Lecture Notes 16

Forwards and Futures

Buzz Words: ***Hedgers, Speculators, Arbitrageurs,***
Cost of Carry, Convenience Yield,
Stock Index Arbitrage, Floating vs. Fixed Rates

I. Readings and Suggested Practice Problems

BKM, Chapter 22, Section 4.

BKM, Chapter 23.

BKM, Chapter 16, Section 5.

Suggested Problems, Chapter 22: 13;   Chapter 23: 3, 25.

II. Forward Contracts

A. Definition

A forward contract on an asset is an agreement between the buyer and seller to exchange cash for the asset at a predetermined price (the forward price) at a predetermined date (the settlement date).

- The asset underlying a forward contract is often referred to as the "underlying" and its current price is referred to as the "spot" price.

- The buyer of the forward contract agrees today to buy the asset *on the settlement date* at the *forward price*. The seller agrees today to sell the asset at that price on that date.

- No money changes hands until the settlement date. In fact, the forward price is set so that neither party needs to be paid any money today to enter into the agreement.

B. Example: Foreign Exchange (FX) Rates

Foreign Exchange Rates

NEW YORK (Dow Jones)--The New York foreign exchange selling rates below apply to trading among banks in amounts of $1 million and more, as quoted at 4 p.m. Eastern time by Dow Jones and other sources. Retail transactions provide fewer units of foreign currency per dollar.

	U.S. Dollar Equivalent		Currency Per U.S. Dollar	
	Tue	Mon	Tue	Mon
Argentina (Peso)	1.0001	1.0001	.9999	.9999
Australia (Dollar)	.6973	.6989	1.4341	1.4308
Austria (Schilling)	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Italy (Lira)	.0005907	.0005900	1693.00	1695.00
Japan (Yen)	.007927	.007957	126.15	125.68
1-month forward	.007962	.007994	125.59	125.10
3-months forward	.008044	.008073	124.32	123.87
6-months forward	.008150	.008185	122.71	122.18
Jordan (Dinar)	1.4134	1.4134	.7075	.7075

1. Yen / dollar transactions

For immediate delivery: 0.007927 $/¥
For delivery in six months: 0.008150 $/¥

Example
Suppose that we need ¥10 Million in six months (the “maturity” date). We can:
(a) Wait and gamble on what the exchange rate will be then.
(b) Buy yen today and invest them until maturity.
(c) Buy yen forward. No money changes hands today. At maturity, we pay ¥10 Million(.008150 $/¥) = $81,500; receive ¥10 Million.

2. *Hedging and speculation*

a. The need for ¥10 Million is an obligation that exposes us to exchange risk. Buying forward *hedges* this risk.

If we did not have any need for yen, the transaction to *buy* yen forward would represent a *speculative* bet that the yen would *rise* relative to the dollar:

– If at maturity, the exchange rate is 0.010000 \$/¥, resell the ¥ for \$:

¥10 Million × 0.010000 \$/¥ = \$100,000

Profit = \$100,000- \$81,500 = \$18,500

– If at maturity, the exchange rate is 0.007000 \$/¥, we get:

¥10 Million × 0.007000 \$/¥ = \$70,000

Profit = \$70,000- \$81,500 = -\$11,500

b. Hedging means "removing risk". It does not mean "guaranteeing the best possible outcome"
(If at maturity the exchange rate is 0.007000 \$/¥
the hedger will regret having locked in the worse rate.)

- *A forward contract is not an option. The buyer must go through with the contract, even if the spot rate at maturity is worse than agreed upon.*
- *No money changes hands until maturity. (There is nothing corresponding to the option premium.)*

3. Transactions from the ¥ perspective

Suppose a Japanese firm needs $ in 6 months.
They face the opposite problem and can buy $ forward.
Such a firm *might* be the counterparty to the U.S. firm's forward transaction.
(In general, either or both sides might be hedger or speculator).

Again, note that:

- One does not "buy a forward contract"
  (no money is exchanged today for a financial asset)

  One "*enters* into a forward contract:"
  "buys yen forward" or "sells dollars forward"

- For everyone who has sold the dollar forward (against the yen), there is someone who has *bought* the dollar forward (against the yen) (like "zero net supply" in options)

  If one side of the forward contract has a profit (relative to the subsequent spot price), then the other side has a loss (like "zero-sum game" in options).

4. The forward FX market: A few details

- Large denomination: $1 Million or more.
  (The hypothetical yen transaction above would be too small.)
- A telephone/computer network of dealers.
- Direct participation limited to large money center banks.
- Counterparties assume credit risk. (If one defaults, the other has to bear the risk)

III. Futures Contracts

Futures Contracts vs Forward Contracts

Forward and futures contracts are essentially the same except for the ***daily resettlement*** feature of futures contracts, called ***marking-to-market***.

Since this is only a technical difference, in our discussion we will not distinguish between the two, and use futures and forwards interchangeably.

(Indeed, for many practical purposes futures and forward prices are very close, and we will take them to be the same).

Plenty information is available from:

Chicago Mercantile Exchange (http://www.cme.com/)
Chicago Board of Trade (http://www.cbot.com/)

IV. Forward-Spot Parity

A. Forward-spot parity is a valuation principle for forward contracts.

Often approximately correct for futures contracts as well.

F_0 forward price
P_0 spot price

$$F_0 = P_0 + \text{“cost of carry”}$$

The idea**:** “buying forward” is equivalent to
“buying now and storing/carrying the underlying”

The cost of carry reflects:

+ cost of financing the position
+ storage costs (insurance, spoilage)
- income earned by the underlying

Example for a Commodity contract

Kryptonite is $10 per gram in the spot market.
It will cost 2% of its value to store a gram for one year.
The annual interest rate is 7%.

Therefore: The percentage cost of carry is $c = 9\%$

Parity implies $F_0 = 10(1.09) = 10.90$

- Suppose F_0=11 (Forward is *overvalued* relative to the spot)

  Today:
  We can borrow $10, buy Kryptonite spot
  and sell Kryptonite forward.

  At maturity:

repay loan	-10.70	
pay storage	-0.20	
receive F_0	+11	(and make delivery of Kryptonite)

  Net cash flow = +0.10

- Suppose F_0 = 10.85 (Forward is *undervalued* relative to the spot)

  Today:
  If we already hold Kryptonite that we won't need for a year, we can
  – Sell Kryptonite in the spot market, Invest $10.
  – Buy Kryptonite forward

  At maturity:
  – Receive loan and interest +10.70
  – Pay F_0 -10.85
  – Take delivery of Kryptonite

  Apparent net loss of 0.15. But we have saved storage fees (0.20), so relative to holding Kryptonite, we have a 0.05 profit.

B. Cost of carry for different underlying assets

- Underlying = physical commodity:

  financing cost
  \+ warehousing cost
  \+ insurance
  \+ spoilage
  \+ transportation charges
  \- convenience yield

- Underlying = stock (or stock index)

  financing cost
  \- dividends received by stockholder

- Underlying = bond

  financing cost
  \- interest received by bondholder

V. Stock Index Forward-Spot Parity

- The carrying cost for the index is

$$c = r_f - d$$

  where

  r_f is the risk-free rate and
  d is the dividend yield.

- Parity: $F_0 = S_0(1 + r_f - d)^T$

  where

  F_0 is the futures price (today),
  S_0 is the stock price (index level) today,
  T is the maturity of the contract

  [This is also sometimes written: $F_0 = S_0(1 + r_f)^T - D$ where D is the total cash dividend on the index.]

- Violations of parity imply arbitrage profits.

Example of Index Arbitrage

$S_0 = 650$, $r_f = 5\%$, $d = 3\%$

Parity: $F_0 = 650(1 + 0.05 - 0.03) = 663$

- If F_0=665, then the futures contract is *overvalued* relative to the spot price.

Arbitrage:	
Borrow $650	+650
Buy index	- 650
Short the futures	0
Net cash flow	0

"Unwind" at maturity:	
Collect divs [3%(650)]	19.5
Sell stock (index)	+ S_T
Settle futures	-(S_T - 665)
Repay loan [1.05x650]	- 682.5
Total	+2

- If F_0 = 660, then the futures contract is *undervalued* relative to the spot.

Arbitrage:	
Long futures	0
Short stock	+650
Invest short-sale proceeds	- 650
Net cash flow	0

At maturity:	
Settle futures	(S_T - 660)
Pay dividends [3%(650)]	-19.5
Repurchase stock	-S_T
Investment matures	682.5
Total	3

VI. Foreign Exchange Forward-Spot Parity

In FX markets, forward/spot parity is called "covered interest parity"

The cost of carry is the cost of borrowing in one currency (e.g., US dollar $) and investing in the other (e.g., the UK pound £).

Example

The spot (E_0) and forward (F_0) rates are *dollar per pound* $/£.

E_0 is the number of dollars you can get today for £1 (E_0 is the spot $/£ exchange rate).

F_0 is the number of dollars you can, contracted upon today, to get in the future for £1 (F_0 is the forward $/£ exchange rate)

(No) Arbitrage enforces the following relation:

$$F_0 = E_0 \left(\frac{1 + r_{US}}{1 + r_{UK}} \right) = E_0 (1 + \text{"} Cost \ \ of \ \ Carry \text{"})$$

Explanation

You can either commit to pay in the future F_0 dollars for £1,

Or

Step 1: You can borrow $E_0/(1+r_{UK})$ dollars to buy £$1/(1+r_{UK})$.
Step 2: The £$1/(1+r_{UK})$ in a pound-based account will compound to £1 next period, and you will owe $[E_0/(1+r_{UK})]\,(1+r_{US})$ dollars in the next period.
(you essentially took a dollar loan to "synthesize" £1).

Therefore, to obtain £1 in the future, as your assets, you will have a future liability of F_0 dollars (if you enter directly into a forward contract to get £1) , or of $[E_0/(1+r_{UK})]\,(1+r_{US})$ dollars (if you settle a loan that allowed you to synthesize £1).
Under no arbitrage, both liabilities must be equal, and hence
$F_0 = [E_0/(1+r_{UK})]\,(1+r_{US})$ as on the previous page.

VII. Swaps

Definition: A swap is a contract for exchange of future cash flows.

(Leading examples are swaps between currency payments or between floating and fixed interest rates.)

Example: Currency Swaps

A U.S. firm has a British £ obligation consisting of 1£ per year for the next 10 years. (This is a *floating* obligation in US$.)

The firm wants to swap this for a *fixed* US$ obligation of amount $*x*. We can refer to $*x* as the *swap* (exchange) *rate*, because by entering into a swap the firms fixes its exchange rate between 1£ and US$. (At the end of each year, the firm will pay to a swap dealer $*x* and receive 1£, so that it can cover its obligations.)

What is $*x*?

- Basic data: r_{US} = 5.6%, r_{UK} = 6.2%
  Spot exchange rate = $P_0 = E_0$ = $1.51/£

Forward Spot (Covered Interest) Parity:

Today's forward price (in $) for delivery of 1 pound in one year:

$$F_0(1) = E_0 \frac{(1+r_{US})}{(1+r_{UK})}$$

Today's forward price for delivery in k years:

$$F_0(k) = E_0 \frac{(1+r_{US})^k}{(1+r_{UK})^k}$$

- As an alternative to the swap, engage in forward contracts to purchase £1 in each of the next 10 years.

The PV of the \$ obligation is :

$$\frac{F(1)}{(1+r_{US})}+\cdots+\frac{F(10)}{(1+r_{US})^{10}}$$

$$=\frac{1}{(1+r_{US})}E_0\frac{(1+r_{US})}{(1+r_{UK})}+\cdots+\frac{1}{(1+r_{US})^{10}}E_0\frac{(1+r_{US})^{10}}{(1+r_{UK})^{10}}$$

$$=E_0\times\left[\frac{1}{(1+r_{UK})}+\cdots+\frac{1}{(1+r_{UK})^{10}}\right]$$

$$=E_0\times\left[PV \text{ of a 1 pound (annuity) obligation}\right]$$

- If we swap this for an \$$x$ 10-year obligation, the PV is simply x APVF(r_{US}, 10)

- So equating: 1.51 APVF(r_{UK}, 10) = x APVF(r_{US}, 10)
  1.51 APVF(6.2%, 10) = x APVF(5.6%, 10)
  1.51 × 7.29 = 11.01 = x APVF(5.6%, 10)

  which implies x = \$1.47

A swap is equivalent to a portfolio of forwards (futures).

VIII. Additional Readings