Lecture Notes 16

Forwards and Futures

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Buzz Words: Hedgers, Speculators, Arbitrageurs, Cost of Carry, Convenience Yield, Stock Index Arbitrage, Floating vs. Fixed Rates
I. Readings and Suggested Practice Problems

BKM, Chapter 22, Section 4.

BKM, Chapter 23.

BKM, Chapter 16, Section 5.

Suggested Problems, Chapter 22: 13; Chapter 23: 3, 25.

II. Forward Contracts

A. Definition

A forward contract on an asset is an agreement between the buyer and seller to exchange cash for the asset at a predetermined price (the forward price) at a predetermined date (the settlement date).

• The asset underlying a forward contract is often referred to as the “underlying” and its current price is referred to as the “spot” price.

• The buyer of the forward contract agrees today to buy the asset on the settlement date at the forward price. The seller agrees today to sell the asset at that price on that date.

• No money changes hands until the settlement date. In fact, the forward price is set so that neither party needs to be paid any money today to enter into the agreement.
B. Example: Foreign Exchange (FX) Rates

Foreign Exchange Rates

NEW YORK (Dow Jones)--The New York foreign exchange selling rates below apply to trading among banks in amounts of $1 million and more, as quoted at 4 p.m. Eastern time by Dow Jones and other sources. Retail transactions provide fewer units of foreign currency per dollar.

<table>
<thead>
<tr>
<th>Currency</th>
<th>U.S. Dollar Equivalent</th>
<th>Currency Per U.S. Dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tue</td>
<td>Mon</td>
</tr>
<tr>
<td>Argentina (Peso)</td>
<td>1.0001</td>
<td>1.0001</td>
</tr>
<tr>
<td>Australia (Dollar)</td>
<td>.6973</td>
<td>.6989</td>
</tr>
<tr>
<td>Austria (Schilling)</td>
<td>.0272</td>
<td>.0282</td>
</tr>
<tr>
<td>Belgium (Franc)</td>
<td>1.5001</td>
<td>1.5000</td>
</tr>
<tr>
<td>Italy (Lira)</td>
<td>.0005907</td>
<td>.0005900</td>
</tr>
<tr>
<td>Japan (Yen)</td>
<td>.007927</td>
<td>.007957</td>
</tr>
<tr>
<td>1-month forward</td>
<td>.007962</td>
<td>.007994</td>
</tr>
<tr>
<td>3-months forward</td>
<td>.008044</td>
<td>.008073</td>
</tr>
<tr>
<td>6-months forward</td>
<td>.008150</td>
<td>.008185</td>
</tr>
<tr>
<td>Jordan (Dinar)</td>
<td>1.4134</td>
<td>1.4134</td>
</tr>
</tbody>
</table>

1. **Yen / dollar transactions**

   For immediate delivery: 0.007927 $/¥
   For delivery in six months: 0.008150 $/¥

*Example*

Suppose that we need ¥10 Million in six months (the "maturity" date). We can:
(a) Wait and gamble on what the exchange rate will be then.
(b) Buy yen today and invest them until maturity.
(c) Buy yen forward. No money changes hands today. At maturity, we pay ¥10 Million(0.008150 $/¥) = $81,500; receive ¥10 Million.
2. **Hedging and speculation**

   a. The need for ¥10 Million is an obligation that exposes us to exchange risk. Buying forward hinges this risk.

      If we did not have any need for yen, the transaction to buy yen forward would represent a speculative bet that the yen would rise relative to the dollar:

      – If at maturity, the exchange rate is 0.010000 $/¥, resell the ¥ for $:

         ¥10 Million \( \times \) 0.010000 $/¥ = $100,000

         Profit = $100,000 - $81,500 = $18,500

      – If at maturity, the exchange rate is 0.007000 $/¥, we get:

         ¥10 Million \( \times \) 0.007000 $/¥ = $70,000

         Profit = $70,000 - $81,500 = -$11,500

   b. Hedging means “removing risk”. It does not mean “guaranteeing the best possible outcome”

      (If at maturity the exchange rate is 0.007000 $/¥ the hedger will regret having locked in the worse rate.)

      • *A forward contract is not an option. The buyer must go through with the contract, even if the spot rate at maturity is worse than agreed upon.*

      • *No money changes hands until maturity. (There is nothing corresponding to the option premium.)*
3. Transactions from the ¥ perspective

Suppose a Japanese firm needs $ in 6 months. They face the opposite problem and can buy $ forward. Such a firm might be the counterparty to the U.S. firm’s forward transaction. (In general, either or both sides might be hedger or speculator).

Again, note that:

- One does not “buy a forward contract” (no money is exchanged today for a financial asset)
  
  One “enters into a forward contract:” “buys yen forward” or “sells dollars forward”

- For everyone who has sold the dollar forward (against the yen), there is someone who has bought the dollar forward (against the yen) (like “zero net supply” in options)

  If one side of the forward contract has a profit (relative to the subsequent spot price), then the other side has a loss (like “zero-sum game” in options).

4. The forward FX market: A few details

- Large denomination: $1 Million or more. (The hypothetical yen transaction above would be too small.)
- A telephone/computer network of dealers.
- Direct participation limited to large money center banks.
- Counterparties assume credit risk. (If one defaults, the other has to bear the risk)
III. Futures Contracts

*Futures Contracts vs Forward Contracts*

Forward and futures contracts are essentially the same except for the *daily resettlement* feature of futures contracts, called *marking-to-market*.

Since this is only a technical difference, in our discussion we will not distinguish between the two, and use futures and forwards interchangeably.

(Indeed, for many practical purposes futures and forward prices are very close, and we will take them to be the same).

Plenty information is available from:
- Chicago Board of Trade ([http://www.cbot.com/](http://www.cbot.com/))
IV. Forward-Spot Parity

A. Forward-spot parity is a valuation principle for forward contracts.

Often approximately correct for futures contracts as well.

\[
F_0 = P_0 + \text{"cost of carry"}
\]

The idea: “buying forward” is equivalent to “buying now and storing/carrying the underlying”

The cost of carry reflects:
+ cost of financing the position
+ storage costs (insurance, spoilage)
- income earned by the underlying

Example for a Commodity contract

Kryptonite is $10 per gram in the spot market.
It will cost 2% of its value to store a gram for one year.
The annual interest rate is 7%.

Therefore: The percentage cost of carry is \( c = 9\% \)

Parity implies \( F_0 = 10(1.09) = 10.90 \)
• Suppose $F_0=11$ (Forward is overvalued relative to the spot)

*Today*:
We can borrow $10$, buy Kryptonite spot and sell Kryptonite forward.

*At maturity*:
- repay loan $-10.70$
- pay storage $-0.20$
- receive $F_0 +11$ (and make delivery of Kryptonite)

$Net\ cash\ flow = +0.10$

• Suppose $F_0 = 10.85$ (Forward is undervalued relative to the spot)

*Today*:
If we already hold Kryptonite that we won’t need for a year, we can
- Sell Kryptonite in the spot market, Invest $10$.
- Buy Kryptonite forward

*At maturity*:
- Receive loan and interest $+10.70$
- Pay $F_0 -10.85$
- Take delivery of Kryptonite

Apparent net loss of 0.15. But we have saved storage fees (0.20), so relative to holding Kryptonite, we have a 0.05 profit.
B. Cost of carry for different underlying assets

- Underlying = physical commodity:
  
  financing cost
  + warehousing cost
  + insurance
  + spoilage
  + transportation charges
  - convenience yield

- Underlying = stock (or stock index)
  
  financing cost
  - dividends received by stockholder

- Underlying = bond
  
  financing cost
  - interest received by bondholder
V. Stock Index Forward-Spot Parity

• The carrying cost for the index is
  
  \[ c = r_f - d \]

  where
  
  \( r_f \) is the risk-free rate and
  \( d \) is the dividend yield.

• Parity: \( F_0 = S_0(1 + r_f - d)^T \)

  where
  
  \( F_0 \) is the futures price (today),
  \( S_0 \) is the stock price (index level) today,
  \( T \) is the maturity of the contract

  [This is also sometimes written: \( F_0 = S_0(1 + r_f)^T - D \)

  where \( D \) is the total cash dividend on the index.]

• Violations of parity imply arbitrage profits.

**Example of Index Arbitrage**

\( S_0 = 650, r_f = 5\%, d = 3\% \)

Parity: \( F_0 = 650(1 + 0.05 - 0.03) = 663 \)
• If $F_0 = 665$, then the futures contract is overvalued relative to the spot price.

**Arbitrage:**
- Borrow $650 + 650$
- Buy index - 650
- Short the futures 0
- Net cash flow 0

**“Unwind” at maturity:**
- Collect divs [3%(650)] 19.5
- Sell stock (index) + $S_T$
- Settle futures -( $S_T - 665$)
- Repay loan [1.05x650] - 682.5
- Total +2

• If $F_0 = 660$, then the futures contract is undervalued relative to the spot.

**Arbitrage:**
- Long futures 0
- Short stock +650
- Invest short-sale proceeds - 650
- Net cash flow 0

**At maturity:**
- Settle futures $(S_T - 660)$
- Pay dividends [3%(650)] -19.5
- Repurchase stock $-S_T$
- Investment matures 682.5
- Total 3
VI. Foreign Exchange Forward-Spot Parity

In FX markets, forward/spot parity is called “covered interest parity”

The cost of carry is the cost of borrowing in one currency (e.g., US dollar $) and investing in the other (e.g., the UK pound £).

Example

The spot ($E_0$) and forward ($F_0$) rates are dollar per pound $/$£.

$E_0$ is the number of dollars you can get today for £1
($E_0$ is the spot $$/£ exchange rate).

$F_0$ is the number of dollars you can, contracted upon today, to get in the future for £1
($F_0$ is the forward $$/£ exchange rate)

(No) Arbitrage enforces the following relation:

$$F_0 = E_0 \left( \frac{1 + r_{US}}{1 + r_{UK}} \right) = E_0 \left( 1 + " \text{Cost of Carry} \" \right)$$
**Explanation**

You can either commit to pay in the future $F_0$ dollars for £1,

Or

*Step 1:* You can borrow $E_0/(1+r_{UK})$ dollars to buy £1/(1+r_{UK}).

*Step 2:* The £1/(1+r_{UK}) in a pound-based account will compound to £1 next period, and you will owe $[E_0/(1+r_{UK})] (1+r_{US})$ dollars in the next period.

(you essentially took a dollar loan to "synthesize" £1).

Therefore, to obtain £1 in the future, as your assets, you will have a future liability of $F_0$ dollars (if you enter directly into a forward contract to get £1), or of $[E_0/(1+r_{UK})] (1+r_{US})$ dollars (if you settle a loan that allowed you to synthesize £1).

Under no arbitrage, both liabilities must be equal, and hence $F_0 = [E_0/(1+r_{UK})] (1+r_{US})$ as on the previous page.
VII. Swaps

Definition: A swap is a contract for exchange of future cash flows.

(Leading examples are swaps between currency payments or between floating and fixed interest rates.)

Example: Currency Swaps

A U.S. firm has a British £ obligation consisting of 1£ per year for the next 10 years. (This is a floating obligation in US$.)

The firm wants to swap this for a fixed US$ obligation of amount $x. We can refer to $x as the swap (exchange) rate, because by entering into a swap the firms fixes its exchange rate between 1£ and US$. (At the end of each year, the firm will pay to a swap dealer $x and receive 1£, so that it can cover its obligations.)

What is $x$?

- Basic data: \( r_{US} = 5.6\% \), \( r_{UK} = 6.2\% \)
  Spot exchange rate \( P_0 = E_0 = $1.51/£ \)

Forward Spot (Covered Interest) Parity:

Today’s forward price (in $) for delivery of 1 pound in one year:

\[
F_0(1) = E_0 \frac{1 + r_{US}}{1 + r_{UK}}
\]

Today’s forward price for delivery in \( k \) years:

\[
F_0(k) = E_0 \frac{(1 + r_{US})^k}{(1 + r_{UK})^k}
\]
• As an alternative to the swap, engage in forward contracts to purchase £1 in each of the next 10 years.

The PV of the $ obligation is:

\[
\frac{F(1)}{(1 + r_{US})} + \cdots + \frac{F(10)}{(1 + r_{US})^{10}}
\]

\[
= \frac{1}{(1+r_{US})} E_0 \left( \frac{1 + r_{US}}{(1 + r_{UK})} \right) + \cdots + \frac{1}{(1+r_{US})^{10}} E_0 \left( \frac{(1 + r_{US})^{10}}{(1 + r_{UK})^{10}} \right)
\]

\[
= E_0 \times \left[ \frac{1}{(1+r_{UK})} + \cdots + \frac{1}{(1+r_{UK})^{10}} \right] = E_0 \times \left[ PV \text{ of a 1 pound (annuity) obligation} \right]
\]

• If we swap this for an $x 10-year obligation, the PV is simply $x \text{ APVF}(r_{US}, 10)$

• So equating:

\[
1.51 \text{ APVF}(r_{UK}, 10) = x \text{ APVF}(r_{US}, 10)
\]

\[
1.51 \text{ APVF(6.2\%, 10)} = x \text{ APVF(5.6\%, 10)}
\]

\[
1.51 \times 7.29 = 11.01 = x \text{ APVF(5.6\%, 10)}
\]

which implies $x = 1.47$

A swap is equivalent to a portfolio of forwards (futures).

VIII. Additional Readings