

10. CONFIDENCE INTERVALS, INTRODUCTION

“Statistics is never having to say you're certain”.

(Tee shirt, American Statistical Association).

The confidence interval is one way of conveying our uncertainty about a parameter.

It's misleading (and maybe dangerous) to pretend we're certain.

It is not enough to provide a guess (*point estimate*) for the parameter.

We also have to say something about how far such an estimator is likely to be from the true parameter value.

With a confidence interval, we report a range of numbers, in which we hope the true parameter will lie. The interval is centered at the estimated value, and the width (“margin of error”) is an appropriate multiple of the standard error.

We can think of the margin of error as “fuzz”, introduced to account for sampling variability.

Eg: “The margin of error in the Goldman Sachs poll was 3.5%, according to the firm.”

We have “confidence” that the method will work, since we can control the probability that such an interval will fail to contain the true proportion of Millennials who prefer to pay for purchases using PayPal.

Definition of Confidence Intervals

Suppose we want to learn (make inferences) about a population mean μ based on a sample of size n .

The sample mean \bar{X} is a point estimator of the parameter μ . Used by itself, \bar{X} is of limited usefulness because it contains no information about its own reliability.

- **Confidence Interval:** An interval with random endpoints which contains the parameter of interest (in this case, μ) with a prespecified probability, denoted by $1 - \alpha$.

The confidence interval automatically provides a margin of error to account for the sampling variability of \bar{X} .

Eg: A machine is supposed to fill “2-Liter” bottles of Pepsi. To see if the machine is working properly, we randomly select 100 bottles recently filled by the machine, and find that the average amount of Pepsi is 1.985 liters. Can we conclude that the machine is not working properly?

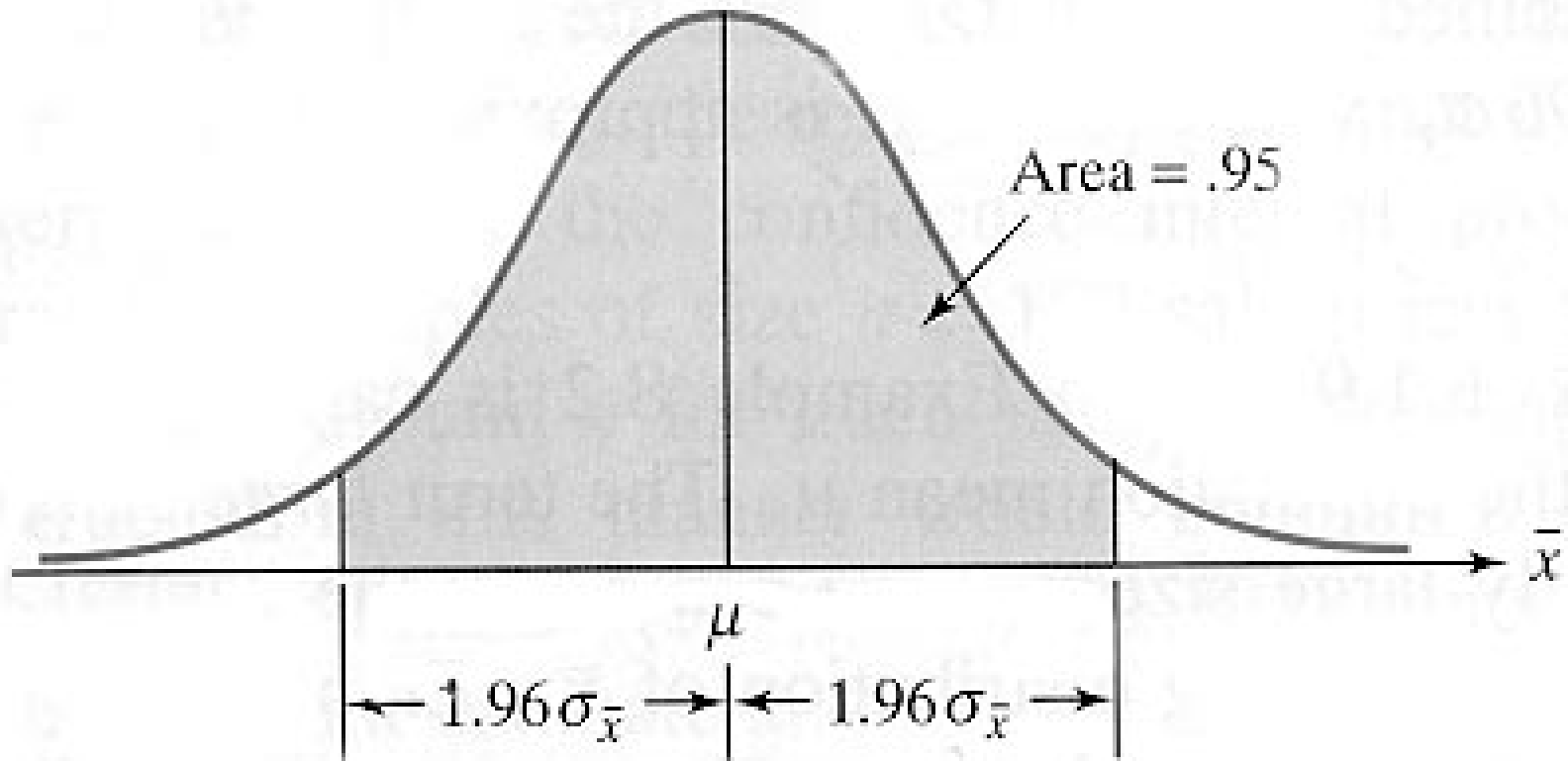
No! By simply reporting that $\bar{x} = 1.985$ liters, we are neglecting the fact that the amount of Pepsi varies from bottle to bottle and that the value of the sample mean depends on the luck of the draw. It is possible that a value as low as 1.985 is within the range of natural variability for \bar{X} , even if the average amount for all bottles is in fact $\mu = 2$ liters.

Suppose we know from past experience that the amounts of Pepsi in bottles filled by the machine have a standard deviation of $\sigma = 0.05$ liters.

Since $n = 100$, we can assume (using the Central Limit Theorem) that \bar{X} is normally distributed with mean μ (unknown) and standard error

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 0.005$$

From the Empirical Rule, the probability is about 95% that \bar{X} will be within two standard errors of its mean.



So the probability is about 0.95 that μ will be within 0.01 liters of \bar{X} .

Thus, the interval $\bar{X} \pm 0.01$ will contain μ with probability about 0.95.

In general, the interval $\bar{X} \pm 2 \frac{\sigma}{\sqrt{n}}$ will contain μ with probability about 0.95.

Therefore, the interval provides (approximately) a 95% confidence interval for μ .