5. EXPECTED VALUE AND VARIANCE FOR DISCRETE RANDOM VARIABLES

For a best 4 out of 7 series between two equally-matched teams, what should the average length of the series be?

<table>
<thead>
<tr>
<th>Duration of series</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.3125</td>
</tr>
<tr>
<td>7</td>
<td>0.3125</td>
</tr>
</tbody>
</table>

Expected Value: \( E(X) = \sum_{x} x p(x) \) if \( X \) is a discrete RV.

• \( E(X) \) is a weighted average of the possible values of \( X \). The weights are the probabilities of occurrence of those values.

• \( E(X) \) is the theoretical mean of the random variable \( X \).

\textbf{Eg 1:} Using the probability distribution for the duration of the World Series for two equally matched teams, the expected length of the series is

\[
4(0.125)+5(0.25)+6(0.3125)+7(0.3125) = 5.8125.
\]

• If we repeatedly observe realizations of \( X \), the long-run average is \( E(X) \).

• Therefore, we can think of \( E(X) \) as a population mean. Here, the population consists of all possible realizations of \( X \). We write

\[
E(X) = \mu.
\]

• The best forecast of \( X \) is the expected value of \( X \), given the available information. We can compute forecasts using formulas or simulations.

• Rational Expectations (Economics): Deviations from perfect foresight are random, with mean zero. Therefore, forecast errors are not predictable.

• “Analysts’ expectations” of EPS are the forecasted earnings, so earnings surprise should be random (with mean zero). If EPS does not meet analysts’ expectations there need not be anything wrong with the company. Random variables with zero mean need not always equal zero.

\textbf{Eg 2:} In the game of craps, for a pass-line bet of $1 without odds, the player wins $1 with probability 0.493. What is the average casino take, per dollar wagered?
**Eg 3:** Compute expected value for a Let’s Make a Deal game where you win $1 if you end up with the red card, and you lose $1 if you end up with a black card. Try two strategies: Never Switch, Always Switch.

**Eg 4:** A 40-year-old male purchases a $500,000 1-year term life insurance policy. If Prob{Still alive after 1 year} = 0.998, what is the insurance company’s expected payout?

- **Variance:** \( \sigma^2 = V(X) = E[(X - \mu)^2] = \sum_{x} (x - \mu)^2 p(x) \)
  
  if X is a discrete RV.

- The variance of X is the expected value of the RV \((X - \mu)^2\).

  \( V(X) = \text{Mean Squared Deviation of X from its own mean, } \mu. \)

- **Standard Deviation:** The standard deviation of X is \( \sigma = \sqrt{V(X)}. \)

  - \( \sigma \) measures the amount of fluctuation in X over a large number of repetitions of the experiment.

  - \( \sigma \) is a measure of risk.

Note: \( \sigma^2 \) and \( \sigma \) defined above are *theoretical* variance and standard deviation of X.

You don’t need any data to compute them. You just need to know the distribution of X.

The mean of a random variable is NOT the same thing as a sample mean. The variance of a random variable is NOT the same thing as a sample variance.

**Eg 5:** What is the standard deviation of the player’s profit for a $1 craps bet?

**[Sampling Lab]**