Preference Discovery in Product Search

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Daria Dzyabura
NYU Stern School of Business
40 West 4th Street
New York, NY 10012
ddzyabur@stern.nyu.edu
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Abstract

Novice consumers, even when rational, sometimes discover and revise their preferences for certain product attributes as they evaluate products. Such preference discovery modifies consumers’ decisions with respect to (1) which products to evaluate in optimal sequential search and (2) which products to purchase. We develop a model of sequential search with preference discovery, and derive several key insights. First, we show that novice consumers may achieve higher rewards net of search cost when there is more noise in initial beliefs about product attributes. For expert consumers, less noise is better. Second, product recommendations can influence the future search path of the consumer, not merely introduce the recommended product into their consideration set. If a product exposes the consumer to a new, previously undiscovered attribute, the consumer may shift his search to a new part of the product space. Third, the value of a product recommendation is not necessarily directly related to the quality of the product. Counter to the literature on recommender systems, it is sometimes better to recommend an inferior product with the goal of helping consumers discover preferences. Non-benevolent agents can direct consumers to profitable products even though the consumer believes he is searching endogenously and rejects the agent’s recommendation. Finally, I present data from an incentive-aligned experiment to suggest that real consumers discover preferences.

Keywords: Product Recommendations, Probability Models, Search, Decision making under uncertainty, Choice models
1. When Consumers Don’t Know What They Don’t Know

Novice consumers might discover and revise their preferences for certain product attributes as they search. When my spouse and I moved to a new city, we knew we needed childcare. We began to interview nannies. Although we were experienced parents, we were novices in hiring nannies. Initially we thought that the most important attribute was experience with daycare or with a previous family. We contacted daycare centers, colleagues whose children had now grown, and other sources of experience. As we searched, we came to value experience less and empathy more. In the end we realized that a willingness to help with nominal household chores was a make or break attribute, not only because it helps our children, but it gives us more quality time with our children. Household chores played no role whatsoever in our early search. As novices in hiring nannies, our search process changed as we learned our preferences, some for completely-unanticipated attributes. If we ever move again, we will be experts in evaluating nannies; our search process is likely to closer to that of optimal search with unchanging preferences. But, then again, we don’t know what we don’t know about childcare for older children.

This anecdote describes a phenomenon that is more general than the search for childcare providers. When high school students begin their college search, they often evaluate only colleges their parents attended, or only those with nationally ranked athletic teams, or only those that are nearby. High school students are well-informed about the attributes they search, but do not realize that other attributes may be more important. Indeed, college counselors often suggest that “students look at schools of various types for the express purpose of helping them refine their thinking about what they want” (personal communication, November 2012). In dating, young men and women do not always know what they will value in a potential partner, even though they usually believe they have well-formed preferences (Finkel et al 2012). Some newer dating services such as Minidates.com and CoffeeMeetsBagel.com offer blind dates to encourage real-life interaction so that daters learn what really matters in a mate. The websites claim that the final outcome is better than it would have been if daters had foregone the discovery process and relied instead on the attributes in online profiles. Other service examples include the choice of a personal lawyer and the choice of a personal care physician.

Preference discovery also applies to products. In a B2B context an entrepreneur needs to
purchase production equipment with little experience in the attributes of, say, automatic screen printing presses. Hauser, Dong, and Ding (2014) present anecdotes and systematic evidence that automotive consumers learn their preferences as they evaluate automotive profiles in conjoint analysis and that these preferences stabilize after they are learned. A recent New York Times article on real estate buyer behavior by Rogers (2013, page F4) suggests “Often people don’t know what they want. […] You may think you want X, but if you’re shown Y, you may love Y better than you ever loved X.” This was reinforced when I spoke to a Boston real estate agent: “Often what people start out thinking they want is not what they end up wanting (private communication, 2012).”

The common thread in all of these examples is that preferences are discovered during search. One on hand, the product or service that is optimal based on a novice consumer’s preferences may not be optimal after the novice becomes more expert (in preferences) as he or she searches. On the other hand, the search process itself may change as preferences change.

This paper explores two aspects of preference discovery. First, I study search with preference discovery in a stylized model, abstracting away from some other aspects of search. In particular, I assume that, for a given set of preferences, the consumer searches optimally as described in classic sequential-search papers such as Weitzman (1979). I then allow preferences to be discovered during search and demonstrate that, unlike optimal search with no preference discovery, the consumer might be better off with noisy information about product attributes than with perfect information about product attributes. I extend the simple model with synthetic data to demonstrate this phenomenon in a more complex setting. The more complex setting enables me to explore how product or service recommendations affect search under preference discovery. I find that a benevolent recommender might recommend an inferior product to help consumers learn their preferences. A non-benevolent recommender can steer the consumer with initial recommendations in such a way that, at the end of a self-directed optimal search, the consumer purchases the product desired by the non-benevolent recommender and is happy with the choice.

Second, I use an incentive-compatible experiment to demonstrate that preference discovery may be a real phenomenon. I first measure preferences for apartment attributes prior to search, allow search, and re-measure after search. Subjects have a reasonable chance of receiving
full rent for a year on an apartment chosen by the experimenter based on subjects’ stated preferences. Although subjects learn both attribute levels and preferences, preference discovery is substantial.

Because preference discovery relies on optimal search theory, I begin with a short review of the optimal-sequential-search equation. I then modify this equation for preference discovery, introduce the stylized model and results, describe the synthetic-data model and results, report the empirical evidence, and discuss related phenomena. The final section suggests further research.

2. Optimal Sequential Search without Preference Discovery

2.1. Representing Products and Services by their Attributes

We adopt the notation of Kohli and Mahajan (1991). Let \( i = 1 \) to \( I \) index the products (or services) in the market. For notational simplicity we assume finitely many products, but the basic concepts apply as \( I \) gets arbitrarily large. Each product is described by a set of attributes indexed from \( n = 1 \) to \( N \). For example, an apartment might have attributes such as rent, commute to campus, condition of apartment, closet space, air-conditioning, outdoor space, etc. As is typical in conjoint measurement, we allow each attribute to have discrete levels indexed by \( l = 1 \) to \( L_n \). For example, the condition of an apartment might have four levels: newly renovated throughout, renovated kitchen only, fair condition, and poor condition. Restricting attributes to finitely many attributes is common in marketing and shares the advantages and disadvantages with conjoint measurement. From a theoretical (but not practical) perspective, we can approximate a continuous attribute with large \( L_n \). If product \( i \) has attribute \( n \) at level \( l \), then set \( x_{i nl} = 1 \). Set \( x_{i nl} = 0 \) otherwise. Let \( \tilde{x}_{in}^t = \{x_{in1}, x_{in2}, ..., x_{inL_n}\} \). Products are represented by \( X_i \), with each column corresponding to an attribute, that is, \( X_i = \{\tilde{x}_{i1}, \tilde{x}_{i2}, ..., \tilde{x}_{iN}\} \). A consumer evaluates products sequentially. Let \( X^t \) be the product that the consumer evaluates at time \( t \), that is, the \( t^{th} \) product to be evaluated. (We do not require each time step to be of the same duration.)

The consumer’s preferences are summarized by preference weights associated with a level of an attribute. (These weights would be called “partworths” in conjoint measurement.) Let \( w_{nl} \) be the preference weight for level \( l \) of attribute \( n \). Let \( \tilde{w}_n^t = \{w_{n1}, w_{n2}, ..., w_{nL_n}\} \) be the

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1 Here the prime (′) indicates that \( \tilde{x}_{in} \) is a column vector.
preference vector for attribute $n$ and let $W = \{\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_n\}$ be the consumer’s preferences. For optimal sequential search without preference discovery, we assume that the consumer knows the $W$. If the consumer purchases product $i$, the consumer gets utility, $U_i$, from product $i$. In our analyses we focus on one-time purchases (initial purchases) of high-value “durable” goods and services such as those in the opening paragraphs of this paper. For such products the consumer is likely to take both search and choice seriously. The consumer derives utility from purchasing a product as given by the standard additive model:

$$U_i(X_i, W) = \sum_{n=1}^{N} \vec{w}'_n \vec{x}_{ni} = \text{trace}(W'X_i)$$

2.2. Optimal Search without Preference Discovery

Weitzman (1979) studies search over products, $i$, and assumes that when a consumer evaluates a product, the consumer learns the utility of the product. Prior to evaluation, the consumer is uncertain about the utility of the product, but has beliefs about the utility of the product as described by a cumulative probability density function, $F_i(U_i)$. Without preference discovery, the consumer is uncertain about the attributes of the product, $X_i$, but not the preference weights, $W$. The uncertainty with respect to $X_i$ induces uncertainty with respect to $U_i$ which, slightly abusing notation, I write at $F_i(W'X_i)$ with the understanding throughout the paper that the argument is $\text{trace}(W'X_i)$.

To describe search, Weitzman partitions the set of available products into those that have already been searched up to time $t$, $S_t$, and those that have not been searched, $\bar{S}_t$. If the consumer terminates and does not purchase a product, the consumer receives a known utility for “the outside good,” $B$. At any point in time the consumer can decide to terminate search and receive utility equal to the maximum over all searched products and the outside option. The utility at $t$ is $U_i^* = \max\{B, \max_{i \in S_t}[W'X_i]\}$. The state of the system at any $t$ is $\{U_i^*, S_t\}$. Let $f(U_i^*, S_t)$ be the value of to the consumer of being in state $\{U_i^*, S_t\}$.

The consumer’s decision to terminate is endogenous. The consumer will continue searching if he or she can gain more from the search than the cost of search. Let $c_i$ be the cost of

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2 With the understanding that the argument is the $\text{trace}(W'X_i)$
evaluating the product $i$. Because we are studying durable goods and services, search happens at a significantly faster clock speed than consumption, hence we do not need to use time discount factors during the search process. Such factors complicate notation but do not affect the insights in this paper because, to a good approximation, they can be incorporated in search costs.

To solve the search problem, the consumer must solve a dynamic program. At each time step, the consumer chooses either the current utility or searches. If the consumer searches a product and that product gives greater utility than $U_t^*$, then the consumer gets to choose between that utility and the utility of continuing to search. If the product gives less utility than $U_t^*$, then the consumer gets to choose between $U_t^*$ and continuing to search. Following standard dynamic programming recursion we write the state transition value function as satisfying the following optimality conditions (the Bellman equation):

\begin{equation}
J(U_t^*, \tilde{S}_t) = \max \left\{ U_t^*, \max_{j \in S_t} \left\{ -c_i + J(U_t^*, (\tilde{S}_t - \{j\})) \cdot \int_{-\infty}^{U_t^*} dF_t(W'X_j) \right. \right. \\
+ \left. \left. \int_{U_t^*}^{\infty} J(W'X_j, (\tilde{S}_t - \{j\})dF_t(W'X_j) \right\} \right\}
\end{equation}

2.3. Optimal Policies

Equation 1 is quite general and, in principle, can be built up recursively by systematic induction, but, as Weitzman (1979, p. 644) cautions: “[t]he actual computation is likely to a combinatoric task of unwieldy proportions unless the number of [products] is very small.” However, if the utility a consumer can expect from evaluating a product depends only on the uncertainty in the attributes of that product, Equation 1 can be used to find a reservation price at which the consumer is just indifferent between evaluating a product and not evaluating a product. In this case, the optimal policy is simple (Weitzman 1979, p. 647).

- If a product is evaluated, it should be the product with the highest reservation price.
- Terminate search whenever the maximum sampled utility, $U_t^*$, exceeds the reservation prices of all unsearched products.

Weitzman shows that the reservation price, $z_{it}$, for product $i$ at time $t$ is given by:
\[ c_i = \int_{t_i}^{\infty} (W'X_i - z_{it})dF_t(W'X_i). \]

2.4. Comments

There are a number of implicit assumptions in Equation 1 that we adopt. The first implicit assumption is that we focus on durable goods. If Equation 1 were used to model frequently purchased goods, then the consumer may learn more about a product each time the product is sampled. There might also be ongoing random shocks over consumption occasions due to changes in circumstance or changes in marketing variables. For frequently purchased products this type of forward-looking behavior might dominate initial search. Recent research suggests that the optimal policy for forward-looking problems has a similar flavor to the optimal policy for Equation 1. At each point in time, compute an index for each product and choose the product with the highest index (Gittins 1989; Lin, Zhang, and Hauser 2014). With the proper independence assumptions, one might replace \( W'X_i \) with its Whittle Index, but that extension is beyond the scope of this paper.

The second implicit assumption is that learning about \( X_i \) does not resolve uncertainty about other products’ attributes, \( X_j \) for \( j \neq i \). In other words, if a consumer evaluates an apartment in Brooklyn, the consumer learns about that apartment’s condition. Equation 1 assumes that the consumer does not update his/her beliefs about the distribution of the conditions of other apartments in Brooklyn or about the distribution of conditions of other apartments in Manhattan. My goal is to establish that preference discovery can be important in, at least, some situations. Relaxing the independence assumption would greatly complicate the model, although it may not change the basic insight.

The third implicit assumption is that either (1) no sales agent is involved or (2) any agent is benevolent. A “benevolent agent” acts in the best interests of the consumer. Benevolent virtual agents are common on the Internet. For example, Liberali, Urban and Hauser (2013) study an automotive agent designed to recommend the automobile that is in the best interests of the consumer. Even human agents might act in the consumer’s best interests if the market is highly competitive and/or if the agent wishes to act to gain the consumer’s trust to benefit a long-term relationship (Morgan and Hunt 1984). Sections §3 and §4 assume no agents. I defer to §5
the discussion of agents.

3. A Model of Preference Discovery

The key concepts of preference discovery are (1) preference weights are learned over time and (2) learning takes place as consumers evaluate products. For example, a novice consumer might begin searching apartments in Brooklyn. Based on the consumer’s beliefs about his/her preferences, the consumer might limit his/her search to Brooklyn and choose the apartments to evaluate almost entirely based on location, size, and condition. The consumer may be vaguely aware of many attributes but may place little or no weight on those attributes. Such attributes might be in-building laundry, the light from windows, wood floors, closet space, high ceiling, in-building gym, central air-conditioning, exposed brick fireplaces, the quality of the maintenance company, outdoor space, soundproof walls, proximity to supermarkets, non-clanging pipes, type of cable television available. During the search, the consumer might see apartments with or without these attributes and change his/her preference weights. For example, seeing an apartment with windows that look out on walls but meters away might cause the consumer to care more about light from windows. Or, seeing an apartment close to supermarkets, gyms, or nightclubs might cause the consumer to value those attributes more. Seeing apartments with differing levels of these attributes is serendipitous; the consumer did not choose to evaluate apartments based on these attributes because the consumer focused only on location, size, and condition.

3.1 Preference Weights Change

In optimal search, attribute weights in the consumer’s decision rule (Equation 1) were constant: $W$ did not change as the consumer searched. In preference discovery, the attribute weights in the decision rule change. Let $V^t = \{\vec{v}_1^t, \vec{v}_2^t, ..., \vec{v}_N^t\}$ be the consumers’ attribute weights at time $t$. Let $W_{true}$ be the preference weights for the consumer if and when the consumer becomes “expert.” In other words, if the consumer searches all $I$ products, his/her preference weights will converge to $W_{true}$. For novice consumers, the $V^1$ likely underweight some attribute levels and overweight other attribute levels relative to $W_{true}$. For expert consumers $V^1 \cong W_{true}$.

3.2. Learning Preference Weights through Product Search
Suppose a consumer, who has hitherto not considered the quality of the maintenance company, visits an apartment and talks to tenants about the maintenance company. The tenants praise the maintenance company and the consumer comes to realize he/she should place a high weight on the quality of the maintenance company. In this case, the visit caused $v_{nl}^{post\ visit} = v_{maintenance\ company, high\ quality}^{post\ visit}$ to change from a small weight (if any) to the final weight, $w_{nl}^{true}$. For simplicity, I model preference discovery as happening with a single visit and changing to the true value. In particular,

$$v_{nl}^{t+1} = \begin{cases} v_{nl}^{t} & \text{if } x_{int}^{t} = 0 \\ w_{nl}^{true} & \text{if } x_{int}^{t} = 1 \end{cases}$$

Preference discovery is a model of surprise. The consumer might anticipate that he/she will learn something by evaluating a product (e.g., visiting an apartment), but the stylized models assume the consumer cannot anticipate that he/she will learn about an attribute that was hitherto not important to the consumer. That is, before visiting the apartment I assume that the consumer did not even consider the quality of the maintenance company as relevant to his/her choice of apartments.

The stylized assumptions imply that there are two types of things that the consumer learns by evaluating a product. The consumer anticipates that he/she will learn the value of an attribute. If attribute $i$ is important to the consumer, resolving the uncertainty in $X_i$ will resolve the uncertainty in $U_i$. Of course, an attribute will only play a role in the decision of which product to search if that attribute is important.

However, the consumer cannot anticipate the change in preference weights, in part, because the consumer has never experienced $x_{int} = 1$. In fact, if $\hat{v}_{n}^{t} = \bar{0}$ for attribute $n$, then, a priori, attribute $n$ will not play a role in choosing which product to evaluate. While this model is stylized, it is more general than it might seem if we were to recode the $\hat{v}_{n}^{t}$ to represent changes in preference rather than preference weights per se. Because the consumer cannot anticipate which attributes preferences will change, the consumer cannot anticipate which product evaluations will change preferences. In the stylized model, the a priori gain due to preference discovery does not vary by product.
The consumer might have a vague feeling that evaluation will teach the consumer something about preferences, but the consumer cannot anticipate the specific \(v_{nl}^t\) that will change. If we want, we can incorporate this vague feeling, which depends on \(t\) but not \(i\), as a change in the cost or benefits of evaluating a product. A constant change in \(c_i\) for all \(i\) does not change the model, so I leave to future research the explicit modeling of the vague feeling about the benefits of preference discovery.

3.3. Uncertainty in the Levels of Product Attributes

The values of the levels of a product’s attributes prior to evaluation are characterized by a belief distribution. Because an attribute is described by finitely many levels, beliefs are modeled with a multinomial distribution with parameters, \(\vec{p}_{in} = \{p_{in1}, p_{in2}, \ldots, p_{inL_n}\}\) for all levels of each attribute \(n\) for product \(i\). Recalling that \(x_{int} = 0\) or \(1\) and that \(\sum_t x_{int} = 1\), the probability mass function is:

\[
\begin{align*}
    f_{in}(\vec{x}_{in}) &= p_{in1}^{x_{in1}} p_{in2}^{x_{in2}} \ldots p_{inL_n}^{x_{inL_n}}
\end{align*}
\]

Belief distributions are independent over attributes, that is, the consumer’s beliefs about a particular apartment’s air-conditioning system is independent of the consumer’s belief about whether or not the apartment is near a supermarket. Independence is clearly an approximation. Having an air-conditioning system might be correlated with the condition of an apartment. If such correlations are high, we combine attributes. For example, we might have an “air conditioning and apartment condition” attribute. Such decisions are empirical in nature and based on consumer surveys as in §6. As in Equation 1, belief distributions are independent over products.

3.4. Optimal Search with Preference Discovery

At any point in the search process, the novice consumer searches with his/her preferences at that point in the process, \(V_t\), rather than \(W^{true}\). The revised Bellman equation at time \(t\) is:
When $V_t \neq W^{true}$, Equation 4 might imply a different product to evaluate than does Equation 1. The consumer evaluates that product and resolves the uncertainty about $X_t^t$. Resolving the uncertainty in $X_t^t$ resolves the uncertainty in $U_t^t = V^{t+1}$. (Recall that $X_t^t$ is the product, if any, that the consumer chooses to evaluate at $t$.)

However, observing $X_t^t$ also updates preferences: $V^t \rightarrow V^{t+1}$. In $t + 1$, the consumer searches with the updated preference weights, $V^{t+1}$, and updated maximum utility, $U_{t+1}^t$. For example, suppose that at time $t$, the consumer chooses apartments to visit without any consideration of the quality of the maintenance company and then, by luck, visits an apartment with an outstanding maintenance company. That consumer updates his/her preferences to place a higher weight on the quality of maintenance companies. When the consumer searches in $t + 1$, he/she chooses the next apartment to visit, this time placing a much higher weight on the quality of maintenance companies. This two-way relationship between examining products and discovering preferences is shown schematically in Figure 1.

\[
J(U_t^t, \bar{S}_t) = \max \left\{ U_t^t, \max_{j \in \bar{S}_t} \left\{ -c_i + J(U_t^t, (\bar{S}_t - \{j\})) \cdot \int_{-\infty}^{U_t^t} dF_t(V^{t+1}X_j) \right\} + \int_{U_t^t}^{\infty} J(V^{t+1}X_j, (\bar{S}_t - \{j\})) dF_t(V^{t+1}X_j) \right\}
\]

(4)
As the consumer moves through the search process in Figure 1, his/her preferences are discovered and the new preference, in turn, changes his preferred product to evaluate.

4. Noisy Information and Preference Discovery

The optimal policy for sequential search without preference discovery was to evaluate the product with the highest reservation price. Although the computation of the reservation price (Equation 2) depends upon the noise in the consumer’s beliefs about the product attributes, there was no clear interaction between the optimal policy and noise. When we allow preference discovery, noise interacts with expertise.

4.1. Analytical Results (Existence) on the Interaction of Noise and Preference Discovery

I motivate the interaction between noise and preference discovery with a particularly simple market of two products with two binary attributes. Product 1 has attribute 1 but not attribute 2 and product 2 has attribute 2 but not attribute 1. Without loss of generality, \( w_{21}^{\text{true}} > w_{11}^{\text{true}}, \) \( w_{22}^{\text{true}} = w_{12}^{\text{true}} = 0, \) and \( B = 0. \) An appendix establishes that, for this market:

Proposition 1. If an (expert) consumer knows his/her preferences \( (V^t = W^{\text{true}}) \), then the expert consumer finds the best product with minimal search cost when there is no uncertainty in product attributes \( (p_{111} = p_{122} = p_{212} = p_{221} = 1) \).

Proposition 2. If a (novice) consumer has not yet discovered his/her preferences \( (V^t \neq W^{\text{true}}) \), then the novice consumer may be better off (utility net of search cost) with noisy prior beliefs, that is, when \( p_{111}, p_{122}, p_{212}, p_{221} < 1 \).

The intuition is simple. If the novice consumer under- or over-values some attribute levels and if there is no noise in the attribute levels, then the novice consumer will act on the incorrect preference weights. With incorrect weights the novice consumer will evaluate the product with the lower true utility and will never evaluate the product with higher true utility. The novice consumer will not know what he or she has missed. For example, the consumer might never visit an apartment with a high quality maintenance company. On the other hand, if there is noise in the attribute levels, there are reasonable conditions where the novice consumer will evaluate the better product, update his/her preference weights, and choose the higher utility product.
Proposition 2 is a new phenomenon that is not possible with optimal search alone. Whether the novice consumer is actually better off depends upon the specific values of the $\tilde{p}_{ln}$’s and the $V^l$’s. Proposition 2 is an existence proof based on a simple market. The proof extends readily to more products, more attributes, and more levels. However, extending the result analytically to more complicated scenarios is difficult. Instead, I turn to synthetic data to handle the complexity and illustrate that the phenomenon is more general than a 2 x 2 market. The synthetic data then allow me to investigate other interesting phenomena.

4.2. Insights Using Synthetic Data: Noise and Preference Discovery

Synthetic data can be used with any number of attributes, attribute levels, and products. In this paper I simulate a product space with four attributes at four levels each. If there were no further restrictions on the product space, there would be $4 \times 4 \times 4 \times 4 \times 4 = 1024$ products. To examine whether the results of §4.1 generalize, the market vary on noise and expertise. To make the markets more realistic, I use a partial factorial design to simulate market efficiency.

4.2.1. Varying noise. Figure 2 illustrates how noise might vary among markets. The market on the left is relatively noisy. Prior to evaluating the product, the consumer believes that each of the four levels of the apartment’s condition are roughly equally likely. Contrast this noise level with that in the market on the right. In that market, the consumer if fairly certain that the apartment has a renovated kitchen.

![Figure 2](image-url)

**Figure 2. Varying Levels of Noise in the Consumer’s Prior Beliefs**
Each graph plots the $p_{ln}$’s for the four levels ($l = 1$ to 4) of the “condition” attribute, $n$, for product $i$. 
To model noise, I draw the \( \mathbf{p}_{in} \) from a Dirichlet distribution. The Dirichlet distribution assures that the \( p_{inl} \)’s are bounded between 0 and 1 and sum to 1.0 over \( l \). Specifically,

\[
f(\mathbf{p}_{in}) = \frac{\Gamma(\sum_{l=1}^{L_n} \alpha_{inl})}{{\prod_{l=1}^{L_n} \Gamma(\alpha_{inl})}^p} p_{in1}^{\alpha_{in1}-1} p_{in2}^{\alpha_{in2}-1} \ldots p_{inL_n}^{\alpha_{inL_n}-1}
\]

where the parameters, \( \alpha_{inl} \)’s, of the Dirichlet distribution control the noise and \( \Gamma(\cdot) \) is the gamma function. The parameters control the noise because, with the Dirichlet distribution, \( E[p_{inl}] = \alpha_{inl}/\sum_{k=1}^{L_n} \alpha_{ink} \). For example, the left graph of Figure 2 (high noise) corresponds to a roughly equal \( \alpha_{inl} \)’s and the right graph of Figure 2 (low noise) corresponds to a situation where \( \alpha_{in3} \gg \alpha_{in1}, \alpha_{in2}, \alpha_{in4} \).

In the highest noise situation I set \( \alpha_{inl} = 1 \) for all \( l \). Noise in the consumer’s prior beliefs decreases as \( \alpha_{inl} \) increases for the true attribute level. Let \( \alpha_{noise} \) control the noise and set \( \alpha_{inl} = \alpha_{noise} \) if \( x_{inl} = 1 \). Otherwise, keep \( \alpha_{inl} = 1 \). In the synthetic data \( \alpha_{noise} \) varies on a logarithmic scale from 1 to 150. Specifically, \( \alpha_{noise} = 1, 3.5, 12.2, 42.3, \) and 150.

4.2.2. Varying Expertise. An expert consumer starts at \( t = 1 \) with \( V^t \approx W^{true} \); a novice consumer starts at \( t = 1 \) with a \( V^1 \) that can be very different from \( W^{true} \). To capture expertise, I draw the initial preference weights, \( v_{nl}^1 \), and the true weights, \( w_{nl}^{true} \), from a joint normal distribution for each of 500 consumers for each synthetic data run. Preference weights can be positive or negative—a consumer may prefer or not prefer to have an apartment with exposed brick, thus, I use a zero-mean distribution with unit variances. (The results are not sensitive to these restrictions.) A correlation parameter, \( \rho \), captures expertise. Highly correlated \( v_{nl}^1 \) and \( w_{nl}^{true} \) correspond to high expertise and uncorrelated \( v_{nl}^1 \) and \( w_{nl}^{true} \) correspond to low expertise. Specifically,

\[
\begin{pmatrix} w_{nl} \\ v_{nl}^1 \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right).
\]

4.2.3. Simulating market efficiency. A market with all 1024 products is not particularly efficient because, for a given set of preference weights, \( W^{true} \), there exist dominated products. It

\footnote{For this specific plot, values \( \alpha_{in1} = \alpha_{in2} = \alpha_{in4} = 1 \) were used for both plots, and \( \alpha_{in3} = 1 \) for the left plot and \( \alpha_{in3} = 50 \) for the right plot.}
is more realistic to allow some attribute levels to be anti-correlated (e.g., homes in a certain neighborhood are old, cars with large engines have low fuel efficiency, Porsche does not make pickup trucks, nannies with daycare experience refuse to do housework). In low market efficiency, all 1024 products would be available. In a more efficient market, some products are deleted based on a symmetric $20 \times 20$ anti-correlation matrix, $C$, with elements, $c_{kj}$. The rows and the columns correspond to attribute levels (5 attributes at 4 levels each). The probability that product $i$ is deleted is the multiplication of the $c_{jk}$ for all pairs of attribute levels in product $i$ for which $x_{int} = 1$. For a given synthetic-data run, I draw the elements of the matrix, $c_{kj}$, from a Beta distribution with parameters 0.1 and 0.8. Results are not sensitive to the choice of parameters. Including these unlikely feature combinations in simulating the market ensures that the consumer will have to make some tradeoffs, and therefore benefits from knowing more than just the level with the highest partworth for each attribute.

4.3. Search Costs and Net Rewards for Novice and Expert Consumers

We begin with expert consumers setting $\rho = 1$ such that search with preference discovery (Equation 4) reduces to the special case of search without preference discovery (Equation 1). Figure 3 plots the average search time and average net payoff for expert consumers when they search optimally. The results are as expected. When there is little noise and consumers know their preferences, they find the best product quickly and earn high net payoffs (utility net of search costs). However, as noise increases, it is more difficult for the consumers to

![Figure 3: Average Search Time and Average Net Payoff for Expert Consumers](image-url)
find the best product. They search longer and do not earn payoffs that are as high. Because the chosen values of $\alpha_{\text{noise}}$ are somewhat arbitrary, the important insight from Figure 3 is that search costs increase monotonically as noise increases and that net payoffs decrease monotonically as noise increases.

Analyses provide an interesting contrast when novice consumers discover their preferences ($\rho = 0$). Search costs increase with noise, but the net payoff is no longer monotonic in noise. Then net payoff increases over the first four levels of noise before decreasing at extremely high noise. (When $\rho = 0.7$, the results are between those of Figures 3 and 4.) For another perspective on the interaction of noise and expertise, I compared the net rewards obtained by novice consumers ($\rho = 0$) when noise is high ($\alpha_{\text{noise}} = 1$) versus when noise is low ($\alpha_{\text{noise}} = 150$). Roughly 60% of the consumers (299 out of 500) obtain higher rewards when noise is high. These results are consistent with the analytic results and suggest that when consumers can discover their preferences through search, noise in prior beliefs can increase net payoffs.

![Figure 4: Average Search Time and Average Net Payoff for Novice Consumers](image)

**5. Benevolent Recommendations and Preference Discovery**

§3 and §4 ignored agents, but agents are sometimes involved in the search for high-value products and services. For example, benevolent consumer-centric recommendation systems are common on the Internet. Websites such as Netflix, Youtube, SiriusXM radio, Kelly Blue Book, and other platforms such as Xbox game console provide recommendations to consumers. Many
of these firms seek to build trust by recommending the products that they believe are in the best interests of the consumer. There has been substantial research in developing recommender systems to aid consumers. Typically, these systems seek to recommend the product that the consumer will prefer (Adomavičius and Tuzhilin (2005), Ghose, Ipeirotis and Li (2012), Pu, Chen and Hu (2011)). Human agents might also be benevolent. For example, Best Buy does not pay commission and instructs their salespeople to help consumers find the best product. Even some realtors in highly competitive markets such as Boston’s Beacon Hill area, claim that they maximize profit by helping the consumer find the best apartment (more in §6). Naturally, not all recommendation systems or agents are benevolent. Nonetheless, I begin by examining how preference discovery interacts with benevolent recommendations. I then discuss how preference discovery enables non-benevolent recommendations to game the system.

5.1. The Effect of an Initial Recommendation

To simulate recommendations, I use the same synthetic data regime, but modify the search process. I allow the recommender (agent or virtual agent) to recommend one product at $t = 1$. The consumer evaluates that product and then proceeds with optimal search using Equation 4. Throughout the process (novice) consumers discover preferences.

I plot the results of optimal search separately for every possible recommendation. The consumer may not, and often will not, choose the recommended product and the recommendation often influences the path of optimal search. For example, the benevolent agent might recommend a product with true utility of $W^{true}X_r = 3.9$, but, after search, the consumer might purchase a product with true utility of $W^{true}X_l = 7.5$.

Figure 5 plots the results for an illustrative consumer. Each point in Figure 5 represents an initial recommendation. The horizontal axis is the utility of the recommended product and the vertical axis is the net payoff (utility net of search costs) that the consumer earns from the product that is chosen after optimal search. In Figure 5 the final choice has an equal or higher payoff than any recommended product. (In general, the final choice might not always be higher because there are search costs and because $V^t$ might not equal $W^{true}$ when search terminates.) The highest utility product is indicated with a solid arrow ("Type A").
Even with preference discovery, it is a good strategy to recommend the highest utility product for this consumer. That recommendation gives close to the highest net reward. However, Figure 5 illustrates other phenomena. For example, a low-utility initial recommendation (“Type B”), as indicated by the dotted arrow, also yields a high reward. That recommendation does well because it helps the consumer to discover his/her preferences. An example would be a realtor showing his client an unacceptable apartment but in a neighborhood that the consumer likes but did not previously know about. Fong (2014) provides evidence from a large field experiment that

Figure 5. Results of Initial Recommendations for an Illustrative Consumer
Each point represents one potential initial recommendation, $X_r$, made at $t = 1$. **Type A**: Highest utility product ($\max_i W^* x_i$). **Type B**: Low utility recommendation, high net payoff. **Type C**: Higher utility recommendation than Type B, but lower net payoff.
demonstrates a similar phenomenon for an online wine retailer: targeted promotional offers for wine that is similar to the consumers’ previous purchases result in less search activity than new genres, which encourage customers to broaden their search. That low-utility recommendation actually yields a higher net reward than a recommendation with substantially higher utility (“Type C”, shown with a dashed arrow). The higher-utility recommendation (Type C) does not encourage as much preference discovery as the lower-utility recommendation (Type B). These phenomena are not specific to the illustrative consumer in Figure 5. Figure 6 repeats the plots for fifteen randomly-chosen consumers. Note that not all consumers are impacted by recommendations: some consumers are able to find the best product without any recommendation, and no matter what they are recommended at time 1, they end up with the best product. I obtain similar insight from all 500 consumers.

5.2. Constrained Recommendations

Suppose that the recommendation system (or agent) were constrained in its ability to recommend products. For example, a realtor might judge that an apartment shopper is not serious and may want to limit recommendations to open houses. Alternatively, a recommender’s database may be limited to a subset of the products on the market.

To illustrate this scenario, suppose that the recommender could only choose between the two products indicated by dotted- and dashed- arrows in Figure 5. In this case, the benevolent agent would recommend the lower-utility product under the understanding that it would help the consumer discover his/her preferences. This contradicts the design of almost all extant virtual recommender systems.

This illustration suggests that recommendation systems that explicitly model preference discovery might serve consumers better. Such research might also address the question as to how the recommendation system might anticipate \( W \) and learn \( V^1 \). It is beyond the scope of this paper to develop such recommendation systems. As a first step, §6 suggests that preference discovery might be observed, although the methods of §6 would need to be developed further if a virtual recommendation system were to anticipate preference discovery.
Figure 6. Results of Initial Recommendations for Twenty-Four Consumers
5.3. Agents that are Not Benevolent

Suppose that an agent was not benevolent. That agent might be rewarded more highly if the consumer chooses from a subset of the available products. For example, some realtors get a higher reward if they are the listing agent, or an online retailer might have higher margins on some products than others (e.g. Ghose, Ipeirotis and Li (2013), Liberali, Donkers, and Stremersch (2014)). However, the non-benevolent agent may not wish to make it explicit to the consumer that the agent is limiting search to a subset of products.

Figures 5 and 6 illustrate nicely that initial recommendations matter. The agent can use those recommendations to steer the consumer while seeming benevolent. For example, by recommending the “Type C” product in Figure 5, the agent can recommend a product with high initial utility (although not the highest). As the consumer searches and discovers his/her preferences, the consumer ends up buying the recommended product. The consumer feels good about the search because the consumer had control of the search after the initial recommendation. The consumer never realizes that the consumer might have found a higher-utility product, because the consumer never updated his/her preferences by experiencing some attributes with low \( v^t \) but high \( w^{true} \). By recommending an initial product, the agent steered the consumer to a profitable product and the consumer is happy.

The agent can be even more insidious. The consumer might suspect the agent is not benevolent and may be skeptical of the agent’s recommendation. Figures 5 and 6 suggest that the non-benevolent agent can recommend an inferior product and, in doing so, cause the consumer to converge to an entirely different product, but a product that is profitable for the agent. During the search process, the consumer endogenously rejects the agent’s recommendation and feels in control of the subsequent search process, but still purchases the product the agent wants the consumer to purchase.

The analysis of non-benevolent agents is within the realm of game theory and is an exciting area to investigate with discovered preferences. The exact results will depend upon the specifics of the game such as the product set, the initial preferences, the true preferences, the profits assigned to each consumer choice, the consumer’s beliefs about the agent’s incentives, and the consumer’s knowledge.
6. Demonstration of Preference Discovery

§3 suggests that the phenomenon of preference discovery affects consumers’ search processes. §4 suggests that noise in consumers’ beliefs might actually benefit consumers who discover preferences. And, §5 suggests that recommendation systems and agents (benevolent or not) are affected by preference discovery. But do consumers really discover their preferences through search. §1 began with anecdotes, but anecdotes are not data. This section reports the results of two empirical studies that are consistent with preference discovery. The first study relied on qualitative interviews with realtors. The second study was a laboratory study in which I measured preferences before and after subjects evaluated apartments.

The basic idea of the apartment-evaluation study was that (1) consumers articulate their preferences for a product category, (2) they search in that product category, and (3) they update their preferences (if appropriate) after search. I expected that some consumers would change their preferences (dorm-living novices without apartment hunting experience) and some would not (experts with substantial apartment-hunting experience). To draw both experts and novices I used subjects from a pool of students attending college in New York City (NYC).

6.1. Qualitative Interviews with Realtors

To understand better the real estate market I interviewed 12 realtors in a densely-populated residential neighborhood in Boston. Many students live in this neighborhood, but it is not exclusively for students. The market is quite competitive among realtors—there are at least fifteen offices with multiple agents along a quarter-mile stretch of a single street in the neighborhood. In this market, realtors are aware of losing clients to other realtors and well-aware of the impact of negative word of mouth. Although I had no way of verifying their stated beliefs, the realtors claimed that the market rewards benevolence.

There were three primary insights from the interviews. (1) Realtors are aware that consumers’ preferences are discovered as they view apartments (condos). (2) Realtors are aware that expert consumers have more-stable preferences than novice consumers. (3) Realtors take preference discovery into account in choosing which apartments to recommend. For example, the following quotes are typical of those I obtained from realtors.
- “Often what people start out thinking they want is not what they end up wanting.”
- “Let’s say they tell you they want three things, like renovated kitchen, pet friendly, and up to $2500. I find them something that has those 3. Then they get there and tell me they hate the view and won’t take it because of that.”
- “People may not think about what common areas look like, but once they actually go out and see it, they realize that they will be affected by it.”
- “Let’s say someone is looking for a 1 bedroom with a good layout. I show them one, and then I walk in the bedroom and open up a French door to a private deck. They love that, and want me to look for more apartments with a deck.”
- “It does happen, not very often, that they see just one [apartment] and take it. Usually with people who have been living (in this neighborhood) for a long time and know the area and know exactly what they want.”

6.2. Incentive Alignment for the Second Study

Realtors believe consumers discover preferences. But do consumers discover preferences? A key challenge in identifying preference discovery is to give subjects sufficient incentives to report their preferences accurately. To address this challenge, I sought to make both preference elicitation, before and after product evaluation, incentive aligned. A task is incentive aligned, rather than the more-formal incentive compatible, if (Ding, et al. 2005, p. 120):

“subjects believe (1) it is in their best interests to think hard and tell the truth; (2) it is, as much as feasible, in their best interests to do so; and (3) there is no way, that is obvious to the subjects, they can improve their welfare by ‘cheating.’” I adopt the incentive alignment procedure and the preference elicitation task from Ding, et al. (2011). In particular, subjects were told that one subject from a small sample would be selected and given a reasonable chance at receiving free rent for one year (up to $20,000). The chosen subject would receive $100 for sure and get to draw two envelopes out of twenty. If both contained winning cards, the subject received the free rent.4

If the subject received free rent, an apartment would be chosen for them by two independent judges using the preferences that the subject articulated in the study. To avoid

4 The chances of drawing two winning cards are 1 in 190. However, subjects perceive the chances as much higher as indicated by the excitement and overwhelming interest in response to a request for subjects from my university’s subject pool. <I don’t know how to get the above line left-justified. I leave that to you.>
elicited preferences that were too specific, the judges would choose from a secret set and the
direct elicitation as described in Ding, et al. (2011).

Subjects were instructed to: “Provide instructions to Dan and Emily who have been hired
to select an apartment for you if you win the lottery. They do not know anything about this study,
and will only read your instructions.” Subjects were given suggestions about their instructions,
such as “State as many instructions as possible, so that the agents have the best information of
your preferences”. When they were done, they clicked “Submit my email.”

6.4. Evaluate Example Apartments

After stating initial preferences, subjects evaluated twelve apartment listings. The listings
were chosen from actual listings in NYC and each included verbal descriptions and images and
varied. The listings varied on attribute levels that had been identified in pretests and by browsing
local listings. Figure 7 is one example. The listings spanned the set of available attributes and
were the type of apartments normally chosen by students at the university.
It was not feasible for subjects to visit the apartments. Instead, I induced careful evaluation as a surrogate for search. Subjects rated each listing and provided “one or two reasons for [his/her] rating.” Pretests indicated that this task induced subjects to evaluate each listing carefully. This surrogate is likely to underestimate preference discovery relative to the evaluation of real apartments.

**Williamsburg, Brooklyn**

**Bedrooms:** 2  
**Price:** $1,850  

- Very large living area  
- Luxurious & very spacious kitchen with stainless steel appliances, french cabinetry and countertop space  
- 2 Nice size bedrooms with big windows  
- Closet space  
- Washer/dryer Hook-up in unit  

Nearby subway stations:  
- Chauncey St (0.18 mi) - J train (1 transfer to get to NYU)  
- Bushwick Av - Aberdeen St (0.19 mi) - L train (1 transfer to NYU)
6.5. Optional Preference Updating Task

One danger with re-measuring preferences is that subject might perceive that the experimenter wants the subject to revise his/her preferences. The preference revision task was designed and pretested carefully to minimize such demand artifacts. Subjects were shown their original letter and were asked “Is there anything about your response that could be improved to help the agent make a better decision on your behalf?” Subjects could select “YES, I would like to update these instructions to better reflect my preferences,” or “NO, these instructions are accurate and complete.” If subjects chose to revise, subjects could submit their revision as additional information, in which case the judge would see both the original and new email. There was no additional reward for completing this task, other than the implicit reward that providing better instructions would help “Dan and Emily” make better choices on the subject’s behalf. Subjects could skip this task if they felt their initial stated preferences still described their preferences. Subjects were not told about this task when they stated their initial preferences. Subjects could not anticipate that revisions were possible.

6.6. Subjects Discovered their Preferences

Of the 79 subjects, 42 revised their preferences substantially. (An additional 6 subjects provided clarifications.) Responses varied in length with an average of 231 words and a standard deviation of 160 words. For example, the following respondent expanded the set of locations that he was willing to consider. Prior to evaluating the listings, he would have limited his search to apartments within walking distance of the university. After search he expanded his search, in part, because he discovered that apartments that were further away had attributes he had come to desire. These attributes were not weighed nearly as heavily prior to preference discovery.
**$V^1$ prior to evaluation of listings.** “Areas that I am interested in are West and East Village, St. Marks, 14th St., Soho, Chinatown, or any other areas within a 15min walking distance from NYU. I would also consider any apartments available uptown that are close to trains.”

**W after evaluation of listing.** Areas that I am interested in are West and East Village, St. Marks, 14th St., Soho, Chinatown, or any other areas within a 15min walking distance from NYU. I would also consider any apartments available uptown that are close to trains. Additionally, I would extend my location preferences around the city preferably only NYC and not NJ if there are places in great neighborhoods with a great price and large room. I would actually prefer Brooklyn or Queens if there is a larger place and better neighborhood than Manhattan.”

As befits the product category, real estate, location was the most-revised attribute. But subjects revised eighteen other attributes as indicated in Figure 8. Detailed responses are available upon request.

![Figure 8. Attribute Preferences that Were Revised Most Often by Subjects](image-url)
In summary, the incentive-aligned study suggests that students revise preferences substantially as the result of a realistic evaluation of apartments in NYC. In this study, the experimenter “recommended” twelve apartments to evaluate. Had subjects not evaluated these recommendations, they would have used their initial stated preferences and, likely, they would not have searched as broadly. With the recommendations, subjects used their revised preferences and, likely, their \textit{(in vivo)} search process would have changed.

7. Comments

The incentive-aligned study likely underestimates preference discovery because subjects evaluated listings rather than real apartments. Furthermore, any unobserved shirking is likely to underestimate preference discovery. Nonetheless, while preference discovery is one interpretation of the data, there are alternative explanations.

The model proposed in §3 is stylized in order to isolate preference discovery. This paper addresses one important mechanism for preference discovery—product evaluation during optimal search. However, there are other ways in which consumers might discover their preferences. For example, many websites recommend to consumers which attributes to consider when evaluating products such as electronics, used cars, dietary supplements, etc. In reality, consumers are likely to seek such information help to form $V^1$. In some cases consumers will find that $V^1 = W^{true}$, but in other cases consumers will experience additional self-discovery as they evaluate products. Even if a trusted source lists important attributes, “learning by doing” or “learning by evaluation” may still be necessary.

Preference discovery is loosely related to the literature in marketing on consideration sets. In that literature consumers use a consider-then choose rule or a cognitively-simple heuristic such as a lexicographic rule (Hauser and Wernerfelt 1990; Jedidi and Kohli 2005) to determine a choice set. Preference discovery can be extended to include decision heuristics. For example, the consumer might discover or reorder attributes in a lexicographic decision process for consideration and discover preference weights for decisions within the consideration set.

8. Summary and Future Directions

The key idea of preference discovery is that, when evaluating products, consumers
experience attribute levels that hither fore had not been “on their radar”, and may revise their decision rule to account for these attributes. Analytically, preference discovery changes the optimality conditions to use $V^c$ rather than $W^{true}$. Preference discovery modifies consumer search and the choice of durable goods and products. For novice consumers, noise in attribute evaluations can help them achieve higher net rewards. Furthermore, the consumer’s search process can be manipulated with recommendations: a product recommendation does not simply introduce the product into the consumer’s consideration set, but it can also change the future search path. Finally, experimental evidence suggests that preference discovery is consistent with changes in consumer preferences after they evaluate apartments.

In this paper, we focused on the implications of preference discovery on recommendations and accuracy of product information. Further work is required to understand how it impacts firms’ other strategic decisions, such as retail assortments, product line optimization, and ad targeting. Additionally, more sophisticated models of preference discovery may lead to interesting results. For example, novice consumers may anticipate that there are some “unknown unknowns”. While they would not have a prior distribution on the unknown attributes, they may have a prior on the likelihood they will stumble upon a new attribute. This likelihood could be low if the consumer is familiar with the category, and high if the consumer knows he is a novice. Fine grained experimental measurement of consumer preference revisions may help shed more light on how consumers revise preferences in response to evaluating a product.
References

Adomavičius, Gediminas and Alexander Tuzhilin (2005), “Towards the Next Generation of Recommender Systems: A Survey of the State-of-the-Art and Possible Extensions,” *IEEE Transactions on Knowledge and Data Engineering*, vol. 17, no. 6


Appendix: Proofs

**Proposition 1:** A consumer who knows his preferences from the start, i.e. $v^1 = w$, is best off when he also knows the product attributes without any uncertainty, i.e. $p_1 = 1$ and $p_2 = 0$.

**Proof:**

At time $t = 1$, the equations for the reservation prices are:

\[
\begin{align*}
    c &= p_1 \cdot (v_1^1 - z_{11}) + (1 - p_1) \cdot (v_2^1 - z_{11}) = p_1 \cdot v_1^1 + (1 - p_1) \cdot v_2^1 - z_{11}, \\
    c &= p_2 \cdot (v_1^1 - z_{21}) + (1 - p_2) \cdot (v_2^1 - z_{21}) = p_2 \cdot v_1^1 + (1 - p_2) \cdot v_2^1 - z_{21}
\end{align*}
\]

The highest possible payoff is $\max\{0, w_2 - c\}$. If $w_2 - c > 0$, the highest payoff corresponds to searching one time and purchasing product 2; otherwise, the highest payoff corresponds to not searching and taking the outside good.

Suppose the consumer knows his preferences from the start, so that $v^1 = w$. Then the reservation prices are:

\[
\begin{align*}
    z_{11} &= p_1 \cdot w_1 + (1 - p_1) \cdot w_2 - c \\
    z_{21} &= p_2 \cdot w_1 + (1 - p_2) \cdot w_2 - c
\end{align*}
\]

If $p_1 = 1$ and $p_2 = 0$, then $z_{11} = w_1 - c$ and $z_{21} = w_2 - c$.

Since $w_2 > w_1$, the highest reservation price is that of product 2 (which is the correct “best” product). If $z_{21} - c < 0$, the consumer does not evaluate this product, stops searching, and takes the outside good. His payoff is 0, which is the maximum possible in this case. If $z_{21} - c > 0$, the consumer evaluates this product. Since the beliefs were already certain, and the weights were correct, they remain unchanged. In the next time period, $U^{2*} = w_2$ and the reservation price of
the remaining product is \( z_{12} = w_1 - c < U^2 \), so he does not search any longer, and purchases product 2. His net payoff is \( w_2 - c \), which is the maximum possible in this case.

**Proposition 2:** If \( v^1 \neq w \), the consumer may be better off with noisy information about product attributes, i.e. \( p_1 < 1 \) and/or \( p_2 > 0 \), than with perfect knowledge of product attributes without uncertainty.

**Proof:** Suppose that the consumer is correct about the weight of the first attribute level, \( v^1_1 = w_1 \), but undervalues the second attribute, \( v^1_2 < w_2 \). Suppose further that \( 0 < v^1_2 < v^1_1 < w_2 \). The highest possible payoff is \( w_2 - c \)

Case 1: \( p_1 = 1 \) and \( p_2 = 0 \).

The reservation prices are

\[
\begin{align*}
z_{11} &= v^1_1 - c = w_1 - c \\
z_{21} &= v^1_2 - c
\end{align*}
\]

Since \( v^1_2 < v^1_1 \), then \( z_{11} > z_{21} \), and the consumer searches the first product at time 1, and at time 2 we have \( U^2 = w_1 \) and the reservation price for product 2 is:

\[
z_{22} = v^1_2 - c < U^2
\]

The consumer terminates the search, purchases product 1, and receives a payoff of \( w_1 - c \).

Case 2: \( p_1 < 1 \) and/or \( p_2 > 0 \)

The reservation prices are:

\[
z_{11} = p_1 \cdot w_1 + (1 - p_1) \cdot w_2 - c
\]
If \( p_2 > p_1 \), then \( z_{21} > z_{11} \), and consumer searches product 2, and \( U^{2*} = w_2 \). At time 2 the reservation price of product 1 is

\[
z_{12} = p_1 \cdot w_1 + (1 - p_1) \cdot w_2 - c < w_2
\]

Since \( z_{12} < U^{2*} \), the consumer terminates search and purchases product 2. His net payoff is \( w_2 - c \), which is greater than \( w_1 - c \) which was the payoff in Case 1.