

Bundling and Competition on the Internet

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ABSTRACT

The Internet has significantly reduced the marginal cost of producing and distributing digital information goods. It also coincides with the emergence of new competitive strategies such as large scale bundling. In this paper, we show that bundling can create "economies of aggregation" for information goods if their marginal costs are very low, even in the absence of network externalities or economies of scale or scope.

We find that these economies of aggregation have several important competitive implications:

1. When competing for upstream content, larger bundlers are able to outbid smaller ones.
2. When competing for downstream consumers, the act of bundling information goods makes an incumbent seem "tougher" to single-product competitors selling similar goods. The resulting equilibrium is less profitable for potential entrants, and can discourage entry in the bundler's markets even when the entrants have a superior cost structure or quality.
3. Conversely, by simply adding an information good to an existing bundle, a bundler may be able to profitably enter a new market and dislodge an incumbent who does not bundle, capturing most of the market share from the incumbent firm and even driving the incumbent out of business.
4. Because a bundler can potentially capture a large share of profits in new markets, single-product firms may have lower incentives to innovate and create such markets. However, bundlers may have higher incentives to innovate.

For most physical goods, which have non-trivial marginal costs, the potential impact of large-scale aggregation is limited. However, we find that these effects can be decisive for the success or failure of information goods. Our results have particular empirical relevance to the markets for software and Internet content.

1. Introduction

1.1 Overview

The Internet has emerged as a new channel for the distribution of digital information such as software, news stories, stock quotes, music, photographs, video clips, and research reports. However, providers of digital information goods are unsure how to price, package and market them and are struggling with a variety of revenue models. Some firms, such as America Online have succeeded in selling very large aggregations of information goods – aggregations that would be economically impractical using conventional media. Others, such as Slate, have made unsuccessful attempts to charge for a more focused “magazine” on the Internet even as similar magazines thrive when sold via conventional paper-based media. Finally, companies such as the Wall Street Journal have successful offerings in both types of media, but their aggregation and pricing strategies differ depending on the medium used to deliver the content. At least some of these differences may be due to the differences in the underlying economics of information goods when they are delivered via different media.

One of the most important effects of the Internet infrastructure has been to radically reduce the marginal costs of reproducing and distributing information goods to consumers and businesses.¹ Information goods may thus enjoy economies of scale in production because of their low marginal costs, or demand-side economies of scale due to network externalities. Such technological economies of scale have important implications for competition, favoring large producers, and can lead to winner-take-all markets (Arthur 1996).

In this paper we analyze a distinct source of “economies of aggregation” for information goods which can be created by certain marketing and pricing strategies. Specifically, we study the effects of large-scale bundling of information goods on pricing, profitability and competition. We show that bundling strategies can offer economies of scale favoring producers that aggregate large numbers of information goods, even in the *absence* of network externalities or economies of scale or scope. While earlier analyses (Bakos and Brynjolfsson, 1999a,b) focused on the implications of bundling and other forms of

¹ In the remainder of this paper, we will use the phrase “information goods” as shorthand for “goods with zero or very low marginal costs of production”. In particular, our basic analysis is motivated by the way the Internet is changing dynamics of publishing (broadly defined to include all forms of digital content).

aggregation for a monopolist providing information goods, this paper considers the effects in competitive settings.

We demonstrate how marketing strategies that exploit these economies of aggregation can be used to gain an advantage when purchasing or developing new content. Firms can employ economies of aggregation to discourage or foreclose entry, even when competitors' products are technically superior. The same strategies can also facilitate predation: a bundler can enter new markets and force a competitor with a higher quality product to exit. Finally, bundling can affect incentives for innovation, decreasing them for firms that may face competition from a bundler while increasing them for the bundler itself. The potential impact of large-scale aggregation will generally be limited for most physical goods. However, when marginal costs are very low, as they are for digital information goods, the effects can be decisive.

Our analysis is grounded in the underlying economic fundamentals of pricing strategy as applied to low marginal cost good. It can help explain the existence and success of very large scale bundling, such as that practiced by America Online as well as the common practice of hybrid publishers, such as the Wall Street Journal of bundling several distinct information goods as part of their online package even as they sell the same items separately through conventional channels.² While our analysis is motivated by the pricing and marketing decisions that publishers face when they make their content available on the Internet, it may also apply in other markets where marginal costs are low or zero, such as cable television, software or even conventional publishing to some extent.

1.2 Bundling large numbers of information goods

Bundling may enable a seller to extract value from a given set of goods by allowing a form of price discrimination (McAfee, McMillan and Whinston, 1989; Schmalensee, 1984). There is an extensive literature in both the marketing and economics fields on how bundling can be used this way,³ but

² For instance, for a flat fee the *Wall Street Journal Interactive* includes not only the content of its paper-based cousin, but also the content of *Barron's* magazine, briefing books on thousands of companies, stock quote and historical charts, Dow Jones News articles and literally thousands other information goods. Delivering all this content via paper would be prohibitively expensive, limiting the size of bundles sold via conventional media.

³ For example, Adams and Yellen (1976), Schmalensee (1984), McAfee, McMillan and Whinston (1989), Hanson and Martin (1990), Eppen, Hanson and Martin (1991), Salinger (1995), Varian (1997). See Bakos and Brynjolfsson (1999a) for a more detailed summary of the pertinent literature for large scale bundling of information goods. In addition, the literature on "tying" and competition is relevant for this paper, especially Whinston (1990) and Burstein (1960).

very few articles have considered the bundling of more than two goods or focused on bundling information goods with zero or very low marginal cost.

Our paper especially builds on prior work by Bakos and Brynjolfsson (1999a). They consider the case of large-scale bundling of information goods and find that it can significantly increase a monopolist's profits. The benefits of bundling large numbers of information goods depend critically on the low marginal cost of reproducing digital information and the nature of the correlation in valuations for the goods: aggregation is less attractive when marginal costs are high or when valuations are highly correlated. We extend this line of research by considering how bundling affects the pricing and marketing of goods in certain competitive settings. Furthermore, by allowing for competition we can consider how large-scale bundling can create a barrier to entry by competitors, enable entry into new product markets and change incentives for innovation. Thus the paper also contributes to the literature on tying, particularly the work of Whinston (1990).

While the focus of this paper is on bundling strategies, the Internet clearly affects competition in many other ways. For instance, lower search costs, network externalities, high fixed costs, rapid market growth, changes in interactivity and other factors significantly affect marketing and competitive strategies. We abstract from these other characteristics of the Internet to better isolate the role of bundling on competition. Furthermore, we focus equilibrium strategies, recognizing fully that many Internet markets are not in equilibrium as we write this paper. As a result, it can be dangerous to extrapolate from current behavior, such as below-cost pricing, sometimes observed on the Internet. By analyzing and understanding the equilibria that result when firms compete in markets for information goods, we hope to gain insight into which outcomes are most likely when any temporary phenomena and disequilibrium strategies have dissipated.

1.3 Approach in this paper

Section 2 reviews the case of a monopolist bundling information goods with independent demands, and provides the necessary background, setting and notation for the analysis of the competitive implications of bundling. Section 3 addresses upstream competition between bundlers to acquire additional information goods, as in the case of competition for content. Section 4 analyzes downstream competition for consumers in a setting with information goods competing in pairs that are imperfect substitutes, including the case of a bundle competing with one or many outside goods. Section 5 explores the implications of the analysis in section 4, discussing how bundling strategies

affect entry deterrence, predatory behavior, and the incentives for innovation. Finally, section 6 presents some concluding remarks.

2. A Monopolist Bundling Information Goods with Independent Valuations

We begin by employing the setting introduced by Bakos and Brynjolfsson (1999a), with a single seller providing n information goods to a set of consumers Ω . Each consumer demands either 0 or 1 units of each information good, and resale of these goods is not permitted (or is prohibitively costly for consumers).⁴ Valuations for each good are heterogeneous among consumers, and for each consumer $\omega \in \Omega$, we use $v_{ni}(\omega)$ to denote the valuation of good i when a total of n goods are purchased. We allow $v_{ni}(\omega)$ to depend on n so that the distributions of valuations for individual goods can change as the number of goods purchased changes.^{5,6}

Let $x_n = \frac{1}{n} \sum_{k=1}^n v_{nk}$ be the mean (per-good) valuation of the bundle of n information goods. Let p_n^* , q_n^* and π_n^* denote the profit-maximizing price per good for a bundle of n goods, the corresponding sales as a fraction of the population, and the seller's resulting profits per good.⁷ Assume the following conditions hold:

A1: The marginal cost for copies of all information goods is zero to the seller.

A2: For all n , consumer valuations v_{ni} are independent and uniformly bounded,⁸ with continuous density functions, non-negative support, mean μ_{ni} and variance σ_{ni}^2 .

⁴ We assume that the producers of information goods can use technical, legal and social means to prevent unauthorized duplication and thus remain monopolists. However, Bakos, Brynjolfsson and Lichtman (1999) have employed a similar framework to study a setting where users share the goods.

⁵ For instance, the value of a weather report may be different when purchased alone from its value when purchased together with the morning news headlines, as they both compete for the consumer's limited time. Similarly, other factors such as goods that are complements or substitutes, diminishing returns and budget constraints may affect consumer valuations as additional goods are purchased. Even certain psychological factors which may make consumers more or less willing to pay for goods the same goods when they are part of a bundle (e.g. Petroschius and Monroe, 1987) can be subsumed in this framework. However, for simplicity, we treat all goods as symmetric; they are all assumed to be affected proportionately by the addition of a new good to the bundle.

⁶ To simplify the notation, we will omit the argument ω when possible.

⁷ For bundles, we will use p and π to refer to *per good* prices and gross profits, i.e., profits gross of any fixed costs. We will use P and Π to denote prices and profits for the entire bundle.

⁸ I.e., $\sup_{n,i,\omega} (v_{ni}(\omega)) < \infty$, for all n, i ($i \leq n$), and $\omega \in \Omega$.

A3: Consumers have free disposal. In particular, for all $n > 1$, $\sum_{k=1}^n v_{nk} \geq \sum_{k=1}^{n-1} v_{(n-1)k}$.⁹

Under these conditions, it can be shown that selling a bundle of all n information goods can be remarkably superior to selling the n goods separately.¹⁰ For the distributions of valuations underlying most common demand functions, bundling substantially reduces the average deadweight loss and leads to higher average profits for the seller. As n increases, the seller captures an increasing fraction of the total area under the demand curve, correspondingly reducing both the deadweight loss and consumers' surplus relative to selling the goods separately. More formally:

Proposition 1:

Given assumptions A1, A2 and A3, as n increases, the deadweight loss per good and the consumers' surplus per good for a bundle of n information goods converges to zero, and the seller's profit per good is maximized.

Proof: This is Proposition 1 of Bakos and Brynjolfsson (1999a).

The intuition behind Proposition 1 is that as the number of information goods in the bundle increases, the law of large numbers assures that the distribution for the valuation of the bundle has an increasing fraction of consumers with "moderate" valuations near the mean of the underlying distribution. Since the demand curve is derived from the cumulative distribution function for consumer valuations, it becomes more elastic near the mean, and less elastic away from the mean. Figure 1 illustrates this for the case of linear demand for the individual goods, showing, for instance, that combining two goods each with a linear demand produces a bundle with an s-shaped demand curve. As a result, the demand function (adjusted for the number of goods in the bundle) becomes more "square" as the number of goods increases. The seller is able to extract as profits (shown by the shaded areas in Figure 1) an increasing fraction of the total area under this demand curve, while selling to an increasing fraction of the consumers.

⁹ This assumption implies that adding a good to a bundle cannot reduce the total valuation of the bundle (although it may reduce the mean valuation).

¹⁰ In the remainder of the paper, our focus will be on "pure" bundling -- offering all of the goods as a single bundle. "Mixed" bundling, which involves offering both the bundle and subsets of the bundle at the same time for various prices will generally do no worse than pure bundling (after all, pure bundling is just a special case of mixed bundling), so our results can be thought of as a lower bound for the profits of the bundler.

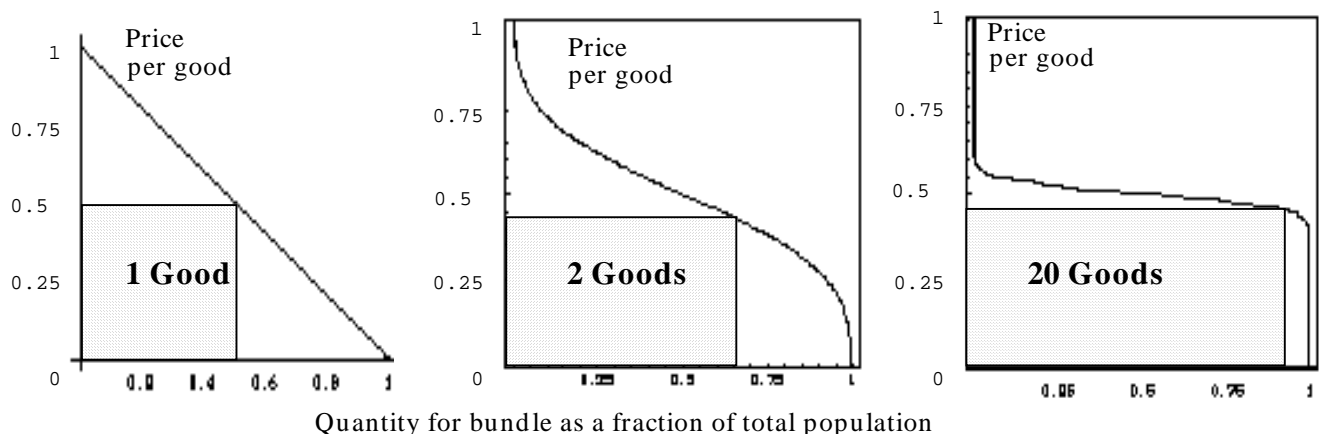


Figure 1 -- Demand for bundles of 1, 2, and 20 information goods with i.i.d. valuations uniformly distributed in [0,1] (linear demand case).

Proposition 1 is fairly general. While it assumes independence of the valuations of the individual goods in a bundle of a given size, each valuation may be drawn from a different distribution.¹¹ For instance, some goods may be systematically more valuable, on average than others, or have greater variance or skewness in their valuations across different consumers. Furthermore, valuations may change as more goods are added to a bundle. As shown by Bakos and Brynjolfsson (1999a), Proposition 1 can be invoked to study several specific settings, such as diminishing returns from the consumption of additional goods, or the existence of a budget constraint.

It is interesting to contrast the bundling approach with conventional price discrimination. If there are m consumers, each with a potentially different value for each of the n goods, then mn prices will be required to capture the complete surplus when the goods are sold separately. Furthermore, price discrimination requires that the seller can accurately identify consumer valuations and prevent consumers from buying goods at prices meant for others. Thus, the conventional approach to price discrimination operates by increasing the number of prices charged to accommodate the diversity of consumer valuations. In contrast, bundling reduces the diversity of consumer valuations so that, in the limit, sellers need charge only one price, do not need to identify different types of consumers, and do not need to enforce any restrictions on which prices consumers pay.

¹¹ The analysis in this section assumes independence to provide a useful baseline for isolating the effects of bundling. As shown by Bakos and Brynjolfsson (1999a) many of the results can be extended to the case where consumer valuations are correlated.

As the number of goods in the bundle increases, total profit and profit per good increase. The profit-maximizing price per good for the bundle steadily approaches the per good expected value of the bundle to the consumers. The number of goods necessary to make bundling desirable, and the speed at which deadweight loss and profit converge to their limiting values, depend on the actual distribution of consumer valuations. In particular, it is worth noting that although the per-good consumers' surplus converges to zero as the bundle grows, the total consumers' surplus from the bundle may continue to grow, but only at a lower rate than the number of goods.

The efficiency and profit gains that bundling offers in our setting contrast with the more limited benefits identified in previous work, principally as a result of our focus on bundling large numbers of goods and on information goods with zero marginal costs. In particular, if the goods in the bundle have significant marginal costs, then bundling may no longer be optimal. To see this, suppose that the costs were great than the mean valuation (but less than the maximum valuation), then selling the goods separately at a price above marginal cost, would be profitable. However, the demand for the bundle at any per-good price great than the mean approaches zero as the bundle size grows, reducing profits. Thus, because of differences in marginal costs, bundling hundreds or thousands of information goods for a single price online can be very profitable even if bundling the same content would not be profitable if it were all delivered on paper.

3. Upstream Competition for Content

In the previous section, we focused on the case of a monopolist selling large numbers of information goods either individually or in a bundle. We now look at the impact of competition. In this section we analyze a setting with firms competing for inputs (e.g., content) and in the next section we analyze downstream competition for consumers in a setting with information goods competing in pairs of imperfect substitutes.

Consider a setting similar to section 2 with n goods. There are two firms selling information goods, denoted as firm 1 and firm 2, to which we refer as the bundlers. These firms can be thought of as publishers selling information goods to the consumers, and we assume they start with respective endowments of n_1 and n_2 non-overlapping goods, where $n_1 + n_2 = n - 1$. We assume that consumers'

valuations are i.i.d. for all goods.¹² Thus different goods offered by the bundlers do *not* compete in the downstream market as they are not substitutes for each other.¹³ More formally we assume that:

A2': Consumer valuations v_{ni} are independent and identically distributed (i.i.d) for all n , with continuous density functions, non-negative support and finite mean μ and variance σ^2 .

In this setting we analyze the incentives of the two firms to acquire the n -th good in order to add it to their respective bundles. Specifically we consider a two-period game. In the first period the bundlers bid their valuations (y_1, z_1) and (y_2, z_2) for good n , where y denotes a valuation for an exclusive license and z denotes a valuation for a non-exclusive license.¹⁴ In the second period the n -th good is acquired by one or both firms depending on whether y_1 , y_2 or $z_1 + z_2$ represents the highest bid, provided that this bid is higher than the standalone profits that can be obtained by the owner of the n -th good. Subsequently, firms 1 and 2 simultaneously decide whether to offer each of their goods to the consumers individually or as part of a bundle (no mixed bundling is allowed), set prices for their offerings, and realize the corresponding sales and profits.

In this setting, if the bundles are large enough, it is more profitable to add the outside good to the bigger bundle than to the smaller bundle. More formally, the following proposition holds:

Proposition 2: Competition between bundlers for goods with i.i.d. valuations

Given assumptions A1, A2', and A3, and for n_1, n_2 large enough, then if $n_1 > n_2$, in the unique perfect equilibrium firm 1 outbids firm 2 for exclusive rights to the n -th good.

Proof: Proofs for this and remaining propositions are in the Appendix.

Proposition 2 builds on Proposition 1 by allowing for competition in the upstream market. It implies that the larger bundler (i.e. the one with the larger set of goods), will always be willing to spend the

¹² The assumption that the goods are identically distributed makes it possible to index the "size" of a bundle by simply counting the number of goods it contains.

¹³ In other words, the bundlers are monopolists in the downstream market, but not monopolists in the upstream market. We disregard any effects by which one monopolist's sales might affect another monopolist via consumer budget constraints, complementarities, network externalities, etc.

¹⁴ Because of the zero marginal cost of providing additional copies of the information good, we want to allow for the possibility that the information good is made available to both bundlers. We thank a referee for this suggestion.

¹⁵ It is easy to see that it would not be an equilibrium for the developer to sell the same good to two competing single-product downstream firms since monopoly profits are always greater than the sum of duopoly profits in the single-good case. Thus the revenue that could be earned by selling an exclusive contract, thereby preserving the monopoly, is greater than the revenue that could be earned by selling the same product to two competitors. However, if the downstream firms are already selling bundles that include other non-competing goods, then the analysis becomes much more complicated and this result does not automatically hold. By requiring exclusive contract by assumption, we defer the issue of downstream competition among bundlers until later in the paper.

most to develop a new good to add to its bundle and will always be willing to pay the most to purchase any new good which becomes available from third parties. Proposition 2 can be easily extended to a setting with more than two bundlers although the incentives for exclusive contracting may diminish as the number of competing bundlers increases.

In settings where the bundlers compete for new or existing goods one at a time, Proposition 2 implies that the largest bundler will tend to grow larger relative to other firms that compete in the upstream market, unless there are some offsetting diseconomies.¹⁶ Of course, if one or both bundlers understands this dynamic and there is a stream of new goods which can potentially be added to the bundles, then each bundler will want to bid strategically to race ahead of its rivals in bundle size and/or to prevent its rivals from growing. In this way, strategies for bundling are very similar to traditional economies of scale or learning-by-doing, as analyzed by Spence (1981) and others. The far-sighted bundler with sufficiently deep pockets should take into account not only how adding a good would affect current profits, but also its effect on the firm's ability to earn profits by adding future goods to the bundle.

In conclusion, large-scale bundling strategies may provide an advantage in the competition for upstream content. Because the benefits of aggregation increase with the number of goods included in the bundle, this provides a competitive advantage to a large bundler in purchasing or developing new information goods, even in the absence of any other economies of scale or scope.

4. Downstream competition for consumers

In this section we consider downstream competition, i.e., competition for consumers, by analyzing a setting where information goods are substitutes in pairs. In a setting similar to the one analyzed in section 2, consider two sets of n information goods A and B , and denote by A_i and B_i the i -th good in A and B respectively ($1 \leq i \leq n$). For all i , goods A_i and B_i are imperfect substitutes (see below). For instance, A_1 might be one word processor and B_1 a competing word processor, while A_2 and B_2 are two spreadsheets, etc. For each consumer $\omega \in \Omega$, let $v_{A_i}(\omega)$ and $v_{B_i}(\omega)$ denote ω 's valuation for A_i and B_i

¹⁶ We conjecture that the implications of Proposition 2 would be strengthened if the goods bundled were complements instead of being identically distributed (e.g., because of technological complementarities or network externalities). In this case, the economies of aggregation identified in Propositions 1 and 2 would be amplified by the advantages of combining complements.

respectively. As before, we will drop the argument ω when possible. To simplify the analysis, we assume that the goods A_i and B_i have independent linear demands with the same range of consumer valuations, which we normalize to be in the range $[0,1]$. Specifically,

A2'': For all i and all consumers $\omega \in \Omega$, all valuations $v_{A_i}(\omega)$ and $v_{B_i}(\omega)$ are independent and uniformly distributed in $[0,1]$.

Even though for each i , the valuations for A_i and B_i are independent, the two goods are substitutes in the sense that a consumer purchasing both goods A_i and B_i , enjoys utility equal only to the maximum utility that would have been received from purchasing only one of the two goods. In other words, the least valued good in each pair does not contribute to the consumer's utility if the other good is also owned. For example, a consumer who prefers Microsoft Excel to Lotus 1-2-3 does not get any additional value if she owns both programs than she would if she only owned Microsoft Excel. More formally,

A4: For all i and all $\omega \in \Omega$, consumer ω receives utility equal to $\max(v_{A_i}, v_{B_i})$ from purchasing both goods A_i and B_i .

Finally, we assume that development of the information goods involves a certain fixed cost:

A5: The production of good A_i , B_i involves a fixed cost of κ_{A_i} , κ_{B_i} respectively.

We consider a two period game with complete information: in the first period the firms invest κ_{A_i} , κ_{B_j} for all goods A_i and B_j that will be produced.¹⁷ In the second period, the firms decide whether to offer each of the goods individually or as part of a bundle (no mixed bundling is allowed), set prices for their offerings, and realize the corresponding sales and profits.

4.1 Competitive and monopoly provision of two substitute goods

We begin by considering the base case in which there is no bundling and thus the goods compete in pairs. This provides a benchmark for the subsequent analysis and allows us to introduce the setting. First we analyze the case when for each pair of goods two separate, competing firms each offer one good in the pair. Because of the independence of consumer valuations for goods i and j if $i \neq j$, competition takes place only between the two goods in each pair. Thus our setting corresponds to n

¹⁷ A good such as B_i that has no substitute can be modeled by setting the fixed cost of the corresponding good A_i to a value that would render its production uneconomical.

separate two-good markets. Dropping the subscripts indexing the pairs, suppose firm A provides good A and firm B provides good B. In this case, if firm A prices at p_A and firm B prices at p_B , the line-shaded areas in Figure 2 show the corresponding sales q_A and q_B , assuming that $p_A \geq p_B$. (Note that the assumption of uniformly and independently distributed valuations implies that consumers are evenly spread throughout the unit square, making it easy to map from areas to quantities demanded.) The increase in sales by firm B from a small decrease in price is gray-shaded and labeled δ .

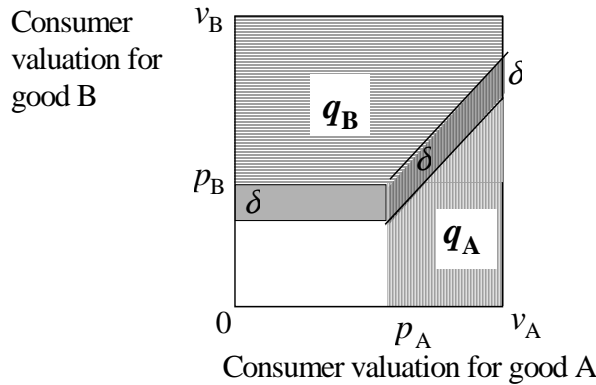


Figure 2: Competing imperfect substitutes

As shown in the Appendix, the unique equilibrium has prices $p_A^* = p_B^* = \sqrt{2} - 1$, quantities $q_A^* = q_B^* = \sqrt{2} - 1$, and corresponding gross profit $\pi_A^* = \pi_B^* = (\sqrt{2} - 1)^2$, or approximately 0.17.

Given the above equilibria in period 2, it is easy to see that any good with fixed cost less than $(\sqrt{2} - 1)^2$ will be produced when both firms offer competing goods. When a competing good is not produced, the remaining seller prices at the monopoly price of 0.5, selling to half the consumers and earning a maximum gross profit from a single good of 0.25. Thus, if a good has fixed cost above 0.25, it will not be produced even by a monopolist. If one good has fixed cost below $(\sqrt{2} - 1)^2$ and the other good has fixed cost above $(\sqrt{2} - 1)^2$, only the low cost good enters. Finally, if both goods have fixed cost between $(\sqrt{2} - 1)^2$ and 0.25, there are two pure-strategy equilibria with either one or the other good entering, but not both.

If a single firm (monopolist) provides both goods A and B, for instance because firms A and B merge, it will set the prices p_A and p_B to maximize its total revenues $p_A q_A + p_B q_B$. As shown in the appendix,

this yields optimal prices $p_A^* = p_B^* = 1/\sqrt{3}$, and corresponding quantities $q_A^* = q_B^* = 1/3$. Revenues are $\sqrt{3}/9$ per good (approx. 0.19). It is worth noting that if the monopolist bundles A and B, he will price the bundle at $1/\sqrt{3}$, and sell quantity $q_A^* + q_B^* = 2/3$, for the same total revenues.¹⁸ Thus the monopolist cannot increase his profits by bundling a single pair of such competing goods.

4.2 Competition between a single good and a bundle

To understand how bundling affects competition, we now analyze how prices and quantities are affected if firm B may include its good in a large bundle. Specifically, assume goods A_1 and B_1 compete as above. In addition firm B offers goods $B_2, B_3 \dots B_n$ and has the option to include any subset of its goods in a bundle. No mixed bundling is allowed, in the sense that each good can be offered either separately or as part of the bundle, but not both ways simultaneously.

Based on Props 1 and 2 of Bakos and Brynjolfsson (1999a), firm B increases its profits by including all goods $B_2, B_3 \dots B_n$ in a bundle; let the optimal bundle price be $p_{B2\dots n}^*$ per good, and the corresponding sales $q_{B2\dots n}^*$. The following proposition holds:

Proposition 3: Competition between a single good and a large bundle

Given assumptions A1, A2'', A3 and A4, for large enough n , in the unique equilibrium firm B will add good B_1 in its bundle, offering a bundle of n goods $B_1, B_2 \dots B_n$.

In the Appendix we show that the optimal quantity for the resulting bundle of n goods will converge to one as the number of goods increases. In other words, Proposition 3 implies that as n increases, firm B's bundle, which includes good B_1 , is ultimately purchased essentially by all consumers.¹⁹ Thus firm A must set its price for good A_1 given the fact that almost all consumers already have access to good B_1 . Figure 3 shows the fraction of consumers that will purchase A_1 at price p_{A1} , and thus firm A will choose p_{A1} to maximize $\frac{1}{2}(1 - p_{A1})^2 p_{A1}$, resulting in price $p_{A1}^* = 1/3$, corresponding sales $q_{A1}^* = 2/9$ and

¹⁸ This is a consequence of the fact that consumers do not derive additional value from their less-preferred good.

¹⁹ Under our assumptions, the quantity sold by the bundler grows monotonically (see Bakos and Brynjolfsson, 1999a, proposition 2). Thus the effect of bundling will be steadily diminished, but not eliminated for smaller bundles.

gross profit $\pi_{A1}^* = 2/27$ or approximately 0.07. As shown in the Appendix, firm B will increase its gross profits by at least 0.28 by adding B_1 to its bundle.²⁰

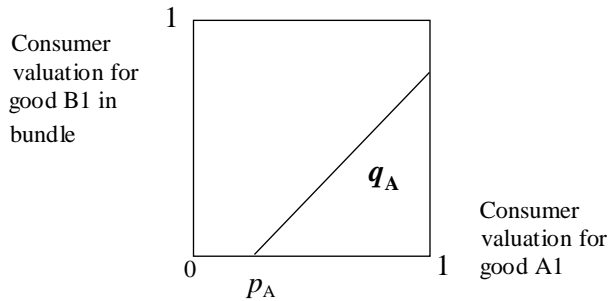


Figure 3: Good A1, sold separately, competes with good B1, part of a large bundle.

Compared to competition in the absence of bundling, firm A has to charge a lower price (.33 instead of .41), be limited to a lower market share (.22 instead of .41), and achieve substantially lower revenues (.07 instead of .17). By contrast, by including good B_1 in a large bundle, firm B will increase the revenues from the bundle by at least .28, and achieve market share close to 100% for good B_1 .

This phenomenon can be observed in the software markets. For instance, Microsoft Office includes numerous printing fonts as part of its basic package. This is easy to do given the low marginal cost of reproducing digital goods. This strategy has drastically reduced the demand for font packages sold separately while allowing Microsoft to extract some additional value from its Office bundle.

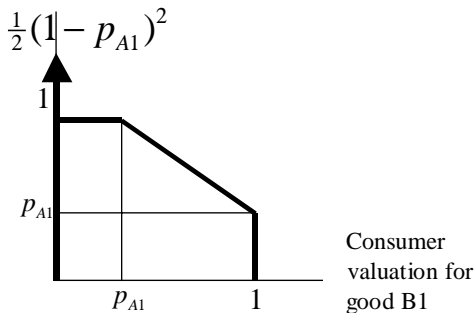


Figure 4: Distribution of valuations for good B1 (including an impulse at the origin), when good A1 is priced at p_{A1} .

²⁰ Although consumers' valuations are uniformly distributed for good B_1 , the actual demand faced by firm B will be affected by the availability and pricing of good A_1 . Figure 4 shows the derived distribution of valuations faced by firm B for good B_1 when firm A prices good A_1 at p_{A1} . The impulse at the origin represents the fact that a fraction $\frac{1}{2}(1 - p_{A1})^2$ of the consumers will purchase A_1 even if B_1 is offered at a price of zero. Given free disposal, we do not allow negative valuations. As expected, consumers' valuations for good B_1 increase as p_{A1} increases.

4.3 Competition between multiple goods and a bundle

We now consider n pairs of competing goods, where one good in each pair may be part of a bundle. In particular, firm B offers goods $B_1, B_2 \dots B_n$ as in the previous section, and has the option to combine them in a bundle. Goods $A_1, A_2 \dots A_n$ are offered independently by firms $A_1, A_2 \dots A_n$. Goods A_i and B_i compete just as A_1 and B_1 competed in the previous section ($0 \leq i \leq n$). As before, there is no private information, and the setting has two periods, with firms $A_1, A_2 \dots A_n$ and B deciding whether to invest $\kappa_{A_i}, \kappa_{B_i}$ respectively in period one. In period two, firm B decides for each good $B_1, B_2 \dots B_n$ whether to offer it as part of a bundle or separately, and all firms set prices and realize the corresponding sales and profits. No mixed bundling is allowed.

The analysis of the previous section is still applicable in this setting with many pairs of substitute goods, because valuations for goods in different pairs are independent. In particular, in each market i , if firm A_i prices good A_i at p_{A_i} , firm B faces a demand derived from the distribution of valuations shown in Figure 4. If good B_i is not offered as part of a bundle, then the equilibrium is as analyzed in section 4.1. If B bundles B_1 with the other goods, $B_2, B_3 \dots B_n$, then Proposition 1 applies and

$\lim_{n \rightarrow \infty} \frac{1}{n} p_{B_{1..n}}^* = \mu_B(p_A^*)$ and $\lim_{n \rightarrow \infty} q_{B_{1..n}}^* = 1$, resulting at the limit in average gross profit of $\mu_B(p_A^*)$ per

good, where $\mu_B(p_A^*)$ is the mean valuation of good B_i given that good A_i is priced at price p_A^* . It can

be seen from Figure 4 that $\mu_B(p_A^*) = \int_{x=0}^{p_A^*} x dx + \int_{p_A^*}^1 (1-x+p_A^*) x dx = \frac{1}{6} + \frac{1}{2} p_A^* - \frac{1}{6} (p_A^*)^3$. As shown in

section 4.2, as n gets large, the seller of a free standing good A_i that competes against good B_i that is offered in a bundle of n goods will maximize profits by charging approximately $p_A^* = \frac{1}{3}$. The bundler

thus faces a demand with mean valuation of about $\mu_B(p_A^*) = \frac{53}{162} \approx 0.33$, and for large n realizes gross profits of approximately .33 per good. Since not bundling a good implies its contribution to gross profits will be at best 0.25 (when there is no competing good), Corollary 1 follows:

Corollary 1: If goods A_i and B_i compete in pairs and only firm B is allowed to bundle, bundling all B_i 's is a dominant strategy.

Goods A_i will be produced if $\kappa_{A_i} < 0.07$. Firm B will include good B_i in its bundle when $\kappa_{B_i} < 1/3$ in the presence of a competing good A_i , and when $\kappa_{B_i} < 1/2$ if there is no competing good. If fixed costs

are between 0.07 and 0.33, then it is profitable for B to produce the good and sell it as part of the bundle even though it would be unprofitable for A to produce a similar good and sell it separately. Thus, a critical result of this analysis is that the bundler has an advantage competing in individual markets against individual outside goods. In section 5.4 we offer a more complete discussion of implications for entry, exit and predation.

4.4 Competition between rival bundles

In the setting of section 4.3, consider the case where goods $A_1, A_2 \dots A_n$ are offered by a single firm A which, like firm B, has the choice of offering each of its goods individually as above, or as a bundle of n goods priced at $p_{B1..n}$ per good, with resulting sales of $q_{B1..n}$. Proposition 1 now applies to both

firms, and $\lim_{n \rightarrow \infty} q_{A1..n}^* = \lim_{n \rightarrow \infty} q_{B1..n}^* = 1$, i.e. almost all consumers purchase both bundles.²¹ The valuation

$v_{A_i}(\omega)$ for good A_i given that consumer ω already owns good B_i is 0 with probability 0.5, and has the probability distribution function $1 - v_{A_i}(\omega)$ for $0 < v_{A_i}(\omega) \leq 1$. The corresponding mean valuation

is $\int_{v_{A_i}=0}^1 (1 - v_{A_i})v_{A_i}dv_{A_i} = \frac{1}{6}$. As n gets large, the optimal price equals the mean valuation, i.e.,

$\lim_{n \rightarrow \infty} \frac{1}{n} p_{A1..n}^* = \frac{1}{6}$ and $\lim_{n \rightarrow \infty} \frac{1}{n} p_{B1..n}^* = \frac{1}{6}$. Thus, if the bundles are very large, in the unique equilibrium both

A and B bundle at a price of $1/6$ per good and almost all consumers buy both bundles.

A consumer ω that owns both goods A_i and B_i , enjoys utility equal to $\max(v_{A_i}(\omega), v_{B_i}(\omega))$, and thus

her expected utility is $\int_{v_{A_i}=0}^1 \left(v_{A_i}^2 + (1 - v_{A_i}) \int_{v_{B_i}=v_{A_i}}^1 \frac{v_{B_i} dv_{B_i}}{1 - v_{A_i}} \right) dv_{A_i} = \frac{2}{3}$ per pair of goods. Thus, in the case of

rival bundles, as n gets larger, the per good deadweight loss converges to zero, and consumers keep average surplus of .33 per pair of goods. If the bundlers merge, on the other hand, they can capture the consumers' surplus per Proposition 1, and in the process double their gross profit.

²¹ Although consumers purchase both bundles, they do not necessarily use all the goods in each bundle. In our setting, a given consumer will use, on average, half the goods in each bundle - i.e. those that have higher valuations than the competing good in the other bundle.

5. Implications for Entry Deterrence, Predation and Innovation

Summarizing the analysis of section 4, Table 1 shows the market shares, prices and resulting revenues contributed by goods A_i and B_i when they compete under the different settings analyzed in section 4.

Setting	A_i	B_i
A_i and B_i compete as standalone goods (section 4.1)	$q_{A_i} = \sqrt{2} - 1 \approx 0.41$ $p_{A_i} = \sqrt{2} - 1 \approx 0.41$ $\pi_{A_i} = (\sqrt{2} - 1)^2 \approx 0.172$	$q_{B_i} = \sqrt{2} - 1 \approx 0.41$ $p_{B_i} = \sqrt{2} - 1 \approx 0.41$ $\pi_{B_i} = (\sqrt{2} - 1)^2 \approx 0.172$
A_i and B_i are both provided by a single monopolist who sells them as standalone goods (section 4.1)	$q_{A_i} = 1/3 \approx 0.33$ $p_{A_i} = \sqrt{3}/3 \approx 0.58$ $\pi_{A_i} = \sqrt{3}/9 \approx 0.19$	$q_{B_i} = 1/3 \approx 0.33$ $p_{B_i} = \sqrt{3}/3 \approx 0.58$ $\pi_{B_i} = \sqrt{3}/9 \approx 0.19$
A_i as a standalone good competes with B_i provided as part of a large bundle of unrelated goods (section 4.2)	$q_{A_i} = 2/9 \approx 0.22$ $p_{A_i} = 1/3 \approx 0.33$ $\pi_{A_i} = 2/27 \approx 0.07$	$q_{B_i} \approx 1$ $p_{B_i} \geq 5/18 \approx 0.28$ $\pi_{B_i} \geq 5/18 \approx 0.28$
Large number of standalone goods A_i competes with a large bundle of goods B_i (section 4.3)	$q_{A_i} = 2/9 \approx 0.22$ $p_{A_i} = 1/3 \approx 0.33$ $\pi_{A_i} = 2/27 \approx 0.07$	$q_{B_i} \approx 1$ $p_{B_i} \approx 52/162 \approx 0.33$ $\pi_{B_i} \approx 52/162 \approx 0.33$
Large bundle of goods A_i each competing with B_i , also provided as a large bundle (section 4.4)	$q_{A_i} \approx 1$ $p_{A_i} \approx 1/6 \approx 0.167$ $\pi_{A_i} \approx 1/6 \approx 0.167$	$q_{B_i} \approx 1$ $p_{B_i} \approx 1/6 \approx 0.167$ $\pi_{B_i} \approx 1/6 \approx 0.167$

Table 1: Equilibrium quantities, prices and revenues in different settings

In this section we discuss the implications of the above analysis for how the strategy of marketing and selling information goods as a bundle can affect competition and profits.

5.1 Bundling and acquisitions

In the setting of proposition 1 where all goods are independent of each other, as the number of goods in the bundle increases, revenues from the bundle increase monotonically. Because there are no fixed costs in this setting, the bundle becomes increasingly profitable. Interestingly, this result carries to the setting of section 4, in which the added goods may compete with other goods in the bundle. In other words, in addition to bundling goods in \mathcal{B} , the bundler will also want to acquire goods in \mathcal{A} . The following proposition holds:

Proposition 4: Monotonic Bundling Profits

Given A_1, A_2, A_3 and A_4 , a bundler of a large number of goods from \mathcal{B} will increase the profits extracted from any good A_i by adding it to its bundle, whether or not the bundle already contains the corresponding substitute good B_i .

Proposition 4 implies that large bundlers of information goods will be willing to acquire related (i.e., competing) as well as unrelated information goods to add them to the bundle. As a result, a bundle may include competing news channels, such as a cable TV bundle that includes both CNN and CNBC. When acquiring both items in a competing pair of goods, a firm will be able to extract more profit by selling the pair as a bundle (0.66 in total profits) than by selling each item in the pair separately (0.38 in total profits).²² Furthermore, a firm's incentives to acquire a second good in such a pair are much higher if it intends to sell it as part of the bundle than if it plans to sell the good separately. The total revenues extracted from the pair of goods by all firms increases by over 0.26 when they are sold as a bundle.²³ The increase in profits is only 0.04 if the two goods continue to be sold separately.²⁴

5.2 Bundling and entry deterrence

We now look at a bundler of a large number of goods facing a single potential entrant. For convenience we will focus on a potential entrant considering whether to offer good A_1 , competing against good B_1 in the bundle. Because B_1 is part of a large bundle, the entrant in effect faces a more "aggressive" incumbent: as shown in section 4.2, it is optimal for the incumbent bundler to price the bundle so that he maintains a market share of almost 100% even after entry. As far as the demand for the entrant's good is concerned, it is as if the incumbent is credibly willing to charge almost zero for good B_1 . As a result, entry will be deterred for a broader range of entry costs.²⁵ For instance, if good B_1 is part of a bundle then it would be unprofitable for a potential competitor to enter and sell A_1 as long as his fixed costs exceed 0.07. In contrast, as shown in sections 4.1 and 4.2, the entrant could profitably enter with fixed costs as high to 0.19 if B_1 were sold as a stand alone good. Thus firms with

22 Contrast this with the case of a bundler of only two competing goods, in which case no additional profits are gained by bundling.

23 Before acquisition, the revenues are no more than 0.33 for the good in the bundle plus 0.07 for the outside good. After acquisition, they are 0.66 when they are sold as a bundle, for a gain of at least 0.26.

24 The sum of the profits of the competing firms is 0.34 while the profits rise to 0.38 if both goods are sold separately by a single profit-maximizing firm.

fixed costs between 0.07 and 0.19 will be deterred from entry if B_1 is made part of the bundle. By continuity, this result can extend to the case where the bundling incumbent has slightly higher production costs than the single-product entrant, a product on the average valued less than the entrant's product (i.e. lower "quality"), or both. In each of these cases, the entrant will not find it profitable to enter despite a superior cost structure or quality level.

This argument also applies in the case of a bundler of n goods facing n potential entrants: If the n goods $A_i \in \mathcal{A}$ in the setting analyzed in section 4.3 are seen as potential entrants competing with the goods B_i in set \mathcal{B} , the following Corollary follows for large values of n :

Corollary 2: Entry Deterrence

Given A_1, A_2, A_3 and A_4 , if the goods A_i cannot be offered as a bundle, then there is a range of fixed costs κ_{A_i} for which none of these goods is produced, even though they would be produced if the products in \mathcal{B} were offered separately.

It should be noted that the "aggressive" pricing of the bundler and the resulting entry deterrence is not based on any threats or dynamic strategies, e.g. artificially lowering prices in the short run to prevent entry. The bundler is simply choosing the price that maximizes profits in the current period. It happens that the optimal pricing policy for the bundler also has the effect of making entry very unattractive. Of course, this can make bundling even more profitable if it reduces competition.

5.3 Coordinated entry by offering a rival bundle

If the potential entrants can offer a rival bundle (e.g., if they merge or can coordinate their entry and pricing) they may be able to profitably enter the incumbent's markets. Specifically, if the entrants all enter simultaneously as a bundle, then they can all gain sufficient market share to stay in business, even if they would have had to exit had they entered separately. The intuition for this result is that when the entrants offer their own bundle of products that compete with the original bundle, then a large number of consumers will find it worthwhile to purchase *both* bundles, because there is no correlation among the valuations of the goods in each bundle, which means that for each pair of

25 This is similar to the result in Whinston (1990) except that in our model, the power of the incumbent's bundle derives from the large number of goods in it, not an inherent characteristic of any particular good.

goods, each consumer is equally likely to find their preferred good in either bundle (see section 4.4 above).

Corollary 3: In the setting of sections 4.3 and 4.4, if the A_i 's can be offered as a bundle, for large n the total revenues for the entrants are maximized by bundling the A_i goods, and the unique equilibrium is characterized by two bundles, each selling at a price of $\frac{1}{6}$ per good, and consumers buying both bundles.

Thus, if they can coordinate and offer a competing bundle of the A_i goods, the A firms will find it profitable to enter even for fixed costs as high as 0.18. When fixed costs are between 0.07 and 0.18, it is unprofitable for the firms to produce the A_i goods and sell them separately, but it is profitable to produce and sell them as a bundle that competes with the B bundle. Consumer welfare is significantly increased by the availability of the competing bundle, increasing from little more zero if only one bundle is sold with no competitors to about 0.33 if both bundles are sold.

5.4 Bundling, predation and exit

We now consider the case in which the incumbent firm sells a single information good and the entrant sells a large bundle of unrelated goods. This case is the reverse of the case in section 5.2 above: If the incumbent sells good A_i and a bundler sells a large bundle of unrelated goods $\mathcal{B} - B_i$, then the bundler can enter the new market by adding good B_i to its bundle, even if it would have been unprofitable to enter the market with a stand-alone good.

The reason is that, as shown in section 4.2, the equilibrium for competition between a bundler and a seller of a single good with identical production costs and quality leaves the bundler with the majority of the market and higher profits while the profits of the single-product firm are reduced. If production were just barely profitable for a single product firm selling good A_i competing with another single product firm selling good B_i , then it would become unprofitable for a single product firm selling good A_i competing with a bundler selling good B_i as part of a large bundle. For instance, in the setting of section 4.2, the act of bundle could force a competing firm to exit if its fixed costs were greater than the 0.07 that would be earned when competing with a bundler. Without bundling, the same firm would not have exited as long as its fixed costs were greater than the 0.17 that could be earned when competing with another single product firm.

Similarly, there is a range of fixed costs for which entry is profitable if and only if the entrant sells a bundle:

Corollary 4: A entrant who sells a large bundle can force a single-product incumbent firm to exit the market even if it could not do so by selling the same goods separately.

If there are fixed costs of production then, the incumbent may find it unprofitable to remain in the market with a reduced market share. As a result, the bundler can successfully pursue a predatory strategy of entering new markets and driving out existing firms.

If the entrant did not have a superior product or cost structure, this strategy would not be credible or successful if the entrant sold its products separately. A threat to temporarily charge very low prices in an attempt to drive out the incumbent would not be credible if the entrant's goods were sold separately. If the incumbent did not exit, the entrant would not be willing to follow through on such a threat; it would not be a subgame perfect equilibrium strategy. However, the mere fact that the entrant has the option of including the new good as part of an existing bundle will immediately make it a more formidable predator. Now, it is credible for the entrant to charge low enough prices to maintain a very high market share. What's more, even with this "aggressive" pricing strategy, the entrant will be more profitable when it bundles its products than if would be if it did not bundle. Thus, even when the incumbent has lower fixed costs, lower marginal costs or higher quality than the entrant, it may be forced to exit when the entrant uses the bundling strategy.

5.5 Bundling and incentives for innovation

Assume that firm A is considering an investment in a market that does NOT currently face any competition. It can create an innovation at some irreversible cost and enter the market. Suppose second firm could, for the same fixed costs and marginal costs and with a similar quality, follow the first firm into the market, leading to a competitive equilibrium among imperfect substitutes, as in section 4.2. For a range of values, this equilibrium will result in sufficient profits for the first firm to undertake the innovation even in the face of potential entry by a similar firm. If fixed costs are somewhat higher, the equilibrium duopoly profits will be sufficiently lower than the monopoly profits

that the first firm will enter the market, but the second firm will not. In either case, the innovation will be undertaken.²⁶

However, now that the second potential entry is a bundler of a set of similar goods in different markets. As in the analysis of predation above, an entrant who is a bundler can quickly capture most of the market share in the new market.

Knowing of this possibility, what are the firm A's incentives for innovating? Clearly, firm A will find it less profitable to innovate and create new markets since it cannot keep as large a share of the returns. Instead of earning the monopoly profits or half the total duopoly profits, firm A will keep only a fraction of the market and much lower profits (0.07 in the setting above). Incentives for innovation by such firms will be reduced, and fewer innovations will be funded and undertaken.

This result is consistent with claims by some entrepreneurs that venture capitalists will not fund their ventures if there is a significant risk that a competing products might be incorporated into a large bundle sold by a potential predator. The investors are rightly especially fearful of a potential competition by a firm that already controls a large bundle of information goods. A potential competitor who merely sells a standalone good cannot as easily take away the market from the innovator.

It is important to note that the effect of bundling on innovation extends beyond the product markets in which the bundler currently competes. If a potential innovator believes that a bundler may choose to enter some *new* market that could be created by the innovator, then the innovator's incentives will be reduced. Some innovations will be unprofitable in this situation even if they would have been profitable to undertake had there been no threat of entry by the bundler or if the only possible entry were by stand-alone goods.

However, this is not the end of the story. While the single-product firms will have reduced incentives to innovate, the bundler will have greater incentives. It can earn greater profits by entering new markets than the single-product firms could. Thus there will be a shift of innovative activity from stand-alone firms to bundlers. Whether the ultimate equilibrium will involve a higher or lower total level of innovation will depend on the ability of the different types of firms to succeed with

²⁶ While we assume that the costs and benefits of the innovation are deterministic, the analysis can be readily generalized to stochastic values.

innovations. Furthermore, the types of innovations that the bundler will undertake can be expected to differ systematically from the innovations pursued by standalone firms.

6. Concluding Remarks

The economies of aggregation we identify are in many ways similar in effect to economies of scale or network externalities. A marketing strategy that employs large-scale bundling can extract greater profit and gain competitive advantage from a given set of goods. Economies of aggregation will be important when marginal costs are very low, as for the (re)production and delivery of information goods via the Internet. High marginal costs render large-scale aggregation unprofitable, which may explain why it is more common for Internet publishing than in publishing based on paper, film, polycarbonate discs or other relatively high-cost media.

Our analysis of bundling and competition showed that:

1. Large bundles may provide a significant advantage in the competition for upstream content.
2. The act of bundling information goods makes an incumbent seem "tougher" to competitors and potential entrants.
3. The bundler can profitably enter a new market and dislodge an incumbent by adding a competing information good to the existing bundle.
4. Bundling can reduce the incentives for competitors to innovate, while it can increase bundlers incentives to innovate.

Although we analyze a fairly stylized setting in order to isolate the effects of large scale bundling on competition, earlier work on bundling (Bakos and Brynjolfsson, 1999a, 1999b) suggest that our framework can be generalized in a number of directions. In particular, Proposition 1 also applies, *inter alia*, to consumers with budget constraints, goods that are complements or substitutes, goods with diminishing or increasing returns, and goods that are drawn from different distributions (Bakos and Brynjolfsson, 1999a). Furthermore, the existence of distribution or transaction costs, which are paid only once per purchase, will generally tend to strengthen the advantages bundlers have of sellers of separate goods (Bakos and Brynjolfsson, 1999b). The effects we analyze are, by design, purely based on using bundling as a pricing and marketing strategy to change the demand for a collection of information goods without (necessarily) any change in any of their intrinsic characteristics on any change in their production technology. Naturally, bundling can be combined with changes in the goods to either reinforce or mitigate the effects we identify.

The development of the Internet as an infrastructure for the distribution of digital information goods has dramatically affected the competitive impact of marketing and selling strategies based on large scale bundling. As we show in this paper, the resulting "economies of aggregation" for information can provide powerful leverage for obtaining new content, increasing profits, protecting markets, entering new markets and affecting innovation, even in the absence of network externalities or technological economies of scale or scope. Large scale bundling was relatively rare in the pre-Internet era, but its implications for marketing and competition are an essential component of Internet marketing strategy for information goods.

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Appendix: Proofs of Propositions

Proposition 1

See the proof for Proposition 1 in Bakos and Brynjolfsson (1999a).

Proposition 2

In the setting of section 3, the following lemma states that if $n_1 > n_2$, then a bundle of n_1 goods will extract more value from exclusive rights to the single good than a bundle of n_2 goods.

Lemma 1: If n_1, n_2 are large enough integers, and if $n_1 > n_2$, then $y_1 > y_2$.

Proof

We first prove that if k is large enough so that the probability distribution for a consumer's valuation for a bundle of k goods can be approximated by a normal distribution, then a bundle with $k+1$ goods will extract more value from a single good than a bundle of k goods. The central limit theorem guarantees that in the setting of section 3, the valuation for a bundle of k goods will converge to a normal distribution. The lemma then follows by induction and the application of an inequality from Schmalensee (1984).

Let x_k be the average (per good) valuation for a bundle of k goods. Denote by $\pi_k(p)$ the per good revenues of a bundler of k goods charging a price p per good and selling to a fraction $q_k(p)$ of consumers. Let p_k^* be the profit-maximizing price, and denote $\pi_k(p_k^*)$ by π_k^* .

For the lemma to hold, it must be $(k+2)\pi_{k+2}^* - (k+1)\pi_{k+1}^* > (k+1)\pi_{k+1}^* - k\pi_k^*$, or $g(k+1) - g(k) > 0$

$$(1)$$

where $g(k) = (k+1)\pi_{k+1}^* - k\pi_k^*$.

For (1) to hold, it suffices that f is increasing in n , i.e., $\frac{dg(k)}{dk} > 0$

$$(2).$$

Let $h(k) = k\pi_k^*$. Then $g(k) = h(k+1) - h(k)$, and $\frac{dg(k)}{dk} = \frac{d}{dk}[h(k+1)] - \frac{d}{dk}[h(k)]$. Thus, for (2) to hold,

it suffices that $\frac{d}{dk}[h(k)]$ is increasing in n , i.e., that $\frac{d^2}{dk^2}[h(k)] > 0$

$$(3).$$

From the definition of h , $\frac{d}{dk}[h(k)] = k \frac{d\pi_k^*}{dk} + \pi_k^*$ and $\frac{d^2}{dk^2}[h(k)] = k \frac{d^2\pi_k^*}{dk^2} + 2 \frac{d\pi_k^*}{dk}$, and thus it suffices to

$$\text{show that } 2 \frac{d\pi_k^*}{dk} + k \frac{d^2\pi_k^*}{dk^2} > 0 \quad (4)$$

where π_k^* maximizes $\pi_k(p) = pq(p) = p(1 - F_k(p))$, and f_k is the pdf for the average valuation of a bundle of n goods, and F_k is the cumulative distribution of f_k .

Next, we show that (4) is satisfied in the case that $f_k = N(\mu, \sigma/\sqrt{k})$.

$$\text{Let } x = \mu/\sigma \text{ and } \alpha_n = \frac{k\mu}{\sqrt{k}\sigma} = \sqrt{k}x.$$

$$\text{Then we can write } \frac{d\pi_k^*}{dk} = \frac{d\pi_k^*}{d\alpha_k} \frac{d\alpha_k}{dk} = \frac{1}{2} k^{-\frac{1}{2}} x \frac{d\pi_k^*}{d\alpha_k} \quad (5)$$

and

$$\begin{aligned} \frac{d^2\pi_k^*}{dk^2} &= \frac{d}{dk} \left[\frac{1}{2} k^{-\frac{1}{2}} x \frac{d\pi_k^*}{d\alpha_k} \right] = -\frac{1}{2} k^{-\frac{3}{2}} x \frac{d\pi_k^*}{d\alpha_k} + \frac{1}{2} k^{-\frac{1}{2}} x \frac{d}{dk} \left[\frac{d\pi_k^*}{d\alpha_k} \right] = -\frac{1}{2} k^{-\frac{3}{2}} x \frac{d\pi_k^*}{d\alpha_k} + \frac{1}{2} k^{-\frac{1}{2}} x \frac{d^2\pi_k^*}{d\alpha_k^2} \frac{d\alpha_k}{dk} \\ \therefore \frac{d^2\pi_k^*}{dk^2} &= -\frac{1}{2} k^{-\frac{3}{2}} x \frac{d\pi_k^*}{d\alpha_k} + \frac{x^2}{4k} \frac{d^2\pi_k^*}{d\alpha_k^2} \end{aligned} \quad (6)$$

Substituting (5) and (6) into (4), we see that (4) is satisfied when

$$k^{-\frac{1}{2}} x \frac{d\pi_k^*}{d\alpha_k} - \frac{1}{2} k^{-\frac{1}{2}} x \frac{d\pi_k^*}{d\alpha_k} + \frac{x^2}{4} \frac{d^2\pi_k^*}{d\alpha_k^2} > 0, \text{ i.e., } \frac{1}{2} k^{-\frac{1}{2}} x \frac{d\pi_k^*}{d\alpha_k} + \frac{x^2}{4} \frac{d^2\pi_k^*}{d\alpha_k^2} > 0.$$

It thus suffices to show that $\frac{d\pi_k^*}{d\alpha_k} > 0$ and $\frac{d^2\pi_k^*}{d\alpha_k^2} > 0$. The first condition follows from inequality (11a) in

(Schmalensee, 1984) and the second condition follows from differentiating that inequality.

Proposition 1 implies that $\lim_{n_1 \rightarrow \infty} y_1 = \lim_{n_2 \rightarrow \infty} y_2 = \mu$. In the case of non-exclusive provision of the single good, if it is provided outside both bundles, Bertrand competition will result in a zero equilibrium price. As n_1 and n_2 get

large enough, both bundles achieve a market share close to 100%. Thus if the good is provided as part of one bundle only, at equilibrium the outside good will realize revenues close to zero while the bundler will be able to incrementally extract lower revenues than when having exclusive rights to the good. If both bundles include the good, neither bundler will be able to extract revenues from it since (almost) all consumers will already have access to it via the other bundle. Thus it follows that as n_1 and n_2 get large enough, $y_1 > z_1$ and $y_2 > z_2$, and at least one of z_1 and z_2 converges to zero. Thus $\lim_{n_1, n_2 \rightarrow \infty} \max(y_1, y_2, z_1 + z_2) = y_1$, which proves the proposition.

Section 4.1, equilibrium for competing substitute goods

At equilibrium, neither firm benefits from lowering its price, taking the price of the other competitor as given. The gray-shaded area in Figure 2 shows the additional sales for firm B if it lowers its price by δ , which equal $\delta p_A + \delta(1 - p_A)$, or δ . Thus if at price p_B firm B sells quantity q_B , by lowering its price by δ , firm B realizes new revenues of δp_B and loses revenues δq_B from its existing sales. At equilibrium, $\delta p_B = \delta q_B$, or $p_B = q_B$. If firm A lowers its price by δ , it realizes new sales of $(1 + p_B - p_A)\delta$, corresponding revenues $(1 + p_B - p_A)\delta p_A$, and loses revenues δq_A from its existing sales. At equilibrium, this yields $p_A = q_A / (1 + p_B - p_A)$.

Then: $q_B = \frac{1}{2} + (p_A - p_B) - \frac{1}{2} p_A^2$ and $p_A = p_B$, and the unique equilibrium is characterized by prices p_A^* and p_B^* that are given by $p_A^* = p_B^* = \sqrt{2} - 1$, corresponding quantities $q_A^* = q_B^* = \sqrt{2} - 1$, and corresponding gross profit $\pi_A^* = \pi_B^* = (\sqrt{2} - 1)^2$, or approximately 0.17.

Section 4.1, monopoly provision of substitute goods

If both goods A and B are provided by a single firm, it will set the prices p_A and p_B to maximize its total revenues $p_A q_A + p_B q_B$. If the monopolist lowers p_B by an amount δ , the additional sales for good B are depicted in the gray-shaded area in Figure 2, and equal new sales of δp_A , and sales $\delta(1 - p_A)$ taken from good A. The corresponding increase in revenues is $\delta p_A p_B + \delta(1 - p_A)(p_B - p_A)$, and must be offset against a loss of revenues δq_B from existing sales of good B. Thus the monopolist will choose p_B so that

$p_A p_B + (1 - p_A)(p_B - p_A) = q_B$. The corresponding condition for p_A is $p_A p_B + (1 - p_B)(p_A - p_B) = q_A$, and it can be seen that $q_A + q_B = 1 - p_A p_B$. In the unique equilibrium the monopolist maximizes $p_A(1 - p_A^2)$, yielding optimal prices $p_A^* = p_B^* = 1/\sqrt{3}$, and corresponding quantities $q_A^* = q_B^* = 1/3$. Revenues are $\sqrt{3}/9$ per good (approx. 0.19), for total revenues of $2\sqrt{3}/9$ (approx. 0.38).

Alternatively, the monopolist can sell only one good at a price of 0.5, with resulting sales of 0.5 and revenues of 0.25. Finally, as consumer valuations for the two goods are i.i.d., consumers do not derive additional value from their less preferred good and the goods have zero marginal costs, if the monopolist bundles A and B he will price the bundle at $1/\sqrt{3}$, and sell quantity $q_A^* + q_B^* = 2/3$, for the same total revenues of $2\sqrt{3}/9$. Thus the monopolist cannot increase his profits by bundling a single pair of competing goods.

Section 4.2

We know from Proposition 1 that $\lim_{n \rightarrow \infty} \frac{1}{n-1} p_{B2...n}^* = \frac{1}{2}$ and $\lim_{n \rightarrow \infty} q_{B2...n}^* = 1$.

Let $q_{B1}(p_{B1}, p_{A1})$ be the demand faced by firm B for good B_1 at price p_{B1} , when firm A sets price p_{A1} for good A_1 . Define $\hat{p}_{B1}(p_{A1})$ so that $q_{B1}(\hat{p}_{B1}(p_{A1}), p_{A1}) = 1/2$. In other words, given a price p_{A1} for good A_1 , $\hat{p}_{B1}(p_{A1})$ is the price at which good B1 achieves a 50% market share.

If $p_{A1} \geq p_{B1}$, then $q_{B1} = \frac{1}{2}(1 + (p_{A1} - p_{B1}) + p_{A1})(1 - p_{B1}) - \frac{1}{2}(p_{A1} - p_{B1})^2 = \frac{1}{2} + p_{A1} - p_{B1} - \frac{1}{2}p_{A1}^2$.

If $p_{B1} > p_{A1}$, then $q_{B1} = 1 - q_{A1} - p_{A1}p_{B1} = \frac{1}{2} - p_{B1} + p_{A1} + \frac{1}{2}p_{B1}^2 - p_{A1}p_{B1}$. Thus, $\frac{\partial q_{B1}}{\partial p_{B1}} = -1$ for $p_{B1} \leq p_{A1}$

and $\frac{\partial q_{B1}}{\partial p_{B1}} = -1 + p_{B1} - p_{A1}$ for $p_{B1} > p_{A1}$. Furthermore, even if $p_{B1} = 0$, a fraction $\frac{1}{2}(1 - p_{A1})^2$ of consumers

will purchase A_1 over B_1 . The resulting distribution of valuations faced by firm B for good B_1 when firm A prices A_1 at p_{A1} is shown in Figure 4, and it has an impulse of measure $\frac{1}{2}(1 - p_{A1})^2$ at the origin. This

distribution has mean $\mu_{B1}(p_{A1}) = \int_0^{p_{A1}} x dx + \int_{p_{A1}}^1 (p_{A1} + 1 - x) dx$, or $\mu_{B1}(p_{A1}) = \frac{1}{6} + \frac{1}{2}p_{A1} - \frac{1}{6}p_{A1}^3$.

For large n if firm B adds good B_1 to the bundle of $B_2, B_3 \dots B_n$ and raises the bundle price by $\hat{p}_{B1}(p_{A1})$, any change in the demand for the bundle be second order, as marginal consumers are equally likely to drop or start purchasing the bundle when B_1 is added at a price of $\hat{p}_{B1}(p_{A1})$. The envelope theorem guarantees that gross profits will be approximately $\left(p_{B2...n}^* + \hat{p}_{B1}(p_{A1})\right)q_{B2...n}^*$, and thus for large n firm B can extract gross profit of at least $\hat{p}_{B1}(p_{A1})$ from good B_1 by including it in the bundle.

Since $\lim_{n \rightarrow \infty} q_{B2...n}^* = 1$, as n increases, good B_1 is eventually made available to essentially all consumers as part of the bundle. Thus firm A must set its price based to the fact that (almost all) consumers already have access to

good B_1 , and will thus choose p_{A_1} to maximize $\frac{1}{2}(1-p_{A_1})^2 p_{A_1}$ as shown in Figure 3, resulting in price

$p_{A_1}^* = \frac{1}{3}$, corresponding sales $q_{A_1}^* = \frac{2}{9}$ and gross profit $\pi_{A_1}^* = \frac{2}{27}$ or approximately 0.07. The corresponding

$\hat{p}_{B_1}(p_{A_1}^*)$ is $\frac{5}{18}$ or approximately 0.28.

Proposition 3

Proof: If the bundle does not contain good B_i and the demands of the goods in \mathcal{B} are independent, the proposition follows from Proposition 2 and Corollary 2a in Bakos and Brynjolfsson (1997). If the bundle contains the corresponding substitute good B_i , the result follows from comparing the settings analyzed in sections 4.1 and 4.3.

Proposition 4

Proof: If B_i is not part of the bundle, this is easily shown based on Bakos and Brynjolfsson (1999a) proposition 2, as adding A_i has the same effect as adding B_i . If both goods are produced, then introducing the second good in the bundle will allow the bundler not to worry about competition and extract nearly the full surplus created by the two goods. If a consumer values one of the two goods at x ($0 \leq x \leq 1$), then his value for both goods is x with probability x , and $\frac{1}{2}(1+x)$ with probability $1-x$, depending on which good is more valued. Thus the mean surplus created by the two goods as part of the bundle is $\int_0^1 (x^2 + (1-x)\frac{1}{2}(1+x))dx = \int_0^1 (\frac{1}{2} + \frac{1}{2}x^2)dx = \frac{2}{3}$. For a large enough n , the bundler can capture virtually the entire surplus by including both goods in the bundle, or gross profits of just under $\frac{2}{3}$. We showed in section 4.2 that good A_i outside the bundle would generate gross profits of approx. 0.07. Since good B_i in the bundle will generate profits less than 0.5 (its mean valuation), the bundler will increase its gross profits by at least 0.16 by adding good A_i to the bundle. Therefore the bundler will be willing to acquire good A_i .

We also need to consider the case where the bundler acquires good A_i , but leaves it outside the bundle. In that case, if the bundler sets a price p_{A_i} for good A_i , the mean valuation for good B_i in the bundle will be

$\mu_{B_i} = \frac{1}{6} + \frac{1}{2} p_{A_i} - \frac{1}{6} p_{A_i}^3$ and the quantity demanded for good A_i will be $\frac{1}{2}(1-p_{A_i})^2$ as shown in section 4.2. The bundler would thus maximize total revenues

$\pi_{A_i} + \pi_{B_i} = \frac{1}{2} p_{A_i} (1-p_{A_i})^2 + \frac{1}{6} + \frac{1}{2} p_{A_i} - \frac{1}{6} p_{A_i}^3 = \frac{1}{6} + p_{A_i} - p_{A_i}^2 + \frac{1}{3} p_{A_i}^3$ by setting $p_{A_i} = 1$, resulting in gross profit of 0.5. In other words the bundler would optimally price the outside good out of the market, maximizing B_i 's contribution to the bundle. Thus the bundler maximizes his profits by including both goods in the bundle.