The work here is based on the data file ANIMALS.mpj.

We would like to find a relationship between Gestation and independent variables \{weight, Tsleep\}.

Gestation is the time in days between conception and birth.
weight is the adult weight in kilograms
Tsleep is the number of hours, out of 24, that the animal sleeps

The conclusions here depend, of course, on what animals were included in the data set.

As a preliminary, animals with incomplete information on these variables were removed.

We get a matrix plot through Graph \(\Rightarrow\) Matrix plot. This reveals that weight needs logs. Using Calc \(\Rightarrow\) Calculator, we create \(\text{LOGE(weight)}\) and name it \(\text{log(W)}\).

Using Stat \(\Rightarrow\) Regression \(\Rightarrow\) Regression, we regress Gestation on \{log(W), Tsleep\}. The residual versus fitted plot suggests that we should take the logarithms of Gestation. The name \(\text{log(G)}\) was used for \(\text{LOGE(Gestation)}\).

Here is the regression of \(\text{log(G)}\) on \{log(W), Tsleep\}:

\[
\text{Regression Analysis: log(G) versus log(W), Tsleep}
\]

The regression equation is
\[
\text{log(G)} = 5.01 + 0.178 \text{ log(W)} - 0.0773 \text{Tsleep}
\]

\[
\begin{array}{l|c|c|c|c|c}
\text{Predictor} & \text{Coef} & \text{SE Coef} & \text{T} & \text{P} \\
\hline
\text{Constant} & 5.0135 & 0.2959 & 16.95 & 0.000 \\
\text{log(W)} & 0.17754 & 0.03585 & 4.95 & 0.000 \\
\text{Tsleepe} & -0.07728 & 0.02424 & -3.19 & 0.002 \\
\end{array}
\]

\[S = 0.652116 \quad \text{R-Sq} = 62.2% \quad \text{R-Sq(adj)} = 60.7%\]

Analysis of Variance

\[
\begin{array}{l|c|c|c|c|c|c}
\text{Source} & \text{DF} & \text{SS} & \text{MS} & \text{F} & \text{P} \\
\hline
\text{Regression} & 2 & 35.709 & 17.854 & 41.98 & 0.000 \\
\text{Residual Error} & 51 & 21.688 & 0.425 & & \\
\text{Total} & 53 & 57.397 & & & \\
\end{array}
\]

Let’s examine the estimated coefficient 0.178. The “official” interpretation is

a one unit change in log(W) leads to a change of 0.178 in log(G), holding Tsleep constant
There’s another way to look at this. Let 
\[ \log(G)[\text{Tsleep}] \] be the residual from regressing \( \log(G) \) on Tsleep
That is, this is the part of \( \log(G) \) that cannot be explained by Tsleep.

In similar fashion, let 
\[ \log(W)[\text{Tsleep}] \] be the residual from regressing \( \log(W) \) on Tsleep
That is, this is the part of \( \log(W) \) that cannot be explained by Tsleep.

Now consider the regression of \( \log(G)[\text{Tsleep}] \) on \( \log(W)[\text{Tsleep}] \). This is

**Regression Analysis: log(G)[Tsleep] versus log(W)[Tsleep]**

The regression equation is
\[ \log(G)[\text{Tsleep}] = -0.0000 + 0.178 \log(W)[\text{Tsleep}] \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coef</th>
<th>SE Coef</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.00000</td>
<td>0.08788</td>
<td>-0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>\log(W)[\text{Tsleep}]</td>
<td>0.17754</td>
<td>0.03551</td>
<td>5.00</td>
<td>0.000</td>
</tr>
</tbody>
</table>

\[ S = 0.645815 \quad R-Sq = 32.5\% \quad R-Sq(adj) = 31.2\% \]

Analysis of Variance

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
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<td>10.429</td>
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<td>0.000</td>
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<tr>
<td>Residual Error</td>
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<td>0.417</td>
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<tr>
<td>Total</td>
<td>53</td>
<td>32.117</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the slope coefficient is 0.178!

In a multiple regression, let
\( Y \) be the dependent variable
\( X \) be one of the independent variables
\( U \) be the set of all other independent variables

In the regression of \( Y \) on \( \{X, U\} \), the estimated coefficient on \( X \) is equal to the estimated coefficient in the simple regression of

\[ \{ \text{residuals in the regression of } Y \text{ on } U \} \]
on
\[ \{ \text{residuals in the regression of } X \text{ on } U \} \]