Targeting in Advertising Markets: Implications for New and Old Media*

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Abstract

We consider a model with many advertising markets (media) and many products (advertisers). A continuum of buyers is distributed across the media markets. The advertisers purchase advertising messages separately on each market. The concentration of similar buyers in a given advertising market is a measure of the targeting ability.

We find that an increase in the targeting ability leads to an increase in the social value of advertisements as the total number of purchases (matches) increases. But an improved targeting ability also increases the concentration of advertisement messages and reduces the number of participating firms in each market. Surprisingly, we find that the equilibrium price is decreasing in the targeting ability over a large range of parameter values.

We trace out the implications of targeting for competing media markets by allowing for display advertisements and sponsored search. We find that the competition across media lowers the price of advertisement on the traditional medium. However, we show that competition by an online (targeted) medium lowers the revenues of old media more than competition by a similar (traditional) channel.

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JEL Classification: D44, D82, D83.

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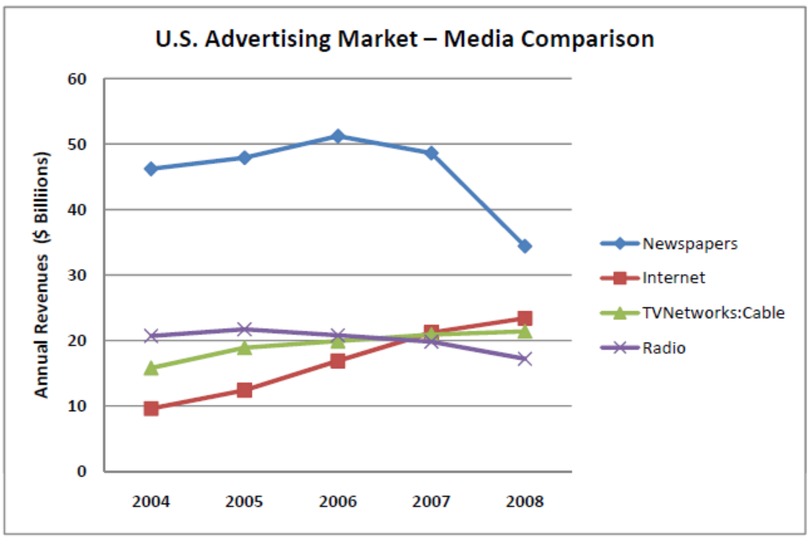
1 Introduction

Over the past decade the internet has become an increasingly important medium for advertising. The arrival of the internet has had important consequences on the market position of many traditional media, such as print, audio and television. For some of these media, most notably the daily newspaper, the very business model is under the threat of extinction due to competition from the internet for the placement of advertising. The following table shows the changes in aggregate spending for advertising on different media between 2004 and 2008.[1]



At the same time, through a variety of technological advances, the internet has allowed many advertisers to address a targeted audience beyond the reach of traditional media. In fact, it has been argued that the distinguishing feature of the internet is its ability to convey information to a targeted audience. In particular, targeting improves the quality of the match between the consumer and the advertisement message, and enables smaller businesses to access advertising markets from which they were previously excluded (the so-called "long tail of advertising" – see Anderson (2006)). While this holds for display advertising, it is even more true for sponsored search, where the individual consumer declares her intent or preference directly, by initiating a query.

[1] Source: Price Waterhouse Coopers annual reports for the Interactive Advertising Bureau.

The objective of this paper is to develop a model of competition between traditional and new media, in which the distinguishing feature of the new media is the ability to (better) target the audience. We are investigating the role of targeting in the determination of (a) the allocation of advertisements across different media, and (b) the equilibrium price for advertising. For this purpose, we first develop a framework to analyze the role of targeting, and then use this to model to analyze the relationship between offline and online advertising.

We present a model in which advertising creates awareness for a product. We consider an economy with a continuum of buyers and a continuum of products. Each product has a potential market size which describes the mass of consumers who are contemplating to purchase the product in question. Each consumer is contemplating only one of the available products and we take the preferred product as the characteristic of the consumer. The role of advertising is to turn a potential consumer into an actual consumer. The placement of an advertisement constitutes a message from the advertiser to a group of consumers. If the message is received by a consumer who is interested in the advertiser's product, the potential customer turns into an actual customer and a purchase is realized. A message received by a customer who is not in the market for the specific product is irretrievably lost and generates no tangible benefit for the advertiser. At the same time, a potential customer might be reached by multiple and hence redundant messages from the same advertiser. Consequently, the probability that a potential customer is turned into an actual customer is an increasing but concave function of the number of messages sent. The role of the advertisement is therefore to facilitate, but not to guarantee, a match between product and consumer.

We begin the analysis with a single advertising market in which all consumers are present and can be reached by any advertiser. It may be useful to think of the single advertising market as a national platform as offered by the nationwide newspapers or the major television networks. We show that in this market only the largest firms, measured by the size of their potential market, purchase any advertising space. We also show that the concentration of consumer types (i.e. the degree of asymmetry in the firms' potential market sizes) has an initially positive, but eventually negative effect on the equilibrium price of messages.

We then introduce the possibility of targeting by introducing a continuum of advertising markets. Each advertising market is characterized by the number of messages that it can send out to its audience. While each consumer is at most present in one advertising market, the likelihood of her presence in a specific market is correlated with her potential interest. For concreteness, the distribution of consumers across advertising markets is assumed to have a triangular structure. Namely, a consumer of type x is located with positive probability in one of the advertising markets labeled $k \leq x$, and is located with zero probability in advertising markets $k > x$. We assume that the distribution of the consumers across the advertising markets is given by an exponential distribution parametrized by γ. As the different consumer segments become more distributed over the different markets, the probability of a match between consumer and advertising is increasing. In consequence, the social welfare is increasing with the ability of the advertisers to reach their preferred audience. We then investigate the equilibrium advertising prices as the degree of targeting improves. While the marginal product of each message is increasing, thus potentially increasing the prices for the advertisement, a second and more powerful effect enters. As consumers become more concentrated, the competition among different advertisers becomes weaker. In fact, each advertiser is focusing his attention on a few important markets and is all but disappearing from the other markets. In consequence, the advertising price is declining even though the value of the advertising is increasing. The participation among advertisers is showing a similarly puzzling behavior. While the total number of advertisers participating across all markets is increasing – in particular, smaller advertisers are appearing with improved targeting – the number of actively advertising firms in each specific market k is decreasing.

In the second part of the paper we introduce competition among advertising markets for the attention of the consumer. Now, each consumer can receive a message from an advertiser on two different media markets. A single message received through either one of the markets is sufficient to create a sale. The "dual-homing" of the consumer across the two media markets may then lead to duplicative efforts by the advertiser. In consequence, the capacities in the competing advertising markets behave as strategic substitutes. We first describe the advertising allocation when the competitors are both traditional media without any targeting ability. In this case, messages on the two media are perfect substitutes, and equilibrium prices are equalized. Furthermore, the allocation of messages only depends on the total supply, not on its distribution across media.

The competition among two offline media markets presents a useful benchmark when we next consider competition between an offline and an online market. Thus, while each consumer is still only interested in one product, he can now receive messages on two media. The online and offline media are thus competing for every advertiser.

We analyze the interaction of offline media – such as newspapers or TV – with both display (banner) and sponsored search advertisements. Display advertisements allow for targeting through superior knowledge of the consumer's preferences (attribute targeting). Sponsored keyword search advertisements allow advertisers to infer the consumer's preferences from her actions (behavioral targeting). We find that the implications of these two online channels are similar, despite very different price formation mechanisms. In particular, behavioral targeting eliminates the redundancy risk on the online medium. As expected, competition lowers the price of advertisement on the traditional medium. However, if the online medium has a large enough capacity, it lowers the revenue of the offline media more than competition by another offline medium of the same size.

In the model with display advertising, we focus our attention on the case of perfect targeting. An important contrast between offline and online media is the representation of advertisers. The offline advertising market displays the same composition of advertisers as it would in the absence of the online market (only a few large advertisers are present), whereas the all advertisers are present in the online market. In other words, competition affects the equilibrium prices and revenues of different media, but not the qualitative properties of the allocation of messages on each medium.

This paper is related to several strands of the literature on advertising. Anderson and Coate (2005) provide the first competing broadcasters model, with exclusive assignment of viewers to channels; their setup is extended by Ferrando, Gabszewicz, Laussel, and Sonnac (2004), and Ambrus and Reisinger (2006) to the case of multiple channels. Anderson and Gabszewicz (2006) and Bagwell (2007) provide important surveys of the advertising and media markets literature. However, the role of targeting for the structure of advertising markets has received scant attention in the literature. The most prominent exception is Iyer, Soberman, and Villas-Boas (2005), who analyze the strategic choice of advertising in an imperfectly competitive market with product differentiation. In Iyer, Soberman, and Villas-Boas (2005), consumers are segmented into different audiences that firms can target with advertising messages. Advertising messages play a similar role in our paper and in Iyer, Soberman, and Villas-Boas (2005). Messages generate awareness of a product and complement consumer preferences in determining sales. However, Iyer, Soberman, and Villas-Boas

(2005) are mostly concerned with the equilibrium prices that result from the competitive advertising strategies. In contrast, we take sales prices as given, and focus our attention on the equilibrium prices of advertising messages themselves. Our results on equilibrium advertising prices and competing media are also in line with recent empirical work by Goldfarb and Tucker (2009). Goldfarb and Tucker (2009) exploit the variation in targeting ability generated by the legal framework. For example, certain advertisers may not be allowed to reach targeted audiences by regular mail. They show that prices for sponsored search advertising are higher when offline alternatives for targeted advertising are not viable. Finally, the theoretical literature on the economics of networks has investigated the profitability of reaching more or less well connected individuals with advertising messages. For example, Galeotti and Goyal (2009) and Campbell (2008) consider a monopolist selling a new product, and relate the properties of the underlying graph (e.g. the degree distribution) to the optimal influence strategy and equilibrium profits.

2 Model

We consider a model with a continuum of products and a continuum of advertising markets. Each product x is offered by firm x with $x \in [0, \infty)$. The advertising markets are indexed by $k \in [0, \infty)$. There is a continuum of buyers with unit mass and each buyer is present in exactly one product market and one advertising market. The consumers population is jointly distributed across products x and advertising markets k according to $F(x, k)$, with a density $f(x, k)$. The market share of product x is given by the marginal distribution

$$s_x \triangleq \int_0^\infty f(x, k)\, dk. \tag{1}$$

Firms are ranked, without loss of generality, in decreasing order of market share, so s_x is decreasing in x. Similarly, the size of the advertising market k is given by the marginal distribution

$$s_k \triangleq \int_0^\infty f(x, k)\, dx. \tag{2}$$

Each buyer is only interested in one specific product x. A sale of product x occurs if and only if the buyer is interested in the product and has received a message of firm x. A message by firm x is hence only effective if it is received by a buyer in segment x. In other words, we adopt the complementary view of advertising (see Bagwell (2007)), in which both the message and the right receiver are necessary to generate a purchase. Each sale generates a gross revenue of \$1, constant across all product markets.

The advertising policy of firm x determines the number of messages $m_{x,k}$ it distributes in advertising market k. Each message of advertiser x reaches a random consumer in advertising market k with uniform probability. Given the size of the advertising market s_k and the message volume $m_{x,k}$, the probability that a given consumer in market k is aware of product x is then given by:

$$\alpha\left(m_{x,k}, s_k\right) \triangleq 1 - \exp\left(-m_{x,k}/s_k\right). \tag{3}$$

The allocation of buyers across product and advertising markets is assumed to be governed by an exponential distribution. In particular, the market share of product x is given by:

$$s_x \triangleq \lambda e^{-\lambda x}. \tag{4}$$

The parameter $\lambda \geq 0$ measures the concentration in the product market, and a large value of λ represents a more concentrated product market. In turn, the conditional distribution of consumers in product segment x over advertising markets k is given by a (truncated) exponential distribution:

$$\frac{s_{x,k}}{s_x} \triangleq \begin{cases} e^{-\gamma x}, & \text{if} \quad k = 0, \\ \gamma e^{-\gamma(x-k)}, & \text{if} \quad k \leq x, \\ 0, & \text{if} \quad k > x. \end{cases} \tag{5}$$

The parameter $\gamma \geq 0$ measures the concentration of the consumers in the advertising markets. A large value of γ represents a more concentrated advertising market. The distributions of consumers across markets and products are conditionally independent. The corresponding unconditional market shares are now given by:

$$s_{x,k} \triangleq \begin{cases} \lambda e^{-(\lambda+\gamma)x}, & \text{if} \quad k = 0, \\ \lambda\gamma e^{-(\lambda+\gamma)x} e^{\gamma k}, & \text{if} \quad k \leq x, \\ 0, & \text{if} \quad k > x. \end{cases}$$

For $\gamma > 0$, we observe that the distribution of consumers over product and advertising markets has a triangular structure. The consumers who are interested in product x are present in all advertising markets $k \leq x$, but are not present in product market $k > x$. As we vary γ from 0 to ∞, we change the distribution and the concentration in each advertising market. With $\gamma = 0$, all consumers are located in the single large advertising market $k = 0$. As we increase γ, an increasing fraction of consumers of type x move from the large market to the smaller markets and as $\gamma \to \infty$, all consumers of type x are exclusively present in advertising market $k = x$. The limit values of γ, namely $\gamma = 0$ and $\gamma = \infty$, represent two special market structures. If $\gamma = 0$, then all consumers are present in advertising market 0 and hence there is a single advertising market. If, on the other hand, $\gamma \to \infty$, then all consumers of product x are present in advertising market x, and hence we have advertising markets with perfect targeting. .

Finally, the supply of messages M_k in every advertising market k is proportional to the size s_k of the advertising market and given by

$$M_k \triangleq s_k \cdot M,$$

for some constant $M > 0$. The constant M can be interpreted as the attention or time that each consumer allocates to receiving messages on the market where he is located. The equilibrium price p_k for messages placed in advertising market k is then determined by the market clearing condition in market k.

3 Single Advertising Market

We begin the equilibrium analysis with the benchmark of a single advertising market. In other words, consumers of all product market segments are present in a single advertising market $k = 0$, which corresponds to the case of $\gamma = 0$. Each firm x can now reach its consumers by placing messages in the single advertising market $k = 0$. Consequently, in this section we drop the subscript k in the notation without loss of generality. The objective of each firm x is to maximize the profit given the unit price for advertising p. The profit π_x is given by:

$$\pi_x = \max_{m_x} \left[s_x \alpha \left(m_x \right) - p m_x \right].$$

An advertising policy m_x generates a gross revenue $s_x \cdot \alpha(m_x)$. The information technology $\alpha(m_x)$, given by (3), determines the probability that a representative consumer is aware of product x, and s_x is the proportion of buyers who are in market segment x. The cost of an advertising policy m_x is given by $p \cdot m_x$. The optimal demand of messages by firm x is determined by the first order conditions and yields a demand function:

$$m_x = \ln \frac{s_x}{p}.$$

It is an implication of the above optimality conditions that firms with a larger market share s_x choose to send more messages to the consumers. In consequence, in equilibrium, the firms with the largest market share choose to advertise. Let $[0, X]$ be the set of participating firms, where X is the marginal firm, and let M be the total supply of messages. The market clearing condition is then given by

$$\int_0^X m_x dx = M. \tag{6}$$

Using the optimal supply of firm x and the formula for product market shares (4), we obtain

$$\int_0^X \left(\ln \frac{\lambda}{p} - \lambda x \right) dx = M. \tag{7}$$

The equilibrium price and participation are now determined by imposing $m_X = 0$ and the market clearing condition in (7). In particular, we obtain the competitive equilibrium (p^*, X^*) as follows:

$$p^* = \lambda e^{-\sqrt{2\lambda M}}, \tag{8}$$

$$X^* = \sqrt{2M / \lambda}. \tag{9}$$

By using these formulas in the equilibrium expressions, we obtain the competitive equilibrium allocation of messages for a single advertising market with a given capacity M,

$$m_x^* = \begin{cases} \sqrt{2\lambda M} - \lambda x, & \text{if} \quad x \leq X^*, \\ 0, & \text{if} \quad x > X^*. \end{cases} \tag{10}$$

To summarize, in the competitive equilibrium, the X^* largest firms enter the advertising market and the remaining smaller firms stay out of the advertising market. With the exponential distribution of consumers across products, the number of messages sent by an active firm is linear in its rank x in the market. The set of participating firms, the number of messages and the equilibrium price change continuously in λ and M. We determine how the equilibrium allocation depends on the primitives of the advertising market, namely λ and M, in the following comparative statics result.

Proposition 1 (Single Market, Comparative Statics)

1. *The equilibrium number of messages m_x^* is increasing in λ for all $x \leq X^*/2$.*
2. *The number of active firms X^* is increasing in M and decreasing in λ.*
3. *The equilibrium price p^* is decreasing in M for all λ.*
4. *The equilibrium price p^* is increasing in λ iff $\lambda < 2/M$.*
5. *The price per consumer reached is increasing in x. It is decreasing in λ for $x \leq X^*/2$.*
6. *The social value of advertising is increasing in λ.*

As the message volume M of the advertising market increases, the number of participating firms X^* also increases. The population of consumers is segmented in many categories. As the market becomes more concentrated in fewer categories, the number of actively advertising firms is decreasing as well. The equilibrium price responds in a more subtle way to the concentration measure λ in the product market. If the product market is diffuse, then an increase in the concentration measure essentially increases the returns from advertising for most of the participating firms. In other words, the demand of inframarginal firms has a larger effect on the price than the demand of smaller firms. If on the other hand, the concentration in the product market is already large, then a further increase in the concentration weakens the marginal firm's demand for advertising. At the same time, as the market share of the large firms is already substantial, their increase in demand for advertising is not sufficient to pick up the decrease in demand of the marginal firm. The additional demand of the large firm is weak because an increase in the already large advertising volume leads to many more redundant messages, which do not generate additional sales.

The dichotomy in the comparative static is thus driven by the determination of the marginal demand for advertising. If the source of the marginal demand is the marginal firm, then the price goes down with an increase in λ, and likewise if the marginal demand is driven by the inframarginal firms, then the advertising price is increasing with λ. In this sense, the non monotonic behavior of prices is not specific to the exponential distribution of firms' market shares. On the contrary, it is a consequence of the natural tension between competition and concentration.

Finally, notice that the competitive equilibrium implements the socially efficient allocation of advertisement messages (given λ). An easy way to see this is that with a uniform unit price of messages, the marginal returns to ads bought by different firms are equalized. A next natural question is how does the social value of advertising depend on product market concentration. Consider holding the allocation m_x^* fixed, and increasing λ. Now the total market share of the participating firms has increased, and fewer messages "get lost." At the new equilibrium, welfare will be even higher, as the allocation is adjusted for the new relative market shares of different products. In consequence, social welfare is increasing in the concentration measure.

One may wonder how relaxing the assumption of perfectly inelastic supply affects the comparative statics result in Proposition 1. For the case of constant supply elasticity $q = Mp^{\varepsilon}$, we can still show that the equilibrium price is first increasing, then decreasing in λ. However, if M is large enough, the equilibrium price will be always increasing in ε, and increasing in λ over a larger range. In particular, when market is very concentrated (so that the marginal demand is low), a more elastic supply reduces the number of active firms in the market. A further increase in concentration may then increase the demand of the active firms, and therefore also the price. For very high values of λ, demand "falls off" fast enough that the equilibrium price decreases. In particular, as λ goes to infinity, both the price and the quantity traded go to zero. However, since an increase in λ causes a drop in the quantity sold, the welfare result is now ambiguous.

The introduction of firm specific profit margins – which may be thought of as the value of a match – affects the equilibrium price on each advertising market. It also affects the distribution of messages and the number of active firms on each market. In the case of exponentially declining profit levels, the rate of decrease of profits plays a role similar to that of the concentration parameter. Intuitively, faster declining profits imply a more skewed equilibrium allocation of messages. As the profit margins are declining faster, the competitive equilibrium displays a decline in the number of participating firms, which is consistent with our hierarchical, mass-to-niche structure of product markets.

4 Many Advertising Markets

We are now in a position to analyze the general model with a continuum of advertising markets. The model with a single advertising market was described by $\gamma = 0$ and we now allow the targeting to be positive. The case of perfect targeting is described by $\gamma = \infty$. We described the distribution of consumers over different advertising markets by a (truncated) exponential distribution. The share of consumers who are active in product category x, and located in advertising market k is therefore given by (5). The share of consumers active in product market x and located in advertising market 0 is given by the residual probability of the product market segment x. As a result, the population size in advertising market $k > 0$ is given by the sum over the population shares,

$$s_{k>0} \triangleq \int_k^\infty \lambda\gamma e^{-(\lambda+\gamma)x} e^{\gamma k} dx = \frac{\gamma\lambda}{\gamma+\lambda} e^{-\lambda k}. \tag{11}$$

For advertising market $k = 0$, it is given by

$$s_{k=0} \triangleq \int_0^\infty \lambda e^{-(\lambda+\gamma)x} dx = \frac{\lambda}{\gamma+\lambda}. \tag{12}$$

The volume of advertising messages is assumed to be proportional to the population size of advertising market k, hence $M_k = s_k M$. The common factor M again expresses the attention devoted to messages by the consumers.

An important implication of the exponential distribution across advertising and product markets is a certain stationarity in the composition over the consumers across the advertising markets. In particular, the relative shares of the product markets are in specific sense constant across advertising markets. Namely, we have

$$\frac{s_{x,k}}{s_k} = (\lambda + \gamma)\, e^{-(\lambda+\gamma)(x-k)} = \frac{s_{x+n,k+n}}{s_{k+n}},$$

for all $x \geq k$ and all $n \geq 0$. Thus, while the exact composition of each advertising market is different, the size distribution of the competing advertisers are constant across advertising markets. The stationarity property allows us to transfer many of the insights of the single advertising market to the world with many advertising markets.

Now we consider the demand function of firm x in market k,

$$m_{x,k} = \arg\max_{m} \left[s_{x,y} \left(1 - \exp\left(-m/s_k\right)\right) - p_k m \right].$$

The first order condition for the firm's problem is given by

$$m_{x,k} = s_k \ln \frac{s_{x,k}}{p_k s_k}. \tag{13}$$

We can use (13), the market clearing condition and with the definition of the marginal firm ($m_{X_k^*,k} = 0$), we obtain the following conditions:

$$\begin{aligned} \int_k^{X_k^*} m_{x,k} dx &= s_k M, \\ \frac{s_{X_k^*,k}}{s_k} &= p_k. \end{aligned}$$

We then obtain the equilibrium prices p_k, the number active firms $X_k^* - k$, and the allocation $m_{x,k}^*$ of messages. In particular, the price and the number of active firms are stationary in the index k of the advertising market, that is:

$$p_k^* = (\gamma + \lambda) \exp\left(-\sqrt{2M(\gamma+\lambda)}\right), \tag{14}$$

$$X_k^* - k = \sqrt{2M/(\gamma+\lambda)}, \tag{15}$$

for all $k \geq 0$. The equilibrium advertising revenues on each market k are given by $R_k^* = s_k p_k^*$. Finally, the allocation of messages is given by

$$m_{x,k}^* = \begin{cases} \gamma\lambda e^{-\lambda k} \left(\sqrt{2M/(\gamma+\lambda)} - (x-k)\right), & \text{if} \quad k > 0, \\ \lambda \left(\sqrt{2M/(\gamma+\lambda)} - x\right), & \text{if} \quad k = 0. \end{cases} \tag{16}$$

Clearly, the larger firms $x \geq k$ receive a higher fraction of the message supply. If in particular we consider firm $x = k$, then the number of messages it receives is also increasing in the targeting ability.

The stationarity of equilibrium prices implies that the marginal utility of an additional message is equalized across markets. We therefore have the following result.

Proposition 2 (Efficiency)

1. *The efficient allocation of a fixed advertising space M is proportional to the size of the advertising market: $M_k = s_k \cdot M$.*

2. *The competitive equilibrium is efficient.*

3. *The social value of advertising is strictly increasing in the targeting ability γ.*

To understand the implications of targeting on social welfare, consider the effect of an increase in γ on the relative size of consumer segment x in advertising market $k = x$:

$$\frac{s_{x,x}}{s_{k=x}} = \gamma + \lambda.$$

We observe that better targeting increases the value that firm x assigns to a message in the advertising market $k = x$. Now let us consider holding the allocation of messages $m_{x,k}$ constant, and increasing the degree of targeting γ. The volume of matched consumers and firms is increasing because of the shift in the relative sizes of advertising markets. Since we know that the competitive allocation of messages is Pareto efficient, the equilibrium (for the new γ) has unambiguously improved the social value of advertising.

The comparative statics results (with respect to λ and M) do not differ qualitatively from the case of a single competitive market. More importantly, the effect of targeting ability γ and product market concentration λ on the equilibrium allocation is remarkably similar. In particular, the response of prices to changes in λ may be generalized as follows:

$$\operatorname{sign} \frac{\partial p_k^*}{\partial \lambda} = \operatorname{sign} \left(\frac{2}{M} - \lambda - \gamma \right).$$

In particular, prices are increasing in λ if both concentration and targeting are low enough. We now focus on the comparative statics with respect to γ, where a higher γ means more precise targeting.

Proposition 3 (Role of Targeting)

1. *The number of messages per capita* $m^*_{x,k}/s_k$ *is increasing in* γ *iff* $x < (k + X^*_k)/2$.

2. *The number of participating firms* $X^*_k - k$ *is decreasing in* γ.

3. *The equilibrium price* p^*_k *is increasing in* γ *iff* $\lambda + \gamma < 2/M$.

4. *The equilibrium revenue* R^*_0 *is decreasing in* γ. *The revenues* $R^*_{k>0}$ *are increasing in* γ *iff* $\gamma < (1 + \sqrt{1 + 2M\lambda})/M$.

The equilibrium number of messages $m_{x,k}$ is increasing in γ for the participating firms larger than the median firm active on each market k. Furthermore, more precise targeting implies a lower number of active firms. The relationship between targeting ability and equilibrium price is generally inverse-U shaped. However, if M or λ are large, then p^*_k is decreasing in γ for all values of γ. In other words, despite the increased social value of advertising, the equilibrium price of advertising is decreasing in the targeting ability over a large range of parameter values. In terms of revenues, it is immediate to see from equations (11) and (12) that an increase in γ leads to an increase in the size of markets $k > 0$ and to a decrease in the size of market 0. Since prices are constant, revenues in market 0 are decreasing in γ. Finally, targeting has the same qualitative effect on the equilibrium revenues in all markets $k > 0$.

We now come back to the similar effects of concentration and targeting. In particular, as with product market concentration, an increase in targeting γ reduces the demand of the marginal firm on each market k. At the same time, better targeting increases the demand of the inframarginal firms. The underlying tension is the one between identifying a consumer segment precisely, and finding several (competing) advertisers who are interested in it. The resulting trade-off between competition and inframarginal willingness to pay applies to a number of contexts, such as generic vs. specific keyword searches, and more or less precise attributes targeting on social networks. The trade-off can be ameliorated when we can maximize revenues by means of menus of contracts. In the context of our model, menu pricing is equivalent to block sales of messages. This additional instrument allows publishers to extract (a fraction of) the inframarginal rents, and therefore to serve a limited number of advertisers without suffering from decreasing marginal returns.

To conclude this section, we should point out that the exponential distribution provides particularly tractable expressions. For robustness, we have derived the main results under the alternative assumption of Pareto-distributed consumer preferences, and Pareto (or exponentially) distributed consumers on advertising markets. We find that the Pareto distribution implies *decreasing* prices p_k as k increases (fat tails mean more competition). With more probability mass on the tails, each market represents a smaller portion of the residual demand. This result holds true even if only the distribution of consumers' tastes is a Pareto distribution. At the same time, we also find that the number of participating firms is larger for smaller markets, where the distribution of customers is more uniform.

5 Different Targeting Parameters

Consumers interested in different products should not be expected to sort into different markets or web sites in a uniform way. For example, customers of smaller, less well-known, brands might be more dispersed, lacking a centralized site or portal; however, these buyers could also be concentrated in a particular market, say in the presence of a specialized "focal" website. Heterogeneity in sorting patterns can have very different revenue implications for sellers of both mass and niche products. In particular, we are interested in the different implications of decreasing vs. increasing degrees of concentration. In terms of our model, we now want to consider modifying the conditional density of consumers of product x in markets k, by allowing γ to depend on the identity of x. For example, we can define $\gamma(x) = \gamma x^n$ and obtain increasing or decreasing targeting ability depending on $n > 0$ or $n < 0$. We define the conditional distribution of consumers x in markets k as

$$s_{x|k} \triangleq \gamma(x) e^{-\gamma(x)(x-k)}.$$

The firms' message demands (13) can now be written as

$$m_{x,k} = s_k \ln \frac{s_x s_{x|k}}{p_k s_k}.$$

Let the marginal firm in market k is denoted by $X := X_k^*$ for ease of notation. We can then use the fact that $m_{X,k} = 0$ to write the market clearing condition as

$$\int_k^X s_k \ln \frac{s_x s_{x|k}}{s_X s_{X|k}} dx = s_k M.$$

We can now insert s_x and $s_{x|k}$, with $\gamma(x) = \gamma x^n$, and simplify as follows:

$$\gamma \frac{k\left(X^{n+1} - k^{n+1}\right)}{n+1} - \gamma \frac{X^{n+2} - k^{n+2}}{n+2} + \left(\frac{\gamma X^n}{2} + \frac{\lambda}{2}\right)(X-k)^2 - n(X-k) + nk \ln \frac{X}{k} = M$$

Similarly, we have the equilibrium price in market k:

$$p_k = \frac{s_X s_{X|k}}{s_k} = \frac{X^n e^{-\gamma X^n (X-k)} e^{-\lambda X}}{\int_k^\infty x^n e^{-\gamma x^n (x-k)} e^{-\lambda x} dx}. \tag{17}$$

We can then show how different targeting levels affect the the number of active firms as we consider smaller and smaller markets.

Proposition 4 (Long Tail Limits)
Let $\gamma(x) = \gamma x^n$, and $k \to \infty$. The number of active firms is given by

$$\lim_{k\to\infty} \left(X_k^* - k\right) = \begin{cases} \sqrt{2M/\lambda}, & \text{if } n < 0, \\ \sqrt{2M/(\gamma+\lambda)}, & \text{if } n = 0, \\ 0, & \text{if } n > 0. \end{cases}$$

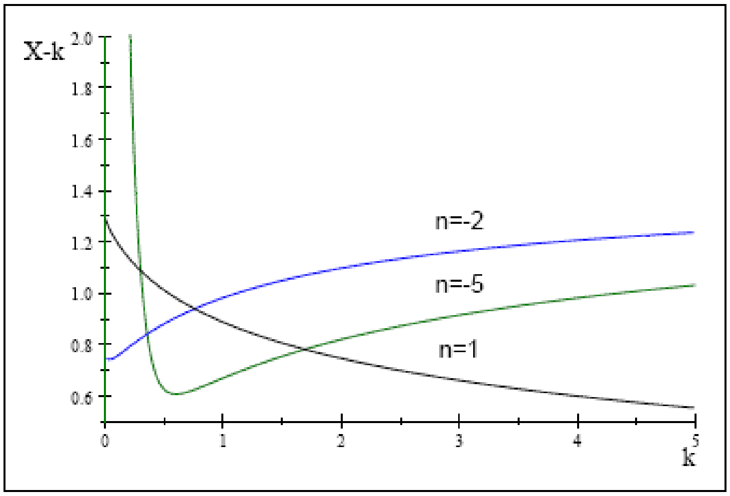
We can then immediately pair this result with a corollary on equilibrium prices.

Corollary 1 (Limit Prices)
Let $\gamma(x) = \gamma x^n$, and $k \to \infty$. The equilibrium prices are given by:

$$\lim_{k\to\infty} p_k^* = \begin{cases} \lambda \exp\left(-\sqrt{2\lambda M}\right), & \text{if } n < 0, \\ (\gamma+\lambda)\exp\left(-\sqrt{2M(\gamma+\lambda)}\right), & \text{if } n = 0, \\ 0, & \text{if } n > 0. \end{cases}$$

To summarize, if the ability to target is higher for smaller firms, these firms are able to reach a larger proportion of their customers on close-by advertising markets. The number of advertisers on large markets should then be relatively higher, as competition is fiercer.

Therefore, increasing targeting ability is more beneficial to smaller firms. The following picture provides an illustration of the equilibrium number of active firms when $n \in \{-5, -2, 1\}$.



If the ability to target is decreasing, then the demands of smaller firms increase even on larger, farther-away markets. However, the rate at which $\gamma(x)$ decreases can have significant effects on the equilibrium concentration of different markets. If $\gamma(x)$ decreases slowly, then "intermediate" markets are relatively highly concentrated, and this reduces competition on large markets, driving down the price and benefiting large firms. However, if $\gamma(x)$ decreases too rapidly, the market shares of firms participating in large markets are relatively homogeneous, with both large and small firms now having a substantial presence. Numerical results show that number of active firms is not necessarily monotonic in k, when $\gamma'(x) < 0$. In particular, some intermediate firms face less competition than large firms, even though their customer base is relatively more dispersed.

6 Market Interaction

So far, each consumer was only present in a single advertising market. We now study how the introduction of various forms of targeted advertising (display, sponsored search) – *in addition to general ads* – affects the equilibrium variables. In particular, we use our analysis of targeted advertising markets as a framework to assess the impact of new media on the distribution of advertisers and on the equilibrium prices on different platforms.

In the remainder of the analysis, we shall weaken the single-homing condition for the consumer to allow for dual-homing and multi-homing. Thus, we consider a model in which each consumer visits two media. A first effect of competition is therefore to multiply the opportunities for a match of an advertiser with a customer. At the same time, we maintain all the assumptions of the previous sections, namely that each buyer is only interested in one product, and that one message is sufficient to generate a sale. Therefore, competition also increases the risk of redundancy of advertisement messages. More precisely, the total awareness level generated by these messages will take the form $(a + b - ab)$, where a and b denote the fractions of a firm's set of customers reached through each advertising channel.

6.1 Two Traditional Platforms

As a benchmark, we consider the case of $\gamma = 0$. This corresponds to two offline platforms, A and B, competing for advertisers. Let M_A and M_B denote the time each consumer spends on each platform. Given an allocation of messages, we denote by a_x and b_x the probability of reaching a customer of product x in each platform. The rest of the analysis parallels the single market case. Each firm solves the following problem:

$$\begin{aligned}\pi_x &= \max_{m_x^A, m_x^B} \left\{ \lambda e^{-\lambda x} \left(\left(1 - e^{-m_x^A}\right) + \left(1 - e^{-m_x^B}\right) - \left(1 - e^{-m_x^A}\right)\left(1 - e^{-m_x^B}\right) \right) - p_A m_x^A - p_B m_x^B \right\} \\ &= \max_{m_x^A, m_x^B} \left\{ \lambda e^{-\lambda x} \left(1 - e^{-m_x^A - m_x^B}\right) - p_A m_x^A - p_B m_x^B \right\}.\end{aligned}$$

Clearly, each firm views advertising on platforms A and B as (perfect) substitutes, because of the loss in the frequency of a productive matches due to the dual-homing of the consumer. The demand for messages in platform A is given by

$$m_x^A = \ln \frac{\lambda (1 - b_x)}{p_A} - \lambda x.$$

Since the messages on the two platforms are perfect substitutes, we immediately obtain

$$\begin{aligned}1 - a_x &= e^{-m_x^A} = e^{\lambda x} \frac{p_A}{\lambda (1 - b_x)}, \\ 1 - b_x &= e^{-m_x^B} = e^{\lambda x} \frac{p_B}{\lambda (1 - a_x)}.\end{aligned}$$

It follows that in equilibrium we must have

$$\begin{aligned} p_A^* &= p_B^*, \\ m_x^A + m_x^B &= \ln \frac{\lambda}{p} - \lambda x, \\ \int_0^X \left(m_x^A + m_x^B\right) dx &= M_A + M_B. \end{aligned}$$

Let $m_x \triangleq m_x^A + m_x^B$, and $p^* = p_A^* = p_B^*$. We then obtain

$$\begin{aligned} p^* &= \lambda e^{-\sqrt{2\lambda(M_A+M_B)}}, \\ X^* &= \sqrt{\frac{2\left(M_A + M_B\right)}{\lambda}}, \\ m_x^* &= \sqrt{2\lambda\left(M_A + M_B\right)} - \lambda x. \end{aligned}$$

In terms of the number of active firms, competition generates a longer tail, simply through an increase in the supply of advertising space. It is interesting to observe that perfect substitutability of messages across the two media leads to indeterminacy of the precise allocation of messages to advertisers on each medium. In particular, both specialization of media – with firms purchasing space exclusively on one market – and proportional representation of advertisers on each medium may occur in equilibrium. Moreover, the allocation of advertisers to markets does not have implications on each medium's revenues, because the consumers' time allocation (M_A and M_B) is assumed to be exogenous. If we considered M_j as a strategic variable – such as capacity choice – then the interaction of similar media would give rise to quantity competition between the two publishers.

6.2 Traditional and Targeted Media

The internet has introduced at least two technological innovations in advertising, namely (a) the ability to relate payments and performance (e.g. pay per click), and (b) an improved ability to practice attributes and behavioral targeting. We now focus on the latter aspect. We are therefore motivated to investigate the allocation of advertising in the presence of a single traditional market, and of a more targeted medium.

We again consider "dual homing" consumers, who spend a total time of M on the offline medium and N in the (new) online medium. More specifically, M is the total supply on the traditional medium, and $s_k N$ is the total supply on each targeted market k. The targeted markets are here effectively thought of as specialized websites, so we refer to the traditional medium as "offline," and to the many targeted markets as "online." If firm x sends a total of m_x non targeted messages and $m_{x,k}$ targeted messages on each market k, its profit function is given by

$$\begin{aligned}\pi_x &= s_x\left(1-e^{-m_x}\right)+e^{-m_x}\int_0^x\left(s_{x,k}\left(1-e^{-\frac{m_{x,k}}{s_k}}\right)-p_k m_{x,k}\right)dk-pm_x\\ &= \int_0^x\left(s_{x,k}\left(1-e^{-m_x+\frac{m_{x,k}}{s_k}}\right)-p_k m_{x,k}\right)dk-pm_x.\end{aligned}$$

Because of the risk of duplication, messages sent online and offline are strategic substitutes for each firm. Clearly, this is not the case for messages sent to two different online channels (markets), as consumers only visit one website (in addition to the offline medium).

The analysis of firms' simultaneous choices of advertising offline and on several online channels is considerably complex. In general, each firm x will advertise on a subset of online markets $k \in [y(x), x]$, and the largest firms will also advertise offline. To obtain some intuition, and to simplify the analysis, we introduce a variation in our modeling framework. We assume the online medium allows to perfectly target consumers. More concretely, we assume that all consumers x are located on market $k = x$. Furthermore, the online medium sells up to $s_k N$ messages on each advertising market k. Consequently, each firm x will only advertise in the online market $k = x$. It will also be the unique competitor, and it will reach a fraction $(1 - e^{-N})$ of its customers.[2] We then ask what is the equilibrium unit price for these $s_k N$ messages, as a function of the firm's demands of messages offline. One advantage of this formulation is that the equilibrium allocation can be characterized easily. In particular, since $s_{k=x} = s_x$, we obtain

$$\begin{aligned}m_{x,x} &= \lambda e^{-\lambda x}N,\\ p_{k=x} &= e^{-N}e^{-m_x}.\end{aligned}$$

Therefore, the more firm x advertises on the offline medium, the lower the corresponding price on the online medium. Finally, the fraction of consumers of product x reached by a

[2] We can therefore think of this targeting technology as an imperfectly effective version of the $\gamma \to \infty$ case in the baseline model.

message in the online market is constant for all x and it is given by $\exp(-N)$. Therefore, the ranking of online market sizes does not modify the order of demands for offline messages. The equilibrium is then very similar to the single market case without competition. In particular, demands for messages are given by

$$\begin{aligned} m_x &= \arg\max_m \left\{ \lambda e^{-\lambda x} \left(1 - e^{-m}\right) e^{-N} - pm \right\} \\ &= \ln \frac{\lambda}{p} - \lambda x - N, \end{aligned}$$

which means that the time spent by the consumer in the online markets acts as a scaling parameter for the equilibrium price in the offline market. In equilibrium, we obtain

$$\begin{aligned} p^* &= \lambda e^{-N} e^{-\sqrt{2\lambda M}}, \\ X^* &= \sqrt{\frac{2M}{\lambda}}, \\ m_x^* &= \sqrt{2\lambda M} - \lambda x. \end{aligned}$$

We find that the equilibrium distribution of messages across participating firms, and the number of active firms, are both unchanged. In other words, market interaction affects prices and revenues, but not the allocation of messages to advertisers. Consistent with intuition, the offline price p^* is decreasing in N, reflecting the drop in each firm's willingness to pay for regular advertisements in the presence of an alternative, better targeted market. However, the equilibrium number of participating firms X^* does not depend on the online supply N: the presence of a targeted online market lowers profits but does not modify the composition of the offline market. Finally, the equilibrium prices on the online market are given by

$$p_k^* = \begin{cases} \exp\left(\lambda k - N - \sqrt{2\lambda M}\right), & \text{if } \quad k \leq X^*, \\ \exp(-N), & \text{if } \quad k > X^*. \end{cases}$$

These prices are increasing in k, because smaller firms buy a lower number of messages offline, and are willing to pay more for a supply of N online messages. They are clearly constant for all those markets (firms) who do not participate in the offline platform.

A natural issue at this point is to compare the effects of competition by an online vs. another offline medium, from the point of view of the traditional platform. In the next result, we compare the equilibrium prices under both kinds of platform competition. Consider the case of perfect targeting, and assume the consumer allocates a fraction α of her time T to the offline medium, or in other words $M = \alpha T$ and $N = (1-\alpha)T$. The equilibrium price on the offline market is given by

$$p^* = \begin{cases} \lambda \exp\left(-\left(\sqrt{2\lambda\alpha T} + (1-\alpha)T\right)\right), & \text{with a targeted competitor,} \\ \lambda \exp\left(-\sqrt{2\lambda T}\right), & \text{with a traditional competitor.} \end{cases}$$

We then establish how competition affects revenues of different media in the following comparison.

Proposition 5 (Price Comparison)
The equilibrium price with a targeted competitor is lower than the price with an identical traditional competitor iff

$$\lambda < (T/2)(1-\alpha)^2 \left(1-\sqrt{\alpha}\right)^{-2}.$$

From the point of view of offline media, equilibrium prices are lower with a perfectly targeted competitor for rather spread-out markets. For highly concentrated markets, a direct competitor is worse, as the interaction between the two publishers is similar to Bertrand competition. Furthermore, the time the consumer spends online $(1-\alpha)$ makes the online medium a relatively stronger competitor, compared to an offline medium with the same capacity.

7 Competition and Behavioral Targeting

Advertising messages sent over the internet are often described as having numerous advantages – at least in principle. Besides being directed at a selected audience, IP tracking and other technological advances often allow servers to keep track of which consumers have been hit with a particular message. In the context of our model, this would mean each firm x has a chance $m_{x,k}s_{x,k}/s_k$ of making a sale in online market k. In other words, the returns to messages are now linear. Therefore, if firm x buys the entire supply s_kN, then it generates awareness in a fraction N of its customers on that market.

With a linear matching technology of messages to consumers, selling ads is remarkably similar to sales of blocks of messages per capita. This in turn is analogous to sponsored search advertising auctions. We will therefore refer to the online medium as a search engine in this section, keeping in mind that our results on the prices of targeted and traditional messages may be easily translated into prices for other kinds of behaviorally-targeted messages.

7.1 Traditional Media and Sponsored Search

We now consider one search engine and one traditional offline medium. Each product is associated with a keyword, and each consumer searches for information about her favorite product via a search engine. Therefore, each consumer in market segment x visits both the generic advertising platform and the search results page for product x. We assume that consumers of product x click on the sponsored link pointing to their desired product (if it appears), irrespective of the search terms they entered and regardless of any messages they might have received offline. In particular, even consumers who are already aware of a product click on the link.

In this context, firms shade down their demand for messages offline, depending on their purchases online, and viceversa. We denote the probability of reaching a consumer of product x in the offline and online markets respectively by

$$\begin{aligned} a_x &\triangleq 1 - e^{-m_x}, \\ b_x &\triangleq \int_0^x \frac{s_{x,k}}{s_x} \frac{m_{x,k}}{s_k} dk. \end{aligned}$$

In other words, if firm x sends $m_{x,k} = s_k$ messages, it will hit all of its customers on market k. The total supply of messages on each page online is $s_k N$. We also assume the search engine displays the messages of G firms on each market. This is in analogy with the number of sponsored links offered on a keyword search results page. For simplicity, we also assume the consumer allocates her time uniformly among the displayed advertisers. Each firm's willingness to pay for each message in market k is then given by $(1 - a_x)\, s_{x,k}/\,(s_x s_k)$. It follows that the marginal firm on each market k is $x = k+G$, and each firm in in the interval $[k, k+G]$ sends $s_k N/G$ messages. The willingness to pay of the marginal firm determines the unit price of messages, which is therefore given by

$$\begin{aligned} p_k &= (1 - a_{k+G}) \frac{\gamma e^{-\gamma G}}{\gamma \lambda e^{-\gamma G}} (\gamma + \lambda) \\ &= (1 - a_{k+G}) (\gamma + \lambda) / \lambda. \end{aligned}$$

The equilibrium price is increasing in γ only if $\gamma < 1/G - \lambda$. If targeting is very precise, for a fixed G, the price is decreasing in γ, because of the decline in the marginal firm's share of each market. We can write the fraction of consumers of type x reached by a message as,

$$b_x = \begin{cases} \left(1 - \gamma e^{-\gamma G}\right) \frac{N}{G}, & \text{if} \quad x > G, \\ \left(1 - \gamma e^{-\gamma x}\right) \frac{N}{G} + \frac{N}{G} e^{-\gamma x}, & \text{if} \quad x \leq G. \end{cases}$$

We only need to remember that consumers of early, large targeted markets do not access messages on G markets, because of the triangular formulation of demand. On the other hand, they can access the larger exterior market $k = 0$. Now define the following constant

$$b \triangleq \frac{N}{G} \left(1 - \gamma e^{-\gamma G}\right).$$

Each firm $x > G$ scales down its demand for offline messages by a constant factor $1-b$, while firms $x \leq G$ reach a variable fraction of their consumers online. On the offline medium, firm x demands the following number of messages:

$$m_x = \ln \frac{\lambda (1 - b_x)}{p} - \lambda x.$$

We assume that the marginal firm offline is $X > G$. This is the case if M is large enough, or equivalently if G is small. In this case, the equilibrium price is given by $e^{-\lambda X} \lambda (1 - b)$, and the market clearing condition is given by

$$\int_0^G \ln \frac{1 - b_x}{1 - b} dx + \int_0^X \lambda (X - x) \, dx = M. \tag{18}$$

The first term on the left hand side of (18) represents the additional offline demand by the largest G firms. If $\gamma > 1$, then b_x is increasing in x, which means the larger firms reach relatively less consumers online. Now define the following function:

$$z(G) \triangleq \int_0^G \ln \frac{1-b_x}{1-b} dx,$$

and note that if $b > b_x$ then $z(G)$ is positive. Therefore, the larger firms effectively reduce offline supply for the smaller ones, by virtue of an increased willingness to pay for offline messages. The number of active firms, the price, and the allocation of messages are given by

$$\begin{aligned} X^* &= \sqrt{\frac{2(M - z(G))}{\lambda}}, \\ p^* &= (1-b)\lambda e^{-\sqrt{2\lambda(M - z(G))}}, \\ m_x^* &= \sqrt{2\lambda(M - z(G))} - \lambda x + \max\left\{\ln \frac{1-b_x}{1-b}, 0\right\}. \end{aligned}$$

Therefore, we find that both the composition and the profits of the offline advertisers are affected by the targeting technology of the search engine. We can now determine the prices of targeted messages online through the relationship $a_x = 1 - e^{-m_x}$:

$$p_k^* = \begin{cases} \frac{\gamma+\lambda}{\lambda} \exp\left(\lambda k - \sqrt{2\lambda(M - z(G))}\right), & \text{if} \quad k + G \leq X^*, \\ (\gamma+\lambda)/\lambda, & \text{if} \quad k + G > X^*. \end{cases} \tag{19}$$

Proposition 6 (Online and Offline Prices)
Let $\gamma \geq 1$ and $N \leq G$, then:

1. *online prices p_k^* are increasing in k if $k \leq X^* - G$, and constant if $k > X^* - G$;*

2. *the offline price p^* depends negatively on N and positively on G;*

3. *the number of active firms offline X^* depends positively on N and negatively on G.*

Once again, prices online are decreasing in the number of messages bought offline by the firms who also advertise online. As a consequence, over the range of markets for which firms only advertise online, prices are constant. They are initially increasing as we move down the tail, since the largest firms reach fewer consumers online, and buy larger amounts of messages online. Furthermore, the offline price depends positively on the number of firms who have access to each online market G, as this stimulates demand offline by the larger firms. Similarly, the number of firms active offline depends negatively on G. When online supply is very diluted, the offline market is highly concentrated, and viceversa.

7.2 Two Search Engines

We now interpret each targeted advertising market as a search engine results page. This allows us to expand our analysis of behavioral targeting by analyzing the equilibrium prices of sponsored search links with competition between two search engines, A and B. Denote by a_x and b_x the probabilities of reaching a consumer on a search engine. Search engines are endowed with a linear technology (behavioral targeting), so the willingness to pay of each firm x per message on keyword k, on search engine A, is determined by

$$(1-b_x)\left(\frac{s_{x,k}}{s_k}-p_k^A\right),$$

and similarly for search engine B. Each search engine allows for G firms to appear on each page k, the consumer spends her time uniformly both across engines and within each results page. The marginal firm is then $x=k+G$, and the unit price on A is given by

$$p_k^A=(1-b_{k+G})\,(\gamma+\lambda)\,e^{-(\lambda+\gamma)G},$$

and similarly for B. The messages reach firm x's customers with probability

$$a_x=\frac{N}{G}\left(1-\max\left\{e^{-(\lambda+\gamma)G},e^{-(\lambda+\gamma)x}\right\}\right).$$

If the search engines have identical targeting abilities $\gamma_A=\gamma_B=\gamma$, the symmetric equilibrium price is

$$p^*=\left(1-\frac{N}{G}\left(1-e^{-(\lambda+\gamma)G}\right)\right)(\gamma+\lambda)\,e^{-(\lambda+\gamma)G}.$$

If the search engines differ in their targeting ability (but offer the same number of links G), then we have

$$a_{x\geq G}=\frac{N}{G}\left(1-e^{-G(\lambda+\gamma_A)}\right),$$

and similarly for B. Hence, for $j\in\{A,B\}$,

$$p_j^*=\left(1-\frac{N}{G}\left(1-e^{-\left(\lambda+\gamma_{-j}\right)G}\right)\right)\left(\gamma_j+\lambda\right)e^{-\left(\lambda+\gamma_j\right)G}.$$

Proposition 7 (Comparative Statics)
The price $p_j^*(\gamma_A, \gamma_B)$ on each search engine j is:

1. *strictly decreasing in its competitor's targeting ability γ_{-j};*

2. *inverse-U shaped in the search engine's own targeting ability γ_j.*

Despite the different mechanisms for price formation and for the allocation of messages to advertisers, the effect of market interaction with a search engine is remarkably similar to that of competition by a targeted advertising platform. In both cases, market interaction affects equilibrium prices, not the allocation of messages within the set of active firms or the composition of this set. The equilibrium price of offline advertisements is decreasing in the ability of the online medium to target customers γ. In the case of behavioral targeting, as we let $\gamma \to \infty$, the offline platform is dominated by the online one (which can reach all the potential customers). However, somewhat surprisingly, the search engine's revenues are decreasing in γ for γ high enough, and converge to zero as $\gamma \to \infty$.

8 The Role of Prominence

In the previous section, the competition between the offline media and the online media lead to dual-homing of the consumer. In the present section, we shall restrict our attention to a single product market, but allow the consumer to be present in many media markets simultaneously, i.e. multi-homing. With a single product market, we allow for many advertising markets which differ in the degree of their centrality or prominence. A consumer who is not present in advertising market k is also not present in any advertising market beyond k. We again use an exponential distribution to represent the distribution of consumer across advertising markets k. In particular, the probability of a consumer being present in markets $[0, x]$ is given by

$$1 - F(x) = e^{-\lambda x}.$$

The parameter λ now represents the rate at which consumers abandon advertising markets. An advertising market with a lower index x is therefore more central as it has consumers who do not show up elsewhere, whereas a market with a higher index has consumers who are also exposed to messages on advertising markets with a lower index. It now follows that the price and the competition for the less central markets is weaker, and more central advertising markets carry higher prices for the same advertising messages. The message volume in market x is given by

$$M_x = (1 - F(x)) M = e^{-\lambda x} M.$$

Consider a consumer who appears in all markets until x. The probability that he has not seen a message is given by

$$\exp\left(-\int_0^x \frac{M_y}{e^{-\lambda y}} dy\right) = \exp(-Mx).$$

The marginal purchase condition in equilibrium is

$$\max_m \left\{ e^{-\lambda x} \frac{\lambda}{M+\lambda} e^{-Mx} \left(1 - e^{-\frac{m}{e^{-\lambda x}}}\right) - pm \right\}, \tag{20}$$

and the average probability of not having seen an add for the population in market x is

$$\frac{\int_x^\infty e^{-My} \lambda e^{-\lambda y} dy}{\int_x^\infty \lambda e^{-\lambda y} dy} = \frac{\lambda}{M+\lambda} e^{-Mx} \tag{21}$$

We can then look at the first order conditions of (20):

$$e^{-\lambda x} \frac{\lambda}{M+\lambda} e^{-Mx} e^{-\frac{m}{e^{-\lambda x}}} e^{\lambda x} - p = 0.$$

Market clearing implies $m_x = e^{-\lambda x} M$, and therefore the equilibrium price in market x is given by

$$\tilde{p}_x = \frac{\lambda}{M+\lambda} e^{-M(1+x)}.$$

We then establish the following properties of the competitive equilibrium allocation.

Proposition 8 (Centrality)

1. *The equilibrium price $\tilde{p}_x$ is higher for more central markets, and thus decreasing in x.*

2. *The equilibrium price $\tilde{p}_x$ is increasing in λ and decreasing in M.*

3. *For all λ and M, $\tilde{p}_x < \exp(-M) = p^*$.*

The fact that prices are declining suggests that the efficient allocation of capacity is not proportional to market size, but skewed in favor of more central markets. Furthermore, prices are increasing in λ. By increasing the rate at which markets shrink, we are reducing the chances of a duplicating messages (it is harder to find the same consumer in subsequent markets), hence increasing the willingness to pay for m_x and the market clearing price. We observe that λ decreases the size of all markets, but does not appear directly in the market clearing equation. Finally, the equilibrium prices in all markets $\tilde{p}_x$ are lower than the equilibrium price p^* in the single market model with a single product firm, or alternatively the price p_x^* in the many market model with perfect targeting.

9 Revenue Maximization

So far, we have studied competitive advertising markets with uniform unit prices for messages. In this section we consider the problem of a monopolist with the ability to price discriminate across different firms. We focus on the single advertising market, and continue to interpret the capacity M as the consumer's attention span. In consequence, a single firm sells M messages to a continuum of advertisers via nonlinear pricing schedules. Advertisers' types are given by their rank in the product markets (x). Given a direct mechanism $(m(x), p(x))$, the indirect utility of type x is given by

$$U(x) = \max_{\hat{x}} \left[\lambda e^{-\lambda x} \left(1 - e^{-m(\hat{x})} \right) - p(\hat{x}) \right].$$

We can write the incentive compatibility constraints for the direct mechanism through the first- and second-order conditions of the advertiser's problem. The seller then solves the following problem:

$$\begin{aligned}
&\max_{m(\cdot),U(\cdot)} \int_0^\infty \left(\lambda e^{-\lambda x} \left(1 - e^{-m(x)} \right) - U(x) \right) dx, \\
&\text{s.t. } U'(x) = -\lambda^2 e^{-\lambda x} \left(1 - e^{-m(x)} \right), \\
&\text{s.t. } m'(x) \leq 0, \\
&\text{s.t. } \int_0^\infty m(x)\, dx \leq M.
\end{aligned}$$

In the absence of production costs, the monopolist wants to sell as many units as possible, hence the capacity constraint will bind. As in the competitive case, the X largest firms participate, and the optimal message allocation is given by

$$m(x) = \lambda \left(X - x + \ln \frac{1 - \lambda x}{1 - \lambda X} \right).$$

The equilibrium set of active firms is given by $[0, X^*]$, where X^* is the solution to

$$\int_0^{X^*} \ln \frac{(1 - \lambda x) e^{-\lambda x}}{(1 - \lambda X^*) e^{-\lambda X^*}} dx = M. \tag{22}$$

The marginal prices charged by the seller can be easily characterized in terms of the non-linear tariff $p(m)$ through the advertisers' first order condition

$$\lambda e^{-\lambda x} e^{-m} = p'(m). \tag{23}$$

We therefore establish the following properties of the revenue maximizing allocation.

Proposition 9 (Revenue Maximization)

1. *The revenue maximizing allocation is given by*

$$m(x) = \begin{cases} \lambda \left(X^* - x + \ln \frac{1 - \lambda x}{1 - \lambda X^*} \right), & \text{if } x \leq X^*, \\ 0, & \text{if } x > X^*, \end{cases}$$

   where X^* is given by the solution to equation (22).

2. *The number of active firms X^* is strictly increasing in M and strictly decreasing in λ. For all λ and M, we have $X^* \in \left[0, \lambda^{-1}\right]$.*

3. *The average unit price $p(m)/m$ is decreasing in m.*

4. *The seller's revenue is increasing in λ and in M, and converges to e^{-1} as either $\lambda \to \infty$ or $M \to \infty$.*

The revenue maximizing allocation differs from the competitive one along several dimensions. First, compared to the competitive allocation, the equilibrium quantities are distorted downwards. For a given X, the revenue maximizing allocation assigns more messages to larger firms, since the term $\ln \frac{1-\lambda x}{1-\lambda X}$ is decreasing. Second, fewer firms participate in equilibrium, and the largest firms receives more messages than in the competitive allocation. This is possible because the seller can price differentially and extract more surplus from the advertisers. Third, the number of active firms bounded from above by $1/\lambda$, and the seller's equilibrium revenue converges to a positive number as $M \to \infty$. This is in sharp contrast with the competitive case, where the number of active firms converges to infinity as $M \to \infty$, and revenues vanish. Finally, unit prices are decreasing. As expected, the seller offers quantity discounts.

The analysis of competition among sellers is a natural question to address at this stage. Suppose that more than one seller is present on the market. As in the analysis of market interaction, suppose that each consumer divides her time equally across two or more media. Suppose further that sellers can adopt menu pricing. Remember that messages bought on different media are perfect substitutes for advertisers. If sellers offer incentive compatible tariffs as the one described above, advertisers will buy their entire supply from a single seller, so as to exploit declining unit prices. Given the menus offered by competitors, each seller will then want to cut prices for the most lucrative market segments, in this case the small advertisers who are paying high unit prices. It follows that in equilibrium all sales must take place at a constant unit price. Given the capacity constraints, the nonlinear pricing problem can now be reduced, for each seller, to a capacity choice.

We are therefore motivated to analyze competition among n identical sellers as a Cournot game, with advertising volume as the strategic variable. Each consumer spends a total time of M/n on each medium. Each seller chooses what level of capacity $q_j \in [0, M/n]$ to place on the market. Clearly, each seller could fill the entire time with messages, or withhold some capacity and keep prices high. Following our analysis in Section 6, we know the equilibrium price of messages is given by

$$p\,(q) = \lambda \exp(-\sqrt{2\lambda\Sigma_j q_j}).$$

The resulting symmetric equilibrium quantities and price are given by

$$\begin{aligned} q^* &= \min\{M/n, 2n/\lambda\}, \\ p^* &= \max\left\{\lambda e^{-2n}, \lambda e^{-\sqrt{2\lambda M}}\right\}. \end{aligned}$$

In the next proposition, we relate the imperfect competition prices and quantities with the competitive equilibrium benchmark.

Corollary 2 (Imperfect Competition)
For any M, there exists $n^*(M) \triangleq \sqrt{\lambda M/2}$ such that, for all $n \geq n^*(M)$, imperfect competition yields the competitive benchmark outcome.

In particular, notice that the capacity constraints imply that imperfect competition will have no impact on the equilibrium outcomes if the number of sellers is high enough. Intuitively, each seller's incentives to withhold capacity are highest in the monopoly case. Thus, our benchmark model may be viewed as describing a framework in which sellers have market power, but the number of competitors is high relatively to the time allocated to the medium.

10 Concluding Remarks

In this paper, we provide a systematic analysis of targeting, and of the trade-offs that arise due to changes in the targeting technology. We adopt a hierarchical framework to rank product and advertising markets of different sizes. We explore in particular the tension between competition and value extraction. This tension appears as general issue as the targeting ability of the various media improve. In this sense, our model can provide insight into the effects of detailed users information in the hands of social networks, on the profitability of IP address tracking, and of allowing sophisticated, differential bidding in keyword auctions.

The analysis we have presented is the outcome of a number of modeling choices which limit the scope of our results. We now conclude by discussing how a simpler, more symmetric setup would provide insights into different aspects of the media competition problem.

In particular, we contrast our hierarchical structure of markets with an alternative, in some respects more symmetric model. Suppose consumers and advertisers are uniformly located around a unit circle. Each location on the circle represents an advertising market, and the matching technology is as before. While the size of each market does not change, the relative distribution of consumers of type x across markets may vary. Suppose consumers of product x are distributed on the entire circle according to a truncated double exponential distribution. In line with our main findings, we obtain that the equilibrium price of advertising is decreasing in γ for M large enough, and single-peaked in γ for low levels of M. The symmetry of the circle model also enables us to analyze competition between two imperfectly targeted online media. In this model, we are able to solve for the equilibrium in closed form. We find that the equilibrium price on each platform is decreasing in the rival platform's targeting ability. Furthermore, it is single-peaked in the platform's own targeting ability for low levels of M.

Appendix

Proof of Proposition 1. (1.)–(4.) The comparative statics results can be derived directly by differentiating expressions (8), (9), and (10) in the text.
(5.) The total expenditure of firm $x \leq X^*$ is given by

$$p^* m_x^* = \lambda e^{-\sqrt{2\lambda M}} \left(\sqrt{2\lambda M} - \lambda x\right),$$

and the total number of consumers reached is

$$s_x \left(1 - e^{-m_x^*}\right) = \lambda e^{-\lambda x} \left(1 - e^{\lambda x - \sqrt{2\lambda M}}\right).$$

Therefore, the price paid by firm x per consumer reached is given by

$$\begin{aligned} \frac{p^* m_x^*}{s_x \left(1 - e^{-m_x^*}\right)} &= \frac{\lambda e^{-\sqrt{2\lambda M}} \left(\sqrt{2\lambda M} - \lambda x\right)}{\lambda e^{-\lambda x} \left(1 - e^{\lambda x - \sqrt{2\lambda M}}\right)} \\ &= \frac{\sqrt{2\lambda M} - \lambda x}{e^{\sqrt{2\lambda M} - \lambda x} - 1} = \frac{z}{e^z - 1}, \end{aligned}$$

which is decreasing in z (with $z = \sqrt{2\lambda M} - \lambda x$), and therefore increasing in x. It is also decreasing in λ if $x < \sqrt{M/2\lambda}$ (which represents the median active firm).
(6.) The average probability of a match, which is equal to the total fraction of consumers reached, is given by

$$\begin{aligned} W(\lambda, M) &= \int_0^{X^*} s_x \left(1 - e^{-m_x^*}\right) dx \\ &= \int_0^{\sqrt{2M/\lambda}} \lambda e^{-\lambda x} \left(1 - e^{-\lambda(X^* - x)}\right) dx \\ &= 1 - \frac{1 + \sqrt{2M\lambda}}{e^{\sqrt{2M\lambda}}}, \end{aligned}$$

which is increasing in λ. ■

Proof of Proposition 2. (1.) – (2.) The competitive equilibrium described in the text leads to uniform prices p_k^* across advertising markets. Therefore, for each firm and for each market, the marginal returns to messages are equalized. Since the match production function is concave in m, this allocation maximizes the probability of a match given the available supply of messages.

(3.) The average probability of a match now takes into account the fraction of consumers reached in the exterior market as well as in the interior markets. It is given by,

$$W(\lambda,\gamma,M)=\int_0^{\infty}\int_k^{X_k^*}s_{x,k}\left(1-e^{-\frac{m_{x,k}}{s_k}}\right)dxdk+\int_0^{X_0^*}s_{x,0}\left(1-e^{-\frac{m_{x,0}}{s_0}}\right)dx,$$

where $m_{x,k}^*$ is given by (16) in the text. Therefore, we obtain

$$W(\lambda,\gamma,M)=1-\frac{1+\sqrt{2M(\lambda+\gamma)}}{e^{\sqrt{2M(\lambda+\gamma)}}},$$

which is increasing in λ and γ. ■

Proof of Proposition 3. (1.) – (4.) These statements follow from differentiation of expressions (14), (15), and (16) in the text. ■

Proof of Proposition 6. (1.) Follows directly from differentiation of expression (19).
(2.–3.) We show that if $\gamma\geq 1$ and $N\leq G$, then

$$\frac{\partial z(G,N)}{\partial G}>0,\text{ and }\frac{\partial z(G,N)}{\partial N}<0.$$

By construction, $b_G=b$, so we have

$$\begin{aligned}\frac{\partial z(G,N)}{\partial G}&=\int_0^G\frac{\partial\ln\frac{1-b_x}{1-b}}{\partial G}dx\\ \frac{\partial z(G,N)}{\partial N}&=\int_0^G\frac{\partial\ln\frac{1-b_x}{1-b}}{\partial N}dx.\end{aligned}$$

We can then show that

$$\frac{\partial\ln\frac{1-b_x}{1-b}}{\partial N}\propto-\left((1-\gamma)e^{-(x-G)\gamma}+\gamma\right)<0,$$

which is negative for all $x\in[0,G]$. Finally, we have

$$\begin{aligned}\frac{\partial\ln\frac{1-b_x}{1-b}}{\partial G}\quad\propto\quad&\Big[(1-\gamma)e^{-(x-G)\gamma}+\gamma+\\ &G\gamma^2\left(1-(N/G)e^{-x\gamma}-(N/G)\left(1-\gamma e^{-x\gamma}\right)\right)\Big],\end{aligned}$$

the second term of which is positive if $(N/G) \leq 1$ and $\gamma \geq 1$. ■

Proof of Proposition 9. (1.) The modified Hamiltonian is

$$H(x, m, U, a) = \lambda e^{-\lambda x}\left(1 - e^{-m}\right) - U - a\lambda^2 e^{-\lambda x}\left(1 - e^{-m}\right) + b(M - m)$$

where $a(x)$ and b are the multipliers on the IC constraint and on the capacity constraint. The first order conditions are given by

$$\begin{aligned} \lambda e^{-\lambda x} e^{-m} - a\lambda^2 e^{-\lambda x} e^{-m} &= b \\ a'(x) &= 1 \\ a(0) &= 0. \end{aligned}$$

Therefore, we have

$$\begin{aligned} a(x) &= x, \\ m(x) &= \ln \frac{(1 - \lambda x)\lambda e^{-\lambda x}}{b}, \\ b &= \lambda e^{-\lambda X}(1 - \lambda X). \end{aligned}$$

From the market clearing condition, we have

$$\int_0^X \ln \frac{(1 - \lambda x) e^{-\lambda x}}{(1 - \lambda X) e^{-\lambda X}} dx = M,$$

and by defining $z := \lambda X$, we obtain

$$\frac{1}{2}z^2 - z - \ln(1 - z) = \lambda M. \tag{24}$$

(2.) The left-hand side of (24) is increasing in z, and therefore z is increasing in M, and $z(M) \in [0, 1]$. Furthermore, $z \to 1$ (so $X \to \lambda^{-1}$) as $M \to \infty$. We can derive the last comparative statics result from

$$\begin{aligned} \lambda X'(\lambda) &= \left(M \frac{z - 1}{z(z - 2)} - \frac{z}{\lambda}\right) \\ &= -(1 - z)\ln(1 - z) + \frac{1}{2}z(z + 1)(z - 2) < 0. \end{aligned}$$

(3.) From the buyer's first order condition, we know that

$$\lambda e^{-\lambda x(m)} e^{-m} = p'(m),$$

with

$$x(m) : \ln \frac{(1-\lambda x) e^{-\lambda x}}{(1-\lambda X) e^{-\lambda X}} - m = 0.$$

Differentiating, we obtain

$$p''(m) = -\lambda e^{-\lambda x(m)} e^{-m} \left(1 + \lambda x'(m)\right),$$

and so

$$p''(m) \propto -\left(1 - \frac{1-\lambda x}{2-\lambda x}\right) < 0.$$

Finally, since $p(0) = 0$, the concavity of the function $p(m)$ implies $p(m)/m$ is decreasing.

(4.) By the IC constraint, advertisers' utility is given by

$$\begin{aligned} U(x) &= \int_x^X \lambda^2 e^{-\lambda x} \left(1 - e^{-m(x)}\right) dx \\ &= \lambda \left(e^{-x\lambda} - e^{-\lambda X}\right) - (1-\lambda X) \lambda e^{-\lambda X} \ln \frac{1-\lambda x}{1-\lambda X}. \end{aligned}$$

Revenues are given by $\int_0^X p(x)\, dx$, which may be written as:

$$\begin{aligned} R &= \int_0^X \left(\lambda e^{-\lambda x} \left(1 - e^{-m(x)}\right) - U(x)\right) dx \\ &= X^2 \lambda^2 e^{-X\lambda} = z^2 e^{-z}. \end{aligned}$$

Since $R(z)$ is increasing for $z \in [0,1]$, we conclude that R is increasing in λ and M. Furthermore, $z \to 1$ implies $R \to e^{-1}$. ■

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