \{i\})|x)$  with probability of permutation  $\frac{1}{I!}$ .
- Sum over all permutations.

**First-Best**

By assumption 11, maximum surplus (for given investment level  $x$ ) is attained in grand coalition  $\underline{S}$  with all assets  $\underline{A}$ . Set  $v(\underline{S}, \underline{A}|x) = V(x)$ . First-best problem is

$$\max_x V(x) - \sum_{i=1}^I C_i(x_i) \quad (1.1)$$

with FOC

$$v^i(\underline{S}, \underline{A}|x_{FB}^*) = C_i'(x_{iFB}^*) \quad (1.2)$$

for all  $i$ . Given our assumptions,  $x_{FB}^*$  is unique interior maximum. FOC equates marginal return (increase in total surplus) and marginal cost of agent  $i$ 's investment.

**Second-Best**

At date 0, agents choose investment levels  $x_i$  non-cooperatively, anticipating date 1-payoffs  $B_i(\alpha|x)$ .

**Theorem 1** For any control structure  $\alpha$ , unique Nash equilibrium is  $x_i^*(\alpha) \leq x_{iFB}^*$  for all  $i$ , i.e. each agent underinvests.

**Proof (Sketch):** Take control structure  $\alpha$  and investments  $x_{-i}^*(\alpha)$  by other agents as given. Agent  $i$  chooses  $x_i$  to maximize expected profits  $B_i(\alpha|x) - C_i(x_i)$ . FOC is

$$\frac{\partial B_i(\alpha|x^*(\alpha))}{\partial x_i} = \sum_{S|i \in S} p(S) [v^i(S, \alpha(S)|x^*(\alpha))] = C_i'(x_i^*(\alpha)). \quad (1.3)$$

But by assumption 12,  $v^i(S, \alpha(S)|x^*(\alpha)) \leq v^i(\underline{S}, \underline{A}|x^*(\alpha))$ , which implies

$$\begin{aligned} \sum_{S|i \in S} p(S) [v^i(S, \alpha(S)|x^*(\alpha))] &\leq \sum_{S|i \in S} p(S) [v^i(\underline{S}, \underline{A}|x^*(\alpha))] \\ &= v^i(\underline{S}, \underline{A}|x^*(\alpha)) \\ &= \frac{\partial V(x^*(\alpha))}{\partial x_i}, \end{aligned} \quad (1.4)$$

i.e. at Nash equilibrium, agent  $i$ 's marginal return on investment  $\frac{\partial B_i(\alpha|x^*(\alpha))}{\partial x_i}$  is less than or equal socially efficient marginal return  $\frac{\partial V(x^*(\alpha))}{\partial x_i} \Rightarrow$  agent  $i$  underinvests. ■

**Remarks**

1. In paper, Hart-Moore refer to result as underinvestment. But proof only shows weak inequality, i.e. second-best investment may be efficient. This is unsatisfactory!
2. Intuition same as in team-production problem: For additional investment, agent  $i$  bears full marginal cost, but loses part of marginal gains to others in bargaining. Since externalities are not taken into account, agents underinvest.
3. Without assumption 12, could have overinvestment (as in Grossman-Hart '86).

**Optimal Asset Ownership**

Theorem 1 shows that for any control structure  $\alpha$ , get underinvestment. Now seek control structure that minimizes inefficiency (i.e. maximizes second-best welfare). In other words, seek ownership arrangement that provides agents with (second-best) optimal incentives to invest.

**Example A:**

- 1 asset: luxury yacht.
- 3 agents: tycoon (T), chef (C), skipper (S).
- Service is to provide gourmet dinner for tycoon during sea cruise at date 1.
- At date 0, chef can invest in preparing fancy Californian cuisine (tycoon went to Stanford and loves Californian cuisine). Cost of investment is 100, value to tycoon is 240  $\Rightarrow$  investment is efficient.
- Substitute for skipper easy to find at date 1  $\Rightarrow$  skipper dispensable.
- Only tycoon values chef's investment  $\Rightarrow$  tycoon indispensable & chef's investment relationship-specific.
- Who should own yacht?

Case 1: Skipper owns yacht.

			<i>T</i>	<i>C</i>	<i>S</i>
<i>T</i>	<i>C</i>	<i>S</i>	0	0	240
<i>T</i>	<i>S</i>	<i>C</i>	0	240	0
<i>S</i>	<i>T</i>	<i>C</i>	0	240	0
<i>S</i>	<i>C</i>	<i>T</i>	240	0	0
<i>C</i>	<i>T</i>	<i>S</i>	0	0	240
<i>C</i>	<i>S</i>	<i>T</i>	240	0	0
			80	80	80

- In each row, entries depict marginal contributions of agents to coalition represented by permutation in same row. Agents can only enter coalitions from the right.
- Take first row and start at left cell. Tycoon can enter coalition  $\emptyset$  (no agent to the left of tycoon). Whether he enters or not, value of coalition is 0. Hence, tycoon’s marginal contribution is 0. Next, take chef. Can leave tycoon alone or form coalition. In either case, value of coalition (and hence chef’s marginal contribution) is 0 because yacht is needed to generate the 240. Finally, take skipper. Can join coalition by tycoon and chef or stay out. If he joins, value of coalition is 240; if he stays out, value is 0. Thus, skipper’s marginal contribution is 240.
- In this example, all three agents are needed to create positive surplus. Therefore, only agent who enters last (third column) has positive marginal contribution.
- Each permutation occurs with probability  $1/I! = 1/6$ . Multiplying each cell in agent  $i$ ’s column with  $1/6$  yields agent  $i$ ’s expected marginal contribution or Shapley value. Here, Shapley value is 80 for each agent.
- Can also derive Shapley value with formula given earlier. Take tycoon. Tycoon can be in four possible coalitions:  $\{T\}$ ,  $\{C, T\}$ ,  $\{S, T\}$ , or  $\{C, S, T\}$ . Marginal contribution to  $\{T\}$ ,  $\{C, T\}$ ,  $\{S, T\}$  is 0; marginal contribution to  $\{C, S, T\}$  is 240. Probabilities are  $p(T) = \frac{(1-1)!(3-1)!}{3!} = 1/3$ ,  $p(C, T) = 1/6$ ,  $p(S, T) = 1/6$ , and  $p(C, S, T) = 1/3$ . Shapley value is  $(1/3)0 + (1/6)0 + (1/6)0 + (1/3)240 = 80$ .
- Intuition for Shapley value: All three agents are needed to generate surplus:
  - Tycoon because he consumes gourmet meal.

- Skipper because he owns yacht.
- Chef because he made investment.

Consequence: Agents split surplus by three - each agent gets 80.

- Will chef invest? No. Expected payoff of 80 does not cover investment cost of 100.

*Case 2: Tycoon owns yacht.*

			<i>T</i>	<i>C</i>	<i>S</i>
<i>T</i>	<i>C</i>	<i>S</i>	0	240	0
<i>T</i>	<i>S</i>	<i>C</i>	0	240	0
<i>S</i>	<i>T</i>	<i>C</i>	0	240	0
<i>S</i>	<i>C</i>	<i>T</i>	240	0	0
<i>C</i>	<i>T</i>	<i>S</i>	240	0	0
<i>C</i>	<i>S</i>	<i>T</i>	240	0	0
			120	120	0

- Chef invests since Shapley value is  $120 > 100$ .
- Intuition: Skipper is dispensable  $\Rightarrow$  no bargaining power. Tycoon and chef divide surplus by two.
- Note: Chef owns yacht in neither case. Nonetheless, investment decision is different whether tycoon or skipper owns yacht. If tycoon is owner, chef must only bargain with tycoon - surplus split by two. If skipper is owner, chef must bargain with both tycoon and skipper - surplus split by three and chef doesn't get enough to cover investment cost.

*Case 3: Chef owns yacht.*

- Same as case 2. Again, only tycoon and chef needed to generate surplus.

In example A, tycoon and chef are always needed to generate surplus: Tycoon because he is indispensable, and chef because he makes investment. Consider now each of these factors separately.

**Theorem 2** If only one agent invests, he should own all assets.

**Proof** Only inefficiency is underinvestment. Thus, choose control structure that maximizes agent  $i$ 's (investor's) incentives to invest. His marginal expected return from investment is

$$\frac{\partial B_i(\alpha)}{\partial x_i} = \sum_{S|i \in S} p(S) v^i(S, \alpha(S)). \quad (1.5)$$

By assumption 12, right-hand side is maximized when  $\alpha(S) = \underline{A} \forall S : i \in S$ . ■

- **Intuition:** As owner, agent has access to all assets. Strongest possible bargaining position ex post  $\Rightarrow$  maximum incentives to invest ex ante.

**Definition 3** Agent  $i$  is **indispensable** to asset  $a_n$  if  $\forall j \in S$  and  $\forall$  sets  $\{A | a_n \in A\}$ ,

$$v^j(S, A) \equiv v^j(S, A \setminus \{n\}) \quad (1.6)$$

if  $i \notin S$ .

That is, agent  $i$  is indispensable to asset  $a_n$  if asset  $a_n$  does not improve marginal productivity of any member in coalition unless agent  $i$  is also in coalition.

**Theorem 3** If agent  $i$  is indispensable to asset  $a_n$ , he should own  $a_n$ .

**Proof** Suppose  $i$  doesn't own  $a_n$  under control structure  $\alpha$ . Consider new control structure  $\hat{\alpha}$  where  $a_n$  is owned by  $i$ . Agent  $i$ 's marginal incentives to invest cannot fall under  $\hat{\alpha}$ . Consider now coalitions that had access to  $a_n$  but did not include  $i$ . None of these coalitions is worse off, because by definition 3,  $a_n$  did not affect marginal productivity of coalition members. Finally, consider coalitions that included  $i$  but had no access to  $a_n$ . These coalitions are better off because by assumptions 10 and 12, marginal productivity of their members rises. All other coalitions are unaffected. ■

**Example B:**

- In addition to chef, skipper can also make investment at date 0 (can learn history of local islands to entertain tycoon with anecdotes).
- Cost of skipper's investment is 100, value to tycoon is another 240. Hence, if both skipper and chef invest, value to tycoon is 480.

*Case 1: Chef owns yacht.*

			<i>T</i>	<i>C</i>	<i>S</i>
<i>T</i>	<i>C</i>	<i>S</i>	0	240	240
<i>T</i>	<i>S</i>	<i>C</i>	0	480	0
<i>S</i>	<i>T</i>	<i>C</i>	0	480	0
<i>S</i>	<i>C</i>	<i>T</i>	480	0	0
<i>C</i>	<i>T</i>	<i>S</i>	240	0	240
<i>C</i>	<i>S</i>	<i>T</i>	480	0	0
			200	200	80

- Given that skipper invests, chef's Shapley value is 200 if he invests and 80 if he doesn't (to see where the 80 come from, replace "skipper" with "chef" in example A, case 1). Hence, chef's marginal gain from investing is 120  $\Rightarrow$  chef invests but skipper doesn't.

*Case 2: Skipper owns yacht.*

- Symmetric to case 1  $\Rightarrow$  skipper invests but chef doesn't.

*Case 3: Tycoon owns yacht.*

			<i>T</i>	<i>C</i>	<i>S</i>
<i>T</i>	<i>C</i>	<i>S</i>	0	240	240
<i>T</i>	<i>S</i>	<i>C</i>	0	240	240
<i>S</i>	<i>T</i>	<i>C</i>	240	240	0
<i>S</i>	<i>C</i>	<i>T</i>	480	0	0
<i>C</i>	<i>T</i>	<i>S</i>	240	0	240
<i>C</i>	<i>S</i>	<i>T</i>	480	0	0
			240	120	120

- Skipper and chef both receive 120 – 100 > 0 as opposed to 0 if they hadn't invested  $\Rightarrow$  both invest.
- Result: If tycoon is indispensable, he should own yacht even though he makes no investment. Shows that investment is not necessary condition for ownership.

**Definition 4** Two assets  $a_n$  and  $a_m$  are **complementary** if  $\forall S$  and  $\forall$  sets  $\{A \mid a_n, a_m \in A\}$ ,

$$v^i(S, A \setminus \{a_n\}) \equiv v^i(S, A \setminus \{a_m\}) \equiv v^i(S, A \setminus \{a_n, a_m\}) \quad (1.7)$$

if  $i \in S$ .

That is, two assets are complementary if they are unproductive unless used together.

**Theorem 4** Complementary assets should be controlled together.

**Proof** Suppose under control structure  $\alpha$ , two complementary assets  $a_n$  and  $a_m$  are not controlled together. Consider new control structure  $\hat{\alpha}$  where  $a_n$  and  $a_m$  are controlled together. Coalitions that used to control  $a_n$  ( $a_m$ ) but not  $a_m$  (not  $a_n$ ) are not worse off by losing  $a_n$  ( $a_m$ ) by definition 4. However,  $\exists$  coalitions that used to control only  $a_n$  ( $a_m$ ) but now control both assets. These coalitions are better off by assumption 12. All other coalitions are unaffected. ■

**More Results (without proof):**

1. Each asset should be controlled by one coalition.
2. **Joint ownership** is suboptimal.
3. If asset that is **essential** to agent is owned by second agent, then former agent's incentives to invest increase if all other assets are also owned by second agent.