Risk in Housing Markets: An Equilibrium Approach*

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Abstract

Homeowners are overexposed to city-specific house price risk and income risks, which may be very difficult to insure against using standard financial instruments. This paper develops a micro-founded equilibrium model that transparently shows how this local uninsurable risk affects individual location decisions and portfolio choices, and ultimately how it affects prices in equilibrium. I estimate a version of this model using house price and wage data and provide estimates for risk premia for different cities, which imply that homes are on average about $20000 cheaper than they would be if owners were risk-neutral. This estimate is over $100000 for volatile coastal cities. Next, I simulate the model to study the effects of financial innovation on equilibrium outcomes. Creating assets that hedge city-specific risks increases house prices by about 20% and productivity by about 10%. The average willingness to pay for completing the market per homeowner is between $10000 and $20000. Welfare gains come both from better risk-sharing and from more efficient sorting of households across cities.

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1 Introduction

Throughout their lifetimes, households face many long-term economic risks, most of which may be difficult to insure against.[1] Social safety nets and insurance mechanisms are particularly limited for long-term income and house price risk. The inability to efficiently spread risk across many households has direct adverse welfare effects since individual households may bear most of the burden of negative income and housing wealth shocks. Also, because of risk-aversion, many talented individuals may forgo brilliant careers and investment opportunities that are deemed too risky to undertake. Effective risk sharing can therefore limit downside risk and also enable individuals to make better choices, which leads to higher productivity and welfare in the economy. Such considerations have motivated Shiller (1993, 2003) and others to call for the creation of financial instruments to enable widespread sharing of risks that are not directly traded in equity markets.

Risks management considerations are especially important when a young household chooses a combined labor and housing market. Since most homeowners live near their workplace, they are exposed to a considerable amount of location-specific income and house price risk. From a portfolio management point of view, households are very much concerned about location-specific risk since a disproportionate share of their wealth is invested in one particular house, and they cannot reoptimize very frequently due to large reallocation costs.[2] To illustrate the house price volatility homeowners face Figure 1 shows the real house price index for a set of 20 major US cities. Although the US national house prices have historically been fairly steady, this is not the case for many cities in the US: individual cities are very volatile.[3] Since much of the cross-sectional volatility is idiosyncratic, standard portfolio theory implies that households would ideally hold a diversified portfolio of homes rather than only owning in one particular city. The under-diversification problem is exacerbated even more by the fact that individual homeowners cannot easily hedge local income and house price by using standard financial instruments.[4] The riskiness of a particular location can

[1]Many studies test and strongly reject the hypothesis that people share risks effectively. Some examples include Zeldes (1989), Cochrane (1991), Hayashi, Altonji and Kotlikoff (1996), Athanasoulis and van Wincoop (2000).

[2]Tracy, Schneider, and Chan (1999) report housing wealth comprises about two-thirds of the typical households portfolio.

[3]While there is little short run volatility in housing markets, the focus here is on long run volatility since homes are traded infrequently.

[4]The correlation between the stock market and average labor income is usually estimated to be close to zero. See for example Cocco, Gomes, and Maenhout (2005). Campbell and Viceira (2002) do find a positive correlation of income with lagged stock returns that is as high as .5 for college graduates. The correlation between housing returns and the aggregate stock market or REITs is also found to be very low (see Flavin and Yamashita (2002), and Hinkelmann and Swindler (2006)). Hinkelmann and Swindler (2006) also find very little correlation between prices of futures contracts (not including the new housing futures) and house price returns, stressing that the creation of housing futures should be very beneficial for hedging purposes. Constructing local stock price indexes for largest

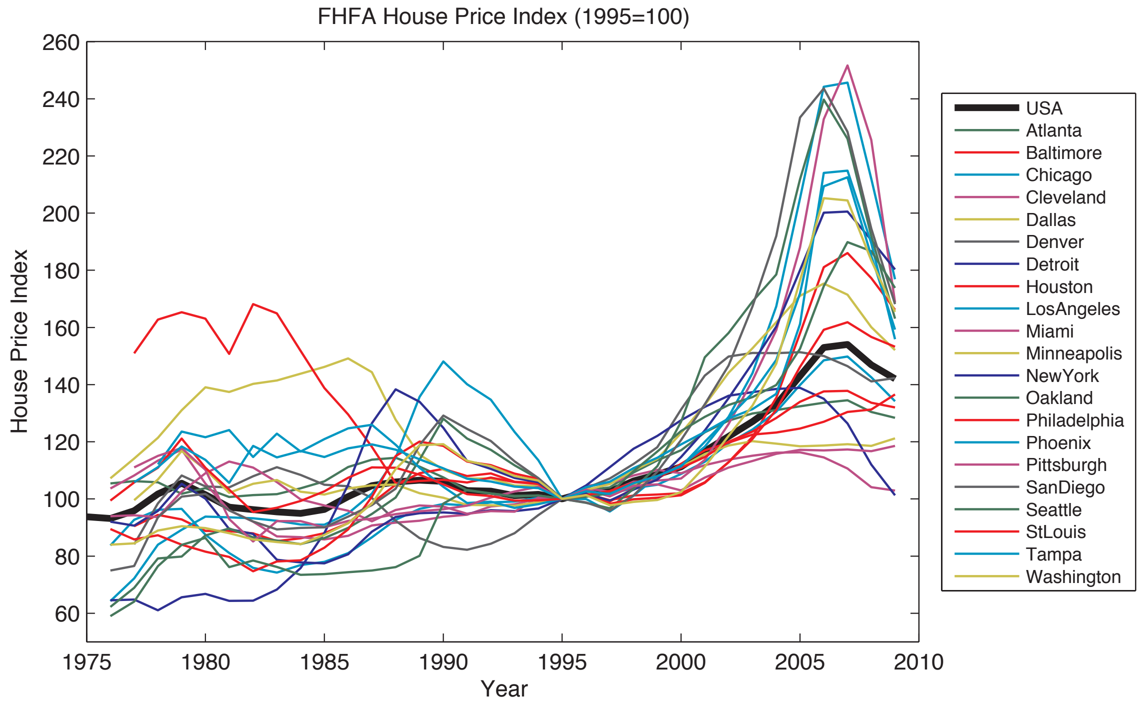

Figure 1: The Real House Price Index for Major US Cities

therefore simultaneously affect households' location and portfolio choice decisions and ultimately housing prices. In turn, financial innovation that allows households to better control their exposure to these risks may have a major impact on labor market choice, home prices and welfare.

The goal of this paper is to show both theoretically and empirically how exposure to uninsurable city-specific risks affects house prices, as well as household location and portfolio decisions. Understanding the nature and magnitude of these effects is crucial in evaluating the benefits of creating new financial instruments that facilitate risk sharing, such as those proposed by Shiller (1993, 2003) . To that end, I first develop an equilibrium theory that shows in a transparent way how location-specific risk is capitalized into house prices and how it affects productivity and welfare in the economy. The model is estimated using wage and price data for individual metropolitan areas in the US, providing empirical support for the idea that risk is priced in housing markets. I then simulate the model to study the benefits from creating financial assets that correlate with city-specific house prices and income.

The backbone of the paper consists of a novel micro-founded dynamic equilibrium model that merges standard methodologies used in urban economics, which study sorting and spatial properties

employers from 16 cities in California, Hizmo (2010) reports a correlation of .36 between the local house price returns and the local stock price index.

of the problem, with models from continuous-time finance that study financial assets. To my knowledge, this is the first flexibly estimable model that simultaneously considers risk-aversion, multiple sources of uncertainty, rich agent heterogeneity, sorting, portfolio choice and asset prices in one unified framework. The main features in the model that drive most of the interesting results are that markets are incomplete, in that there is no perfect hedging of risks, houses are indivisible, and agents are heterogeneous in terms of their productivity and risk preferences.

The general setting of the model is very similar to that in Ortalo-Magne and Prat (2010) (OMP hereafter).[5] Each instant, a generation of households are born and live for T periods. Households initially choose a single labor and housing market from a system of cities. Once households choose a city to live in, they are assumed to live there until the end of their life. Unlike the main specifications of OMP, I model houses as indivisible assets. Each household must buy one house in order to live in a city, which gives them access to local wages and amenities.[6] Households also have access to a risk-free asset and the stock market, and can adjust their portfolios continuously. At the end of their lives, households sell their homes and their financial assets, and consume their terminal wealth. Because they cannot hold shares of homes in different cities, households have to resort to using stocks to insure against their income and house price risks.[7] Because stocks may not be correlated with every dimension of risk that households face, households may face city-specific uninsurable risks.

The solution to the household problem involves two steps. Conditional on location choice, households decide how to invest their wealth in the financial market given their local income and house price process. Taking into account the utility derived from these optimal portfolio decisions, households choose the city that provides them with the highest expected utility. The portfolio choice

[5]OMP is the first paper to theoretically show the links between location and portfolio choice in an equilibrium model. Building on their elegant work, I extend the theory in number of important dimensions. Since the main focus in OMP is not empirical, income in different cities follows a random walk (in discrete time) with no drift and so do stock prices. I consider city-specific income processes that are richer and match empirical patterns of wage processes. Here, the growth in wages is modeled as a city-specific mean reverting process. Also, because of the continuous time setting, this model can handle stock prices that follow geometric Brownian motions with drift, which is in line with the modern continuous-time finance literature. In addition, this model also includes amenities, heterogeneity in preferences for local amenities, and heterogeneity in risk-aversion. In terms of the equilibrium, OMP conjecture a price process and show the existence of an equilibrium, while saying relatively little about about uniqueness. This paper proves that a linear stationary equilibrium is unique in its class, and does not rely on the existence of “hyper-marginal” households.

[6]The assumption that everyone owns is made for tractability purposes. Focusing on the owner-occupied rather than the rental market and could be justified by the observation that the homeownership rate in the US is around 70%.

[7]Since agents do not migrate to different cities every period they are concerned only about long-term house price risk, or the change in house prices from the time when they bought the home to the time they will sell it before they die.

problem with exogenous income that households face in this model is studied extensively in the literature. Generally the solutions are either found numerically and/or under the assumption that markets are complete.[8] Using transformations of the Hamilton-Jacobi-Bellman equation similar to Henderson (2005), I derive a closed-form solution of the portfolio choice with housing, exogenous income and incomplete markets. The explicit solution is crucial here since it is used to solve for the equilibrium allocation of individuals across space.

Given optimal portfolio decisions, I then solve for the spatial allocation of households. The equilibrium concept used here follows in the tradition of classic urban models of Rosen (1979) and Roback (1982) where spatial equilibrium is found by equalizing utility differences across cities. The more modern version of these models is the static horizontal sorting model of Bayer, McMillan, and Rueben (2005) or Bayer and Timmins (2005). Under a static setting, these models prove existence and uniqueness of a spatial equilibrium under very general assumptions about household heterogeneity. I make direct use of these horizontal sorting results in my model.[9]

Merging closed-form results from the optimal portfolio choice problem with a horizontal sorting model of individuals across space, I prove the existence and uniqueness of a linear stationary equilibrium. One key theoretical result is that home prices are derived to be a closed-form function of the underlying productivity of the economic base of a city minus a city-specific premium, which is a function of agent heterogeneity, sorting and risks in the particular city. Home prices can also be interpreted as the expected discounted sum of future dividends of the marginal household in that city plus a risk premium. In equilibrium, risk premia of homes takes a linear factor structure where the tradable part of risk is priced at the market price of risk. The non-insurable part of the local risk is also priced in equilibrium but its price is the risk-aversion parameter of the marginal person who lives there. In general, cities exposed to higher amounts of non-insurable risks have lower home prices in order to compensate residents for the extra risk they are taking.

Financial asset portfolio decisions are also found to be a generalized version of classic results in portfolio choice in finance. Stock holdings consist of a classic myopic term as found in Merton (1969), and a hedging demand term that depends on the correlation of the particular stock with

[8] Examples that use numerical methods to solve the portfolio choice problem with housing and exogenous income include Cocco (2004), Yao and Zhang (2005), and Van Hemert (2009). Because these models focus on numerical solutions to single agent problems, they allow for general income processes and utility function, as well as own/rent decisions. Kraft and Munk (2010) solve the optimal portfolio choice in closed-form but they assume that all individuals are renters and markets are complete.

[9] In terms of dynamic settings, the closest models are Glaeser and Gyourko (2010) and Van Nieuwerburgh and Weill (2010). These models are designed to study cross-sectional and time-series properties of house prices and wages. This paper differs from these models as it includes risk-aversion, portfolio choice and asset markets in the typical location choice equilibrium model.

income in the city a household lives in. The equilibrium rate of return for a particular stock also depends on its correlation with income in all of the cities and the distribution of the risk-aversion parameter over the population.[10] Stocks that are negatively correlated with income and house prices are in large demand for hedging purposes. In order for the stock market to clear the returns for these stocks are lower in equilibrium.

An interesting insight to emerge is that households sort not only on income and amenities, but also on the uninsurable local risk. Households that are more risk-averse are more likely to locate in cities with less uninsurable risk. Sorting on risk leads in turn to misallocation of human capital. Risk-averse households do not move to the labor market where they are most productive because they may find that city too risky. Instead, some other less risk-averse and less productive household will move to the risky city in question. The creation of financial instruments that can be used as a hedge against noninsurable local risk can therefore reduce the incentives to sort on risk and increase productivity and welfare in the economy. In the limiting case where all risk is tradable, households do not sort on risk and they locate in the labor market where their productivity is highest.

I estimate a version of the model using wage and house price data for 216 metropolitan areas in the US under the assumption that agents are homogeneous.[11] Instead of looking for all the underlying factors that drive income and house prices in the US I focus on three factors that capture a large share of the common variation across metropolitan areas. These factors are constructed in the spirit of the three Fama-French factors. The first factor (HMKT) is the average price growth across metropolitan areas. The second (SMBH) is price growth in high-price metropolitan areas minus growth in low-price ones. The third (HMLH) is growth in metropolitan areas with high price-to-wage ratios minus growth in ones with low price-to-wage ratios. The national HMKT factor turns out to have a correlation of .6 with aggregate REIT returns, while the other two factors are uncorrelated with REITs or the three Fama-French factors. This suggests that the national factor may be spanned by traded assets while the other two most likely are not. I estimate the model under different assumptions about the factors' tradability and I find that areas with higher nontradable variance demand a house price risk premium, which is reflected in lower house prices.

[10]Other studies have also found that the presence of nontradable income and housing wealth in the household's portfolio can affect asset prices. Examples that study the impact of housing decisions and prices on financial asset prices include OMP, Piazzesi, Schneider, and Tuzel (2007), Lustig and van Nieuwerburgh (2005), and Yogo (2006). The interaction of labor income risk and asset prices is studied in Constantinides, Donaldson, and Mehra (2002), Santos and Veronesi (2006), and Storesletten, Telmer, and Yaron (2007).

[11]The model could be fully estimated using individual migrations decisions data available from the Decennial Census or the PSID. Using individual migration decisions allows for direct estimation of heterogeneity in preferences and in productivity. In a companion paper, I estimate the full structural model using a two-stage procedure closely related to the popular random coefficient logit model of Berry, Levinsohn and Pakes (1995).

The estimated risk premia imply that on average homes are about \$20000 cheaper than they would be if owners were risk-neutral, although these estimates vary from \$135000 for San Francisco, CA to -\$20000 for Denver, CO. To my knowledge no other estimates of housing risk premia exist in the previous literature.

Using the estimates for 216 US metropolitan areas, I then simulate the model to study the effect of financial innovation on house prices, household sorting across space and on overall productivity and welfare in the economy. I start from the baseline where only the national HMKT factor is spanned by traded stocks. I then consider the cases where new financial instruments are created that correlate with the two other factors proposed above, and the case when the financial instrument allows for perfect insurance. The creation of tradable financial instruments that correlate with housing and income risk improves households' ability to hedge risk and consequently lowers housing risk premia. Due to heterogeneity in risk preferences, this leads to a different sorting of households across space. In the new sorting equilibrium, human capital is allocated more efficiently, leading to higher overall productivity welfare.

For a reasonable range of risk-aversion parameters, I find that completing markets can increase home prices by about 20 percent, increase productivity in the economy by 10 percent, and significantly improve welfare.[12] The average willingness to pay for access to financial instruments that correlate with all of the sources of risk in the economy is between \$10000 and \$20000 per homeowner.[13] The willingness to pay is, however, much higher for households that are very risk-averse, or households who live in locations with high noninsurable volatility. Overall, these findings do depend on the distribution of the risk-aversion parameter over the population. When heterogeneity in risk-aversion is large, productivity increases, but prices and welfare respond less to completing markets because of sorting effects. When households are homogeneous, all of the benefits of completing markets are directly reflected into higher house prices, and there is no gains in productivity.

The rest of the paper is organized as follows: Section 2 develops a joint equilibrium theory of location and portfolio choice. In Section 3, I describe the estimation procedure and fit the model to annual wage and house price data for 216 metropolitan areas in the US for the last 25 years.

[12]Under the popular "standard incomplete markets" framework in macroeconomics, Heathcote, Storesletten and Violante (2008) also find that insuring wage risk has large welfare and productivity implications. The gain in productivity is through a different channel in their model. Under market incompleteness the less productive workers work too much while high productivity agents work too little. Similar results are also found in Pijoan-Mas (2006). My paper proposes a different channel through which market completeness affects productivity. Completing markets increases productivity because workers are matched with jobs more efficiently because of sorting effects.

[13]To my knowledge there exist no previous estimates in the literature for the willingness to pay for access to these new financial instruments.

Section 4, uses the model to conduct a series of general equilibrium counter-factual simulations designed to study the impact of creating new financial instruments that allow individuals to hedge housing risk; and Section 5 concludes.

2 The Model

The classical Capital Asset Pricing Model (CAPM) states that only variation related to the market factor should be priced in equilibrium. In contradiction with CAPM, Hizmo (2010) and Case et al. (2010) find that factors not related to market returns and idiosyncratic risks are priced in housing market returns. The likely reason why CAPM fails to describe housing returns fully is that housing markets are characterized by many frictions that limit arbitrage. Because houses are large and indivisible, most households only own a home in one city, which exposes them to a large amount of local risk. This, combined with households' inability to perfectly hedge income and house price risks, gives rise to a unique problem that typical asset pricing models are not well-suited to tackle.

This paper develops a micro-founded dynamic equilibrium model that accounts for all of the frictions mentioned above and fits the deviations from the CAPM. This flexible model simultaneously considers risk-aversion, multiple sources of uncertainty, rich household heterogeneity, sorting, portfolio choice and asset prices in one unified framework. In equilibrium, risk premia of homes turns out to take a linear factor structure where the non-tradable part of the local risk is priced. Interestingly, equilibrium financial asset returns are also affected by the frictions in housing markets. Because of the explicit nature of the solution, the main equations of the model can be directly estimated using house price and income data. The model is also well suited for counter-factual simulations since the derived equilibrium is unique for any given set of parameters. In the rest of this section, I describe the setting of the model, prove existence and uniqueness of the equilibrium, and discuss the theoretical results.

2.1 The Setting

2.1.1 Cities and population

There is a measure one of households born at time t and they live until $t + \overline{T}$.[14] I consider an overlapping generations model where a new cohort of people is born in every instant. Households

[14] Allowing for random life spans would complicate the model since households would want to hedge their mortality risk on top of their income and house price risk. We abstract from these complications in the current setting since our main goal is to understand the effects of wage and house price risk.

must buy only one house to live either in the countryside or in any city. They choose a location after they are born and live there until the end of their lives, at which point they sell their house to the new incoming generation. Such transactions occur continuously since at every instant in time a generation is born and a generation dies.

Suppose there are L cities denoted $l = 1, ..., L$ with countryside $l = 0$. Each city l has n^l houses available for people to move into at any given period of time and cities are fixed in size. The total supply of a city is $\int_0^{\overline{T}} n^l ds = \overline{T} n^l$. Assume that one household must occupy exactly one house. Not all of the households from the same cohort can locate in cities since housing is scarce there. This means that a share of the population must locate in the countryside. The reason why the countryside is important here is that it serves as the outside choice that can be used the set the level of utility and prices in the spatial equilibrium. This will become obvious when we discuss uniqueness of the equilibrium.

2.1.2 Financial asset market

Suppose that the whole economy is driven by m independent Brownian motions $\mathbb{B}_t = \left(B_t^1 ... B_t^m\right)'$. Also suppose that people can invest in n risky financial assets and one risk-less asset.[15] The risky assets do not pay any dividends.[16] Households can hold any amount of these assets and there are no transaction costs or other frictions. The evolution of the risky asset prices is given by:

$$dP_t^i / P_t^i \quad = \quad \mu_t^i dt + \sigma_{i1} dB_t^i + \sigma_{i2} dB_t^i + ... + \sigma_{im} dB_t^i \qquad i = 1, 2, ...n$$

For compactness, we write this in matrix form as:

$$D_{P_t}^{-1} dP_t = \mu_t dt + \Sigma_P d\mathbb{B}_t$$

where $P_t = \left[P_t^1, ..., P_t^m\right]$, $D_{P_t}$is a $n \times n$ matrix with diagonal equal to P_t, $\mu_t = \left[\mu_t^1, ..., \mu_t^m\right]$, Σ_P is the $n \times m$ matrix whose rows are the volatilities of P_t^i.

Although we could potentially deal with a more general case, for simplicity assume that the

[15] Notice that depending on m and n the stock market could be complete or incomplete. We will have $m > n$ in our model which will imply that markets are not complete.

[16] The assumption that stocks do not pay any dividends is made for analytical simplicity and is standard in the finance literature. For the purpose of our model it doesn't really matter if returns from the financial assets are coming from dividends or from price appreciation. All we are interested in is what returns are and how they are correlated with other assets in the economy.

coefficients $\mu_t^i = \mu^i$ and $\sigma_{ijt} = \sigma_{ij}$ are constant over time so we have:

$$D_{P_t}^{-1} dP_t = \mu dt + \Sigma_P d\mathbb{B}_t$$

The matrix Σ_P, which captures the exposure of stocks to the underlying risk factors $\mathbb{B}_t$, is given exogenously. In contrast, the expected return vector μ is an equilibrium object. Given the volatility matrix Σ_P, I solve for the equilibrium rate of return vector μ that clears the financial asset market.

For now, we do not consider other kinds of assets or derivatives in this economy. The inclusion and pricing of other sorts of assets or derivatives could also be easily handed in this framework because of the standard normality assumptions that we are making as to how shocks evolve in the economy. Therefore, the set of stock used here can be thought of as the basis that spans all the other financial assets in the economy that we do not consider.

2.1.3 The housing dividend

Households must buy only one house in order to live either in the countryside or in any city. As mentioned, they decide where to live when they are born and live in the same place until the end of their lives. The countryside does not pay any wages and does not offer any amenities.[17] When a household moves to a city, it gets utility from the local amenities as well as wages from local firms. The per period housing dividend a household living in city l receives in terms of dollars is:

$$D_{it}^l = w_{it}^l + \beta_i M^l$$

where w_{it} is the wage household i receives at time t and $\beta_i M^l$ is the value it receives from amenities in city l.[18] The taste parameter β_i capture heterogeneity in the population of preferences for city-specific amenities, such as school quality, crime or weather. The wages household i receives are given by:

$$w_{it}^l = y_t^l + \xi_i + \varepsilon_i^l$$

[17]This assumption is not as stringent as it may seem. House prices and utility in equilibrium will be relative to the outside choice i.e. the countryside. The utility and prices of the outside choice are set to zero for simplicity. In principle, the outside utility could be set to any level and this does not affect the equilibrium allocation of households or asset prices. Changing the level of the outside choice utility only increases the price level in every city by a constant. This is a standard property of standard discrete choice spatial models.

[18]Note that this should be interpreted as the net dividend from living in one house, which is the part of wages and amenities left over after households pay taxes and other costs to live in the city.

where y_t^l is time-varying city productivity, ξ_i is a worker-specific fixed effect that does not vary across cities and ε_i^l is a city-worker match fixed effect. We can think of y_t as the part of the city-specific productivity that does not depend on who works there. The fixed effect ξ_i can be thought of as the effect of an individual's general education or expertise on wages, which does not depend on where the household locates. The last term ε_i^l can be interpreted as the effect of industry or firm-specific human capital on wages. Workers specialized in the auto industry will have a higher ε_i^l in Detroit, MI, and workers who specialize in the high tech industry will have a higher ε_i^l in San Jose, CA.

The only part of wages that is stochastic from the household's point of view is y_t. Suppose that the productivity of city l at time t denoted by y_t^l evolves according to:[19]

$$\begin{aligned} dy_t^l &= s_t^l dt \\ ds_t^l &= \phi^l \left(m^l - s_t^l \right) dt + \Sigma_s^l d\mathbb{B}_t \end{aligned}$$

This process means that the city-specific productivity y_t grows with a stochastic drift s_t. This drift follows a mean-reverting stochastic process also known as the Ornstein-Uhlenbeck process, which is the continuous-time analog of a discrete time AR(1) process. The stochastic shocks that govern this process are the underlying sources of risk in the economy $\mathbb{B}_t$. The vector Σ_s^l, which is $m \times m$, governs the exposure of city l to these underlying sources of risk. The city-specific parameter m^l is the long-run average of the drift s_t, while the parameter ϕ^l governs the speed at which the process reverts to the mean m^l. High values of ϕ^l imply that the process reverts to the long run mean quickly, and low values imply that the process is more likely to wander off far from the mean for extended periods of time. This process implies that income will on average increase by the average amount m^l, but there will be "business cycles" where income increases faster or slower than the long run average m^l.

2.1.4 The utility specification

A household is born at some time t, buys a house in a city, starts working and receives wages, and continuously invests his wealth in financial assets. At the end of his lifetime, the household sells his home and all of his assets and consumes all of his accumulated wealth. It is assumed that there

[19] This exact form of the evolution of the city-specific productivity is not essential for the model. The choice of this particular stochastic process is motivated by empirical patterns of the metropolitan area-specific average income series, which are used to estimate of the model in the next section.

is no intermediate consumption; all of the wealth is consumed at the end of his life. Allowing for intermediate consumption complicates the solution of our problem to the point where analytical solutions are not possible. In fact, there are no known analytical solutions in the literature to the portfolio choice problem with exogenous income and incomplete markets, unless the income process is extremely simplistic.[20]

At birth time t_0, households maximize their lifetime utility given by:

$$V\left(X_{t_0}, w_{t_0}, s_{t_0}, l\right) = \underset{\theta_t, l}{sup} - E_{t_0} e^{-\gamma_i \left[X_{t_0+\overline{T}} + M^l \beta_i \frac{1}{r}\left(e^{r\overline{T}}-1\right)\right]} \tag{1}$$

where $X_{t_0+\overline{T}}$ is the financial wealth accumulated through the lifetime and the second term is the utility accumulated from access to the local amenities. Each household chooses the optimal location l in which to live and each period chooses the optimal θ, which is the dollar amount invested in stocks. The above maximization problem is constrained by the wealth evolution equation, which for $t \in (t_0, t_0 + \overline{T})$ is:

$$dX_t = \theta_{it} \frac{dP_t}{P_t} + r\left(X_{it} - \theta_{it}\right) dt + w_{it} dt$$

where X_t is total wealth and θ_t is amount of money invested in stocks. Suppose an individual will die at time T, which means he is born at $t_0 = T - \overline{T}$. Using the constraint, we can solve for the terminal wealth and write the optimization problem for any $t \in (t_0, T)$ as:

$$V_t^l \quad = \quad \underset{\theta_t, l}{sup} - E_t e^{-\gamma_i \left(e^{r(T-t)} X_t + \int_t^T e^{r(T-u)}\left(\theta_u' D_{P_u}^{-1} dP_u - 1'\theta_u r du + w_u du\right) + \frac{1}{r}\left(e^{r(T-t)}-1\right)\beta_i M^l + p_T^l\right)} \tag{2}$$

Households therefore choose a city l to work and live in, and a series of stock holdings θ_t that maximize their lifetime utility, which is defined over their terminal wealth. This wealth is composed of their capital gains on the home they bought, the wealth accumulated from wages they received, the wealth accumulated from investing in the stock market, and the dollar value of the utility accumulated from having access to local amenities.

[20]Merton (1971) solves this problem by assuming deterministic income. Svensson and Werner (1993) provide solutions in the case of infinitely lived agents with intermediate consumption in the case where income is drawn from an iid normal distribution, meaning that all of the shocks to income are temporary. See Henderson (2005) for a more detailed discussion.

2.2 Equilibrium

An equilibrium in this economy is a set of home prices, asset prices and portfolio and location decisions such that households maximize utility and the asset markets and the housing markets clear. I consider a stationary equilibrium where the mass of houses available to a new cohort n^l is constant over time.[21] This means that there are n^l households of every possible age in one city.

Because of the complexity of the problem considered, I focus only on equilibrium prices that are linear in the states:

$$p_t^l = A^l y_t^l + B^l s_t^l + C^l \tag{3}$$

where A^l, B^l, and C^l are city-specific constants to be determined in equilibrium. Starting with prices of this form, I show that there exists a unique equilibrium and that the values for these three vectors $\mathbf{A} = [A^1, A^2, ..., A^L]$, $\mathbf{B} = [B^1, B^2, ..., B^L]$ and $\mathbf{C} = [C^1, C^2, ..., C^L]$ are determined uniquely in equilibrium. The price for the countryside is normalized to zero.[22]

Several steps are needed to prove that there exists a unique equilibrium . First, conditional on a household having located in a specific city, I solve for optimal value function and for optimal portfolio choice. Next, I show that given vectors $\mathbf{A}$ and $\mathbf{B}$, the equilibrium stock returns are uniquely determined by asset market clearing. Then, I turn to the location decision which gives unique values for $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ and a unique sorting equilibrium. These vectors $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ ensure equilibrium both in the housing and the asset market.

2.2.1 Portfolio choice and stock market equilibrium

Conditional on living in some location l, each household continuously chooses where to allocate his money. The problem given in equation (2), conditional on a choice of l, is the optimal portfolio choice problem for an investor facing imperfectly hedgeable stochastic income. The returns on income and stocks are imperfectly correlated, so the market is incomplete. This is a complicated class of problems to solve and very few instances in the finance literature have been successful in finding a closed-form solution.[23]

[21]The assumption of fixed supply is not empirically realistic for many metropolitan areas, although it may fit the most volatile and supply constrained coastal metropolitan areas. Allowing for elastic supply is first on the list of many extensions to the current model.

[22]The assumption on the linearity of equilibrium prices is not as limiting as it seems. The equilibrium prices found from this assumption turn out to be the expected discounted sum of future dividends of the marginal person that lives in the city plus a city-specific risk premium as shown later in a proposition.

[23]Henderson (2005) is the closest example to this paper in providing closed-form results. Under the special case of iid and normally distributed income, Svensson and Werner (1993) obtain explicit results in an infinite horizon problem. Similar results are also found by Duffie and Jackson (1990) and Tepla (2000) in a finite horizon problem.

To find the value function, I follow the logic laid out by Henderson (2005).[24] First, I take the first order conditions of the Hamilton-Jacobi-Bellman (HJB hereafter) and substitute out θ_t and X_t. This gives rise to a nonlinear partial differential equation we need to solve. The nonlinear HJB equation is then transformed to a linear partial differential equation. At this point, we do a change of measure and use the Feynman-Kac theorem to get the value function with the portfolio choice substituted out. Taking the expectation, we achieve the result in the following proposition, which is proved in the appendix.

Proposition 1. (Value Function) *The value function of an household at some time $t \in (t_0, T)$ conditional on being in city l is:*

$$V(t, y, s, l) = -e^{U_t^l} \tag{4}$$

with:

$$\begin{aligned} U_t^l = & -\gamma_i \left[e^{r(T-t)} X_t + \frac{1}{r} \left(e^{r(T-t)} - 1 \right) \left(\beta_i M^l + \xi_i + \varepsilon_i^l \right) + \hat{k}_{t1}^l y_t^l + \hat{k}_{t2}^l s_t^l + \hat{k}_{t3}^l + \int_t^T \hat{k}_4 \hat{k}_{u2}^l \sqrt{\Sigma_{ss}^l} du \right] \\ & + \frac{1}{2} \gamma_i^2 \left(\Sigma_{ss}^l - \Sigma_{sP}^l \Sigma_{PP}^{-1} \Sigma_{Ps}^l \right) \int_t^T \left(\hat{k}_{u2}^l \right)^2 du - \frac{1}{2} \left(\mu_t' - 1'r \right) \Sigma_{PP}^{-1} \left(\mu_t - 1r \right) (T - t) \end{aligned}$$

where $\hat{k}_{t1}^l ... \hat{k}_4$ are defined as

$$\hat{k}_{t1}^l = A^l - \frac{1 - e^{r(T-t)}}{r}$$

$$\begin{aligned} \hat{k}_{t2}^l = & \left\{ A^l \frac{1 - e^{-\phi^l (T-t)}}{\phi^l} + B^l e^{-\phi^l (T-t)} \right. \\ & \left. - \frac{1 - e^{r(T-t)}}{\phi^l r} + \frac{e^{-\phi^l (T-t)} - e^{r(T-t)}}{\phi^l \left(r + \phi^l \right)} \right\} \end{aligned}$$

Other cases that study the portfolio choice problem with exogenous income either solve the model numerically or make the assumption that markets are complete. Examples that use numerical methods include Cocco (2004), Yao and Zhang (2005), and Van Hemert (2009). Kraft and Munk (2010) solve the optimal portfolio choice in closed-form but they assume that markets are complete.

[24] The problem is different from that in Henderson (2005) since here risks and stocks are multidimensional, the income process is governed by a state variable that does not depend on the level of income, and agents hold a risky house in their portfolio.

$$\hat{k}_{t3}^{l} = m^{l}\left\{-\hat{k}_{t2}^{l} + A^{l}(T-t) + B^{l} + \frac{t\left(1-e^{r(T-t)}\right)}{r} - \frac{1}{r^{2}}\left(rT + 1 - e^{r(T-t)}(rt+1)\right)\right\} + C^{l}$$

$$\hat{k}_{4}^{l} = -\frac{\Sigma_{sP}^{l}\Sigma_{PP}^{-1}(\mu - 1r)}{\sqrt{\Sigma_{ss}^{l}}}$$

and $A^{l}, B^{l}, C^{l}, m^{l}, \phi^{l}, \varepsilon_{i}^{l}$ *and* Σ_{s}^{l} *are city-specific constants and* $\Sigma_{KL} = \Sigma_{K}\Sigma_{L}'$.

Notice that the value function above is the exponential of a linear function in states. All of the functions $\hat{k}_{1t}^{l}...\hat{k}_{4}^{l}$ do not depend on the states or individual heterogeneity parameters. This simple structure is very helpful in solving the portfolio and location decisions given in the next proposition and proven in the appendix.

Proposition 2. (Portfolio Choice) *A household of type* i *living in city* l *at time* $t \in (t_0, T)$ *holds stocks in the dollar amount given by the vector:*

$$\theta_{ilt} = (\Sigma_{PP})^{-1}\left[\frac{(\mu - \mathbf{1}r)}{\gamma_{i}e^{r(T-t)}} + \frac{\hat{k}_{t2}^{l}}{e^{r(T-t)}}\Sigma_{Ps}^{l}\right] \tag{5}$$

where $\hat{k}_{t2}^{l}$ *is a city-specific nonrandom function of time that depends on vectors* $\mathbf{A}$ *and* $\mathbf{B}$.

The optimal portfolio in the stock, θ_{ilt} given in (5) is comprised of two components. The first is the same term that Merton (1969) finds as an optimal strategy in the absence of stochastic income. This strategy is myopic as it is the portfolio choice for an investor who ignores income risk and only looks one period ahead. Since income is risky and is correlated with stock returns, the optimal portfolio also includes a hedging component. This second term hedges the change of stochastic income and can be interpreted as the inter-temporal hedging demand as in Merton (1971).[25] Notice that the second term does not depend on the risk-aversion parameter. This is because income is normally distributed and it is instantaneously riskless, which means that investors do not hedge income but the change in income s_t.

Now we turn to equilibrium in the stock market conditional on values of $\mathbf{A}$ and $\mathbf{B}$. There is a fixed supply of measure one of stocks and in equilibrium this should be equal to the aggregate demand for stocks. The aggregate demand for stocks of every household type i at age t in city l

[25] A similar hedging demand is also found in Henderson (2005), Duffie and Jackson (1990) and Tepla (2000).

should equal the total stock supply:

$$\sum_{l=1}^{L}\left\{\int_{\gamma_i:V^l\geq V^k,\forall k}\int_0^T \Sigma_{PP}^{-1}\left[\frac{(\mu-\mathbf{1}r)}{\gamma_i e^{r(T-t)}}+\frac{\hat{k}_{t2}^l}{e^{r(T-t)}}\Sigma_{Ps}^l\right]dt d\Gamma(\gamma_i)\right\}=\mathbf{S}$$

where $\Gamma(\gamma_i)$ is the cumulative density function for γ_i, n^l is the number of homes available in city l, and $\mathbf{S}$ is the net supply of assets, which could be zero. Simplifying and solving for μ, we get the equilibrium market returns.

Proposition 3. (Stock Market Equilibrium) *Given a set vectors* $\mathbf{A}$ *and* $\mathbf{B}$ *the stock returns are uniquely determined in equilibrium*:

$$\mu=\frac{r\Sigma_{PP}\mathbf{S}+r\sum_{l=1}^{L}\left(n^l\Sigma_{Ps}^l\int_0^T\frac{\hat{k}_{2t}^l}{e^{r(T-t)}}dt\right)}{\left(1-e^{-r(T-t)}\right)\int_{\gamma_i}\frac{1}{\gamma_i}d\Gamma(\gamma_i)}+1r \tag{6}$$

where $\hat{k}_{2t}^l$ *is a city-specific constant that depends on vectors* $\mathbf{A}$ *and* $\mathbf{B}$ *and the age* t *of the individual.*

The equilibrium rates of returns for stocks in this case do depend on the distribution of the risk-aversion parameter over the population but do not depend on the location decisions of individuals.[26] Equilibrium returns do not only depend on the variance-covariance matrix of the stock but also on the covariance of that particular stock with the local shocks in each city and the size of those cities. This finding is a deviation from standard equilibrium models where local nondiversifiable risk is ignored. If stocks are positively correlated with local shocks, they will have a higher rate of return than otherwise in order for the market to clear. Further interpretation of this result is postponed until after we solve for the equilibrium value of $\hat{k}_{2t}^l$, which makes this equation much more transparent.

2.2.2 Housing market equilibrium

We turn now to equilibrium in housing markets. At birth time t_0, households maximize the value function in equation (4) with initial wealth $X_{t_0}^l=X_{0i}-p_{t_0}^l=X_{0i}-A^l y_{t_0}^l-B^l s_{t_0}^l-C^l$. Given their optimal portfolio decisions given by Proposition 2, at time t_0 they choose the optimal location l that maximizes their lifetime utility in equation (4). Any monotonic transformations of the value

[26] The reason that equilibrium rates of return do not depend on the location decisions of individuals is that, once the move to a city, all individuals face the same exposure to the city's risk factors given by Σ_s^l. If I were to allow the variance of wages to be individual specific then the location decisions of individuals would affect asset prices directly. I have solved the model in that case but do not present it here since allowing for individual heterogeneity in exposure to factors makes the solution much clumsier and harder to interpret.

function do not change the maximization problem a household faces. After taking the log of the value function and dropping any terms that are the same across choices, the maximization problem for a household born at time t_0 the log value function takes a very simple form.

Proposition 4. (Optimal Location) *A household i born at time t_0 chooses his/her optimal location by solving for the maximum log value function:*

$$max_l U_{t_0 i}^l = \beta_i M^l + k_1^l y_{t_0}^l + k_2^l s_{t_0}^l + k_3^l - rC^l - \gamma_i k_4^l + \varepsilon_i^l \tag{7}$$

where $k_1^l ... k_4^l$ are location-specific constants that do not depend on states, individual heterogeneity, or the vector $\mathbf{C}$*:*

$$k_1^l = 1 - rA^l$$

$$\begin{aligned} k_2^l &= \frac{r}{\left(e^{r\overline{T}} - 1\right)} \left\{ A^l \frac{1 - e^{-\phi^l \overline{T}}}{\phi^l} + B^l \left(e^{-\phi^l \overline{T}} - e^{r\overline{T}} \right) \right. \\ & \left. - \frac{1 - e^{r\overline{T}}}{\phi^l r} + \frac{e^{-\phi^l \overline{T}} - e^{r\overline{T}}}{\phi^l \left(r + \phi^l \right)} \right\} \end{aligned}$$

$$\begin{aligned} k_3^l &= \frac{r}{\left(e^{r\overline{T}} - 1\right)} \left[A^l m^l \overline{T} - m^l \frac{1}{r^2} \left(r\overline{T} + 1 - e^{r\overline{T}} \right) \right] \\ & -m^l k_2^l - \Sigma_{sP} \Sigma_{PP}^{-1} \left(\mu - 1r \right) \int_0^{\overline{T}} k_5^l du \end{aligned}$$

$$k_4 = \frac{1}{2} \left(\Sigma_{ss}^l - \Sigma_{sP}^l \Sigma_{PP}^{-1} \Sigma_{Ps}^l \right) \int_0^{\overline{T}} \left(k_{5u}^l + Be^{ru} \right)^2 du$$

$$\begin{aligned} k_{5u}^l &= \frac{r}{\left(e^{ru} - 1\right)} \left\{ A^l \frac{1 - e^{-\phi^l u}}{\phi^l} + B^l \left(e^{-\phi^l u} - e^{ru} \right) \right. \\ & \left. - \frac{1 - e^{ru}}{\phi^l r} + \frac{e^{-\phi^l u} - e^{ru}}{\phi^l \left(r + \phi^l \right)} \right\} \end{aligned}$$

The constant $k_1^l ... k_4^l$ are found by setting $T - t_0 = \overline{T}$ in the constants$\hat{k}_{1t}^l ... \hat{k}_4^l$ in Proposition 1. The $\hat{k}_{it}$ in Proposition 1 depend on time t because they are functions of the number of years left until death $T - t$. Since at birth there are $\overline{T}$ years left until death, the constants $k_1^l ... k_4^l$ do not depend on time but only on total lifespan $\overline{T}$.

Given the maximization problem (7), household i chooses location l if the utility it receives from this choice exceeds the utility it receives from all other choices, or when:

$$U_i^l \geq U_i^k \quad \Rightarrow \quad \bar{U}_i^l + \varepsilon_i^l \geq \bar{U}_i^k + \varepsilon_i^k \quad \Rightarrow \quad \varepsilon_i^l - \varepsilon_i^k \geq \bar{U}_i^k - \bar{U}_i^l \qquad \forall l, k$$

where $\bar{U}_i^l = U_i^l - \varepsilon_i^l$. The location-worker specific fixed effect ε_i^l is drawn from some probability distribution for each household. Because we have a continuum of households, every possible value of ε_i^l is drawn in each generation. Therefore we can interpret the mass of individuals with ε_i^l in some given range as the probability of drawing ε_i^l from that range. If we interpret ε_i^l as a random variable, the probability that a household chooses location l can be written as a function of everything that enters in U_i^l:[27]

$$Pr_i^l = f_l\left(\beta_i, \gamma_i, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{K}\right) \tag{8}$$

Aggregating these probabilities in equation (8) for all households gives the demand for each city:

$$D_t^l = \int_{\beta_i, \gamma_i} f_l\left(\beta_i, \gamma_i, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{K}\right) dF(\beta_i, \gamma_i)$$

where $F(\beta_i, \gamma_i)$ is the joint distribution of the β_i and γ_i parameters over the population. In equilibrium, demand for each city has to equal supply in each city:

$$D_t^l = n^l \quad \forall l \quad \Rightarrow \int_{\beta_i, \gamma_i} f_l\left(\beta_i, \gamma_i, \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{K}\right) dF(\beta_i, \gamma_i) = n^l, \quad \forall l \tag{9}$$

The sorting problem described here is very similar to horizontal sorting models in urban economics. The closest models in structure are those developed by Bayer, McMillan and Rueben (2005) (BMR) and Bayer and Timmins (2005). At birth, which is when households choose where to locate, $k_1^l ... k_4^l$ are city-specific characteristics. This means that the current framework maps directly to the BMR sorting model with the vector $\mathbf{C}$ serving as a price that clears the market. If demand exceeds supply, C^l is bid up until demand equals supply, and vice versa. Just as in the BMR model, given fixed city characteristics, there is a unique "price" vector $\mathbf{C}$ that leads to a unique sorting equilibrium.

Proposition 5. **(Sorting)** *Given a set of vectors* $\mathbf{A}$ *and* $\mathbf{B}$ *and a set of city characteristics* $k_1^l ... k_4^l$*, if* ε_i^l *is drawn from a continuous distribution, a unique vector* $\mathbf{C}$ *solves the system of equa-*

[27] Again the probability that a household of type i chooses location l is the same as the mass of households of type i that choose location l. I interpret this as a probability only to relate these results to the urban horizontal sorting models as shown below.

tions given in (9). Moreover, the equilibrium spatial allocation of households is unique (within the class of linear stationary equilibria).

The first part of the Proposition 5 is proved in Proposition 1 of BMR and the second part in Proposition 2 of Bayer and Timmins (2005). Their proofs will not be reproduced here.

Now we turn to determine the unique values of the vectors **A** and **B**. Note that the distribution $F(\beta_i, \gamma_i, \varepsilon_i)$ is not time-dependent. The quantity of available houses in each neighborhood n^l is also constant across time by assumption. That means that the same set of heterogeneous individuals will be drawn each period and they will sort across the same set of homes. The sorting problem will be identical in each period except that the states y_t and s_t may change. Because **A** and **B** are constant across time, in order for the constant vector **C** to clear the markets each period, we need the utility from living in a city not to depend on states y_t and s_t. If the utility function given in (7) depended on the states, the vectors **A**, **B** and **C** could not be constant. Setting $k_1^l = k_2^l = 0, \forall l$ gives the unique values for **A** and **B**. The result is given in the next proposition which is proved in the appendix.

Proposition 6. (Equilibrium Prices) *Given a time invariant distribution of household characteristics $F(\beta_i, \gamma_i, \varepsilon_i)$, the equilibrium house prices are given by:*

$$p_t^l = \frac{1}{r} y_t^l + \frac{1}{r\left(r+\phi^l\right)} s_t^l + \frac{\phi^l m^l}{r^2\left(r+\phi^l\right)} + \pi^l \tag{10}$$

where $\pi^l = C^l - \frac{\phi^l m^l}{r^2\left(r+\phi^l\right)}$. This price function gives a unique sorting equilibrium in housing markets as shown in Proposition 5 and gives unique equilibrium rates of return for stocks as shown in Proposition 3.

Proposition 6 shows that the price function given in (10) ensures market clearing in both the stock and the housing market. Not only is this equilibrium price unique among the 'linear-in-the-states' class of functions, but it also results in a unique equilibrium in the housing and asset markets.

2.3 Analysis of the Equilibrium

2.3.1 Equilibrium house prices

The linear structure of equilibrium house prices is not as limiting as it may first seem. The price function given in (10) has also the appealing feature that it can be expressed as the expected discounted sum of productivity in one city plus a city-specific constant, π^l, which captures risk premia and heterogeneity in preferences in each city.

Proposition 7. *Under the assumed processes for the evolution of y_t and s_t, the equilibrium price function (10) is equivalent to the price function:*

$$p_t = E_t \left(\int_t^{\infty} e^{-r(u-t)} y_t^l dt \right) + \pi^l$$

where π^l is a city-specific constant determined in equilibrium that captures housing risk premia and the effect of individual heterogeneity in prices.

In order to prove the proposition above, I plug in the assumed process for y_t^l and solve the double integral that discounts the cash flows to the present and takes the expectation. Given the properties of the y_t^l process, we can switch the order of integration and solve for the double infinite integral to get equation (10). The details of the proof can be found in the appendix.

2.3.2 Sorting and the factor structure for housing risk premia

The analysis so far has been conducted under a general form for Σ_P, which is the co-movement of stock returns in relation to the underlying sources of uncertainty. Without loss of generality, consider the case where stock i's returns are driven by only one underlying Brownian motion. Suppose there are n such stocks and that stock i is only driven by Brownian motion B_t^i. This means that all the off-diagonal elements of Σ_P are zero. There are a total m Brownian motions that drive the economy so if $m > n$, perfect hedging cannot be achieved. The reason for this transformation of the model is that it greatly simplifies the interpretation of the results, and it delivers equations that I can easily estimate using house price and wage data.

Proposition 8. *If the n stocks are driven by the first n sources of risk $B^1 \dots B^n$ and all the off-diagonal elements of Σ_P are zeros, then the location-specific value function is:*

$$max_l U_i^l = \beta_i M^l - r\pi^l - \sum_{j=1}^{n} \lambda_j \sigma_s^{lj} \frac{1}{r\left(r+\phi^l\right)} - \gamma_i \sum_{j=n+1}^{m} \left(\sigma_s^{lj}\right)^2 \frac{\left(e^{r\overline{T}}-1\right)}{4r^2\left(r+\phi^l\right)^2} + \varepsilon_i^l \tag{11}$$

where λ^j is the Sharpe's ratio of stock j given in equilibrium by:

$$\lambda^j = \frac{\mu^j - r}{\sigma_P^j} == \frac{r\sigma_P^j \mathbf{S}^j + \overline{T}\sum_{l=1}^{L}\left(\sigma_s^{lj}\frac{n^l}{(r+\phi^l)}\right)}{\left(1-e^{-r\overline{T}}\right)\int_{\gamma_i}\frac{1}{\gamma_i}d\Gamma(\gamma_i)} \tag{12}$$

and σ_P^j and σ_s^{lj} are the (j,j)th and jth element of Σ_p and Σ_s^l respectively.

Proposition 8 can be derived by plugging the equilibrium house price function in equations (6) and (7). First note how exposure to a source of risk affects utility and in turn prices. A city can be exposed to an underlying source of risk that is spanned by traded stocks or to one that is not hedgeable by traded assets. If the underlying source of uncertainty j is spanned, the standard deviation σ_s^{lj} of the city's exposure to it enters utility in a linear form and is multiplied by the price of risk (Sharpe's ratio) for that source of uncertainty. Otherwise, the unspanned risk factors enter jointly in the term that is multiplied by the risk-aversion parameter γ_i. Notice that if all of the sources of risk were spanned, the latter term would drop out of the utility function and not affect prices in equilibrium. If this is not the case, however, idiosyncratic risk specific to a city that is not spanned by the traded assets will affect utility and it will be priced in equilibrium.

The equilibrium market prices of risk λ^j for stock j in this case depend not only on the interest rates and the volatility σ_P^j, but also on its correlation with the income and house price processes in all of the cities captured by σ_s^{lj}, which is weighted by the size of the city n^l. If stock j is positively correlated with local shocks for a large share of the homes in the economy, they will have a higher rate of return and a higher market price of risk than otherwise. This is because most people will want to short the stock in question for hedging purposes. For every person who shortsells the stock, there must be someone who holds it. Therefore, in order for it to be worth it for people to hold this stock, the rate of return and the market price of risk need to be higher. The market price of risk does not depend on the location decisions of individuals since σ_s^{lj} is not individual specific. If it was, the allocation of individuals across space would directly affect equilibrium asset returns. The distribution of the risk-aversion heterogeneity also affects the market prices of risk as can be seen from the integral in the denominator.

2.3.3 Talent allocation

In this equilibrium, individuals do not always choose the location where they are most productive or where their ε_i^l is the highest. As a result, productivity in the economy is not maximized and there are two reasons for this. Firstly, workers do not go where they are most productive because of amenities that different places possess. A worker who is very productive in low amenity city A but not so productive in a high amenity city B may decide to locate in B if he places a lot of weight on amenities.

The same kind of productivity inefficiencies also arise when cities have different levels of nondiversifiable risk. An individual who is very risk-averse may not decide to locate in a city with a high level of nondiversifiable risk. If all of the sources of uncertainty were hedgeable, however, this kind of sorting inefficiency would not be present. In our counterfactual simulations in the section 4, we look at how much more productive the US would be as a result of creating assets that span presently hedgeable sources of risk.

3 Estimation

The equilibrium model developed in the previous section can be fully estimated by combining individual migration data from the decennial Census, housing price data from FHFA, wage data available from the Bureau of Economic Analysis, and amenity data such as crime, weather and quality of schooling from other sources. Instead of using migration data, I take a simple approach in this paper and estimate the model assuming homogeneity in preferences. I later use the estimated parameters of the model in a series of counterfactual simulations in which I allow for various degrees of preference heterogeneity.

I estimate the model using house price and wage data at the metropolitan area level for the US. Home prices are constructed by using the annual FHFA home price indices at the metropolitan area level, which are scaled up by using median home price levels from the 2000 Census. The wage data used consist of the personal income per capita measure made available by the Bureau of Economic Analysis. I also construct several proxies for amenities, which are: crime levels from the FBI's publication "Crime in the United States", population, population density, and house density from the decennial Census, and a series of weather variables from the Area Resource File maintained by the Quality Resource Systems. The final sample analyzed here consists of annual data for the period from 1985 to 2008 for 216 metropolitan areas (MSAs hereafter).

Under the assumption of homogeneity in preferences for amenities and for risk, the price equation becomes linear in risk factors. The main estimating equation therefore is:

$$p_t^l = \frac{1}{r} y_t^l + \frac{1}{r\left(r+\phi^l\right)} s_t^l + \frac{\phi^l m^l}{r\left(r+\phi^l\right)} + \pi^l \tag{13}$$

where

$$\pi^l = \frac{1}{r} M - \sum_{i=1}^{n} \lambda_i \frac{\sigma_s^{il}}{r^2\left(r+\phi^l\right)} - \gamma \sum_{i=1}^{n} \frac{\left(\sigma_s^{il}\right)^2\left(e^{rT}-1\right)}{4r^3\left(r+\phi^l\right)} + \varepsilon \tag{14}$$

Empirically, p^l is the price level in any particular metropolitan area l, y_t^l is the net income received,

s_t is the instantaneous growth in income, and m^l is the expected growth in income. The price premium π^l is composed of the effect of amenities M, the effect of the exposure to traded sources of risk, and the exposure to local noninsurable sources of risk. Our goal is to estimate all of the parameters of the price equation in order to get estimates for risk premia. Estimating the price equation is achieved in several steps. First, I use time-series regressions to estimate ϕ, as well as to identify common risk factors $\mathbb{B}_t$ that drive the economy. Using these factors, I can estimate the factor loadings σ_s^{il} for each metropolitan area. In the second step, I use cross-sectional regressions to estimate equation (14) and get estimates of the market prices of risk λ_i and the risk-aversion parameter γ. Using these parameters, I then calculate the implied risk premia and amenities for each of the MSAs.

3.1 Time-Series Factor Decomposition

In order to estimate the model, we would first need estimates of the underlying factors that drive the economy in each metropolitan area, which are given by the Brownian motions $\mathbb{B}_t$ in the model. Using price and wage data, we can only identify the total effect that these factors from time $t-1$ to time t as given by

$$\begin{aligned} F_t^l &= \int_{t-1}^{t} \Sigma_s^l d\mathbb{B}_t \\ &= \sigma_1 \int_{t-1}^{t} dB_t^1 + \sigma_2 \int_{t-1}^{t} dB_t^2 + ... + \sigma_m \int_{t-1}^{t} dB_t^m \end{aligned} \tag{15}$$

Breaking this total effect up into the actual fundamental factors, as in the second line of the above equation, would require a deeper analysis into the forces that drive the local industry of each metropolitan area, which is beyond the scope of this paper. If we have an estimate of F_t^l we could use any unobserved factor model, such as principal factor analysis, to estimate the factors $\int_{t-1}^{t} dB_t^i$ and the factor loadings σ_i. While this procedure would be rigorous, the estimated unobserved factors would be very hard to interpret. To avoid interpretation issues, here I first estimate the total effects of these underlying factors F_t^l for each metropolitan area, and then I decompose the total effects into common empirical factors that are constructed in the spirit of the Fama-French factors for the stock market.

3.1.1 Total Effect of Factors

First, I estimate the total effect of the underlying risk factors on wages and prices F_t^l. As in the model I assume that the growth in average income in each metropolitan area evolves according to:

$$
\begin{aligned}
dy_t^l &= s_t^l dt \\
ds_t^l &= \phi\left(m^l - s_t^l\right) dt + \Sigma_s^l d\mathbb{B}_t
\end{aligned}
$$

which means that the change in income dy_t^l has a mean reverting drift s_t^l. The instantaneous income drift s_t^l is driven by the exposure of the metropolitan area to the underlying factors $\mathbb{B}_t$. The parameters of this model cannot be identified by using only income data unless income is observed continuously over time. Using equation (13), however, we can estimate everything by exploiting both wage and house price data even though they are observed over discrete time intervals.

The estimation procedure is straight-forward and follows a series of linear regressions. Note that rearranging the equation (13), we can define x_t as the gap between prices and wages, which is the observable part of our model:

$$x_t^l = p_t^l - \frac{1}{r}y_t^l = \frac{1}{r\left(r+\phi^l\right)}s_t^l + \frac{\phi^l m^l}{r^2\left(r+\phi^l\right)} + \pi^l$$

The variables on the left hand side are the observable annual prices and wages in city l. All of the variables in the right hand side are unobservable and are to be estimated. Since s_t^l is assumed to follow an Ornstein-Uhlenbeck process, we know that the observable x_t^l should also follow an Ornstein-Uhlenbeck process. The evolution of x_t^l can be described by the equation:

$$x_{t+1} = \bar{x}\left(1-e^{-\phi}\right) + e^{-\phi}x_t + \frac{1}{r\left(r+\phi\right)}\int_t^{t+1} e^{-\phi(t+1-u)}\Sigma_s^l d\mathbb{B}_t \tag{16}$$

where $\bar{x} = \frac{1}{r(r+\phi)}m - \frac{\phi m^l}{r^2(r+\phi)} - \pi^l$. The equation above looks like a simple AR(1) process. We can estimate the equation (16) by running the following regressing for each MSA:

$$x_{t+1}^l = a^l + b^l x_t^l + u_t^l \tag{17}$$

We can estimate the ϕ and by setting the parameters:

$$\hat{\phi} = -ln(\hat{b})$$

Notice also that we can get an estimate of $\hat{F}_t^l = \left(\widehat{\int_t^{t+1} \Sigma_s^l d\mathbb{B}_t}\right)$, which is the total sum of factors that drive the economy from t to $t+1$, from the residual of the regression above. The estimate is:

$$\hat{F}_t^l = r^2 \left(r - ln(\hat{b})\right)^2 \sqrt{\frac{-2ln(\hat{b}^l)}{1 - \left(\hat{b}^l\right)^2} \frac{1}{\hat{b}^l}} \hat{u_t}^l = c^l \hat{u_t}^l$$

Using annual house price and income data for 216 metropolitan areas in the US for the period of 1985-2008, I estimate the parameters ϕ, κ^l and a vector F_t^l for each metropolitan area. I deflate prices and wages by the CPI with base year 2008 in order to eliminate any effects of inflation. Although inflation risk can significantly affect lifetime wealth, I abstract from it in the model and estimate the equations of interest using real variables. Also, since the model abstracts from time-varying interest rates, I set interest rate to $r = 0.04$.

The first regression that I estimate separately for each metropolitan area is equation (17), which substituting for what x_t is in terms of data reads:

$$Hprice_t^l - \alpha Income_t^l = a^l + b^l \left(Hprice_{t-1} - \alpha Income_{t-1}^l\right) + c^l F_t^l \tag{18}$$

where $Hprice^l$ is the median house price level in MSA l and $Income_t$ is the per capita income. In the model α would be equal $1/r$ if all of the income that an individual receives can be saved or consumed. In other words the model states that the coefficient on *net* income should be 1/r. In reality only a portion of the labor income can be considered as net benefit since individuals pay taxes and also incur utility losses from sacrificing leisure in order to receive that income. Estimating the parameter α directly can be difficult since wages are correlated with unobserved amenities in each MSA leading to serious endogeneity problems. Because nicer places generally offer higher wages the estimated parameter will be biased upwards. For my sample, the estimated parameter is 3.7, which is very high, implying very negative amenities for many metropolitan areas that have relatively low price to wage ratios. In this analysis I set α to 2.5, which implies reasonable levels and ranking of amenities across metropolitan areas as it will be shown later. The qualitative nature of the results does not depend on the exact value of this parameter. Setting α between 2 and 3 yields very similar results.

In principle, ϕ could be estimated separately for each MSA. Here, I fix ϕ to be the same across metropolitan areas in order to keep the interpretation of the other parameters of the model simple.

The estimate parameter here is $\hat{\phi} = .0315$ and is significantly different from zero at the 99% level. The interpretation of this parameter is that x_t, which is the difference between prices and wages, does not revert very quickly to the mean. If the difference between prices and wages is too high, we would expect it to remain high for quite some time before it eventually reverts back to the normal level. The parameter b from the discrete time AR(1) regression (18) is .9689 (.0041) meaning that x_t is close to being non-stationary. The total variance parameter κ varies widely across metropolitan areas. The most volatile MSAs are San Francisco-San Mateo-Redwood City, CA (105.35), Santa Ana-Anaheim-Irvine, CA (97.88) and Honolulu, HI (97.21). The least volatile MSAs are South Bend-Mishawaka, IN-MI (3.98), Huntington-Ashland, WV-KY-OH (4.47) and McAllen-Edinburg-Mission, TX (5.26). I also estimate F_t^l for each metropolitan area from the residual of the regression (18). While there is no particular interest in the values of F_t^l, decomposing it into different factors is crucial for the estimation of the model.

3.1.2 Three Common Factors for Housing Markets

In order to estimate equation (14), we need estimates for σ_i^l which are the MSA's exposure to the underlying factors that drive the economy. This means that we need to decompose F_t^l in several factors and estimate the factor loadings σ_i^l for each metropolitan area as in equation (15). As it is very difficult to uncover all of the factors that drive the local economy for each city, I decompose F_t^l into a few factors that capture much of the variation in the data.[28] These factors are constructed very similarly to the popular factors that Fama and French (1993) use to explain common variation in the financial asset markets.[29]

The first factor, denoted housing market (HMKT hereafter), is the annual house price returns for the whole US housing market. The second factor, denoted housing small-minus-big (SMBH), is defined as the average returns in MSAs in the bottom half of the price level distribution in a given year minus and the average return in MSAs in the top half. This factor replicates a self-financing diversified portfolio that holds houses in low priced metropolitan areas and shorts houses in high

[28] Hizmo (2010, 2) shows that the same three factors considered here explain about 90 percent of the time-series and cross-sectional variation in the house price returns for twenty five diversified housing portfolios constructed by sorting metropolitan areas on price level and price over wage ratios. For individual metropolitan areas these three factors explain about 50 percent of the variation.

[29] The Fama/French factors are constructed using 6 portfolios formed on size (market capitalization) and book to market value ratios. The first factor is the excess return on the stock market. The second factor, small minus big, is the average return on the three small portfolios minus the average return on the three big portfolios. The third factor, high minus low, is the average return on the two value portfolios minus the average return on the two growth portfolios. These three factors combined can explain over 90% of the time-series and cross-sectional variation of any well diversified stock portfolio.

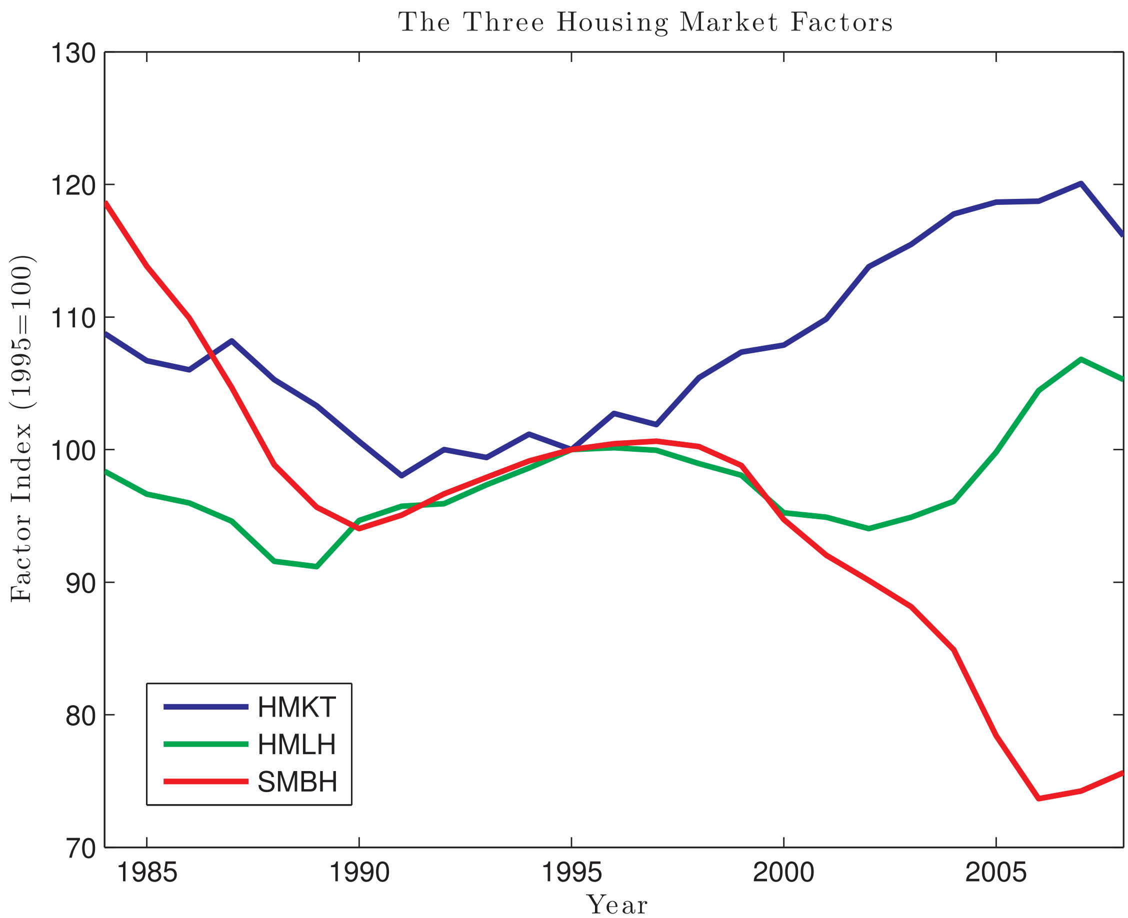

Figure 2: The Factor Decomposition of Local Risk

price ones. The third factor, denoted housing high-minus-low (HMLH), is defined as the average returns in MSAs in the bottom 30 percent of the price level distribution in a given year minus the average return in MSAs in the top 30 percent. This factor replicates a self-financing diversified portfolio that holds houses in metropolitan areas with high price-to-wage ratios and shorts houses in areas with low price to wage ratio. Similar to the small-minus-big Fama-French factors, SMBH intends to capture size effects in MSA price returns. On the other hand, HMLH is intended to capture growth effects since a high price to wage ratio can be an indicator of high expected growth.

For ease of interpretation I orthogonalize the three factors through a series of regressions. First, I regress HMKT on HMLH and SMBH and redefine HMKT as the regression residual. Then I regress HMLH on SMBH and redefine HMLH as the regression residual. This procedure gives three factors that are orthogonal to each other. The qualitative nature of the results is not affected by this orthogonalization. The three factors are displayed in Figure 2.

I decompose the local growth term F_t^l down into the effect of three factors and a residual term

ε. The estimated regression for each metro area is:

$$F_t^l = \alpha^l + \sigma_{HMKT}^l HMKT_t + \sigma_{SMBH}^l \cdot SMBH_t + \sigma_{HMLH}^l \cdot HMLH_t + \varepsilon_t^i \tag{19}$$

The results are summarized in Table 1. The coefficients in this table are presented at the mean across metropolitan areas. In parentheses I show the number of times a coefficient is found to be significantly different from zero at the 90% level. The distribution of the estimated R-squared is summarized by its mean, minimum and maximum value. While the magnitude of these coefficients is hard to interpret, their sign is straightforward. On average, when the housing market factor HMKT increases, so do wages and house prices. The higher the increase in this factor the more prices will deviate from wages. A similar result holds for the HMLH factor. If the growth in high price to wage metropolitan areas is higher than in low price to wage ones, then on average prices and wages will increase, and the price-wage gap will increase. The opposite is found when low priced MSA appreciate faster than high priced MSAs as it can be seen by the average coefficient on the SMBH factor. The HMKT factor alone on average explains about 13% of the time-series variation, the SMB factor about 22%, and the HMLH factor about 16%. The three factors combined explain about 50% of the time-series variation on average, although the R-squared for particular metropolitan areas ranges from .02 to .90.

Table 7 in the Appendix displays all of the factor loadings for the three factors and the estimated R-squared coefficients for all of the 216 metropolitan areas. The MSAs with high R-squared coefficients are generally metropolitan areas with high volatility, growth, price levels, population and density. The opposite is true for MSAs with low R-squared coefficients. The top MSA's in terms of the R^2 are Washington-Arlington-Alexandria, DC-VA-MD-WV (.90), Bakersfield, CA (.87) Riverside-San Bernardino-Ontario, CA (.86). The bottom MSA's are South Bend-Mishawaka, IN-MI (.026), Charlotte-Gastonia-Concord, NC-SC (.027) and Elkhart-Goshen, IN (.047). In terms of factor loadings, cities like New York, Boston, Washington DC, San Francisco have large positive loadings on the HMKT factor, and large negative loadings on SMBH. MSA's near California or Florida generally have high loadings on the HMLH factor, while other large MSAs in Massasuchets, Connecticut or New York have negative loadings. Overall there is a large amount heterogeneity in the exposure of particular metropolitan areas to the three factors, which fits the general observation that local housing markets are very different from each-other, and do not usually follow the national market.

Table 1: Time-Series Regressions of the Local Economic Base on Three Risk Factors

	(1)	(2)	(3)	(4)
HMKT	7.8257			7.8363
	(102)			(147)
SMBH		-11.7965		-11.7404
		(107)		(121)
HMLH			7.7780	7.7919
			(98)	(115)
R^2 distribution				
Mean	.1290	.2172	.1583	.5049
Min	.0001	.0001	.0001	.0264
Max	.4810	.7466	.7392	.9036
Groups	216	216	216	216

Note - The dependent variable in all of the regressions is the estimated process F that drives wages and prices in each metro area . Specifications (1)-(4) show the mean coefficients from time-series regressions that are run separately for each MSA. In parentheses is shown the number of times a coefficient is found to be significantly different from zero at the 90% level. The sample consists of annual data from 1985 to 2008.

3.1.3 Are the Factors Spanned by Traded Assets?

While the three proposed factors do explain a large share of the variance in the underlying local growth term F_t^l, it may not be possible for homeowners to use them to hedge house price and income risk. First of all, it is not feasible for any household to hold multiple homes in several metropolitan areas to replicate any of the three factors. Perhaps, the only feasible way to hedge against these sources of risk would be to invest in financial assets that correlate with the three factors. In order to get an idea about how much of the variance of these three factors can be spanned by using stock returns I regress each of the three factors on the three original Fama-French factors and on REIT returns.

The REIT returns used here come from the aggregate NAREIT index for REITs that invest in mortgage issued on real estate and construction. The results are almost identical if we use Equity or Hybrid REIT indexes. The Fama-French factors are three portfolios constructed using 6 portfolios formed on market capitalization (size) and book-to-market value ratios. The first factor, Mkt-Rf is the excess return on the stock market. The second factor, small minus big (SMB), is the average return on the three small portfolios minus the average return on the three big portfolios. The third factor, high minus low (HML), is the average return on the two high book-to-market ratio portfolios minus the average return on the two low book-to-market ratio portfolios. These three factors combined can explain over 90% of the time-series and cross-sectional variation of any well diversified stock portfolio.

The results are presented in Table (2). The housing market factor HMKT is the only one that seems to be very correlated with traded assets. In the first specification, HMKT is regressed on REIT returns. The coefficient on REIT is positive and statistically significant with the magnitude and standard error of .0396 (.0116). The correlation coefficient between the HMKT and the REIT returns is about 0.6. As it can be seen in the second specification, adding the Fama-French factors does not increase the R-squared and all of the coefficients on these factors are not statistically significant. In both specifications, the coefficient on REIT is positive and statistically different from zero. When the same regressions are estimated for the SMBH and HMLH factor, all of the coefficients are found to be small in magnitude and not statistically significant from zero. The R-squared coefficients from these regressions are also very low. I interpret these results as evidence that homeowners are able to hedge most the HMKT factor risk by using tradable assets. On the other hand, tradable assets do not seem to be helpful at all at hedging the risk that is due to the

Table 2: Predicting Housing Factors with Stock Returns

	HMKT		SMBH		HMLH	
REIT	.0396**	.0488**	.0033	.0097	.0028	.0059
	(.0116)	(.0130)	(.0191)	(.0206)	(.0140)	(.0155)
Mkt-Rf		-.0111		-.0174		.0039
		(.0213)		(.0338)		(.0253)
SMB		.0028		.0150		-.0091
		(.0332)		(.0526)		(.0394)
HML		-.0081		-.0565		-.0242
		(.0276)		(.0438)		(.0328)
R^2	0.345	0.355	0.001	0.084	0.002	0.042
Years	24	24	24	24	24	24

Note - The dependent variables are the three housing factors. The independent variables are aggregate mortgage REIT returns from the FTSE NAREIT series, and the three Fama-French factors downloaded from Kenneth French's website. The sample consists of annual data from 1985 to 2008. The standard errors are given in parentheses.

** statistical significance at the 95% level

SMBH and the HMLH factors.

3.2 Market Prices of Risk and Risk Premia

After estimating the factor loadings in the previous section, we have all of the ingredients to estimate the cross-sectional equation (14). For this we need an estimate of π^l, which can be given by:

$$\begin{aligned} \pi &= E\left(p_t^l - \alpha w_t^l - \frac{1}{r\left(r+\phi^l\right)} s_t^l + \frac{\phi^l m^l}{r^2\left(r+\phi^l\right)}\right) \\ &= E\left(p_t^l - \alpha w_t^l\right) - \frac{m^l}{r^2} \\ &= E\left(p_t^l - \alpha w_t^l - \frac{p_t^l - p_{t-1}^l}{r}\right) \end{aligned}$$

where the second equality uses the fact that $E(s_t^l) = m^l$ and the third equality uses the fact that $E\left(p_t^l - p_{t-1}^l\right) = m^l/r$. Instead of first estimating π and then estimating equation (14), I do it all in one step by using a between-effects panel data estimator that only uses cross-sectional variation in the data. The estimating equation is:

$$\pi_t^l = \frac{1}{r}\beta M^l - \frac{\lambda_1 \hat{\sigma}_{HMKT}^l + \lambda_2 \hat{\sigma}_{SMBH}^l + \lambda_3 \hat{\sigma}_{HMLH}^l}{r^2 \left(r + \hat{\phi}\right)} - \gamma \frac{\left(\hat{\sigma}_{err}^l\right)^2}{4r^3 \left(r + \hat{\phi}\right)} + e_t^l \tag{20}$$

where

$$\pi_t^l = p_t^l - \alpha w_t^l - \frac{p_t^l - p_{t-1}^l}{r}$$

The parameters $\hat{\sigma}$ are the factor loadings from the previous section and σ_{err} is the standard deviation of the residual from regression (19). The variable M captures amenities, which in this case are average temperature in January and July, average hours of sun in January, average humidity in July, a crime index that weights violent crimes ten times more than non-violent ones, population size, population density, housing density and percent are of water of MSA. The parameters to be estimated are β, λ and the risk-aversion parameter γ.

I estimate equation (20) to get estimates for the market prices of risk for the three proposed factors and for the parameter of risk aversion. The results are presented in Table 3. Specification (1) assumes that all the three factors are spanned and the residual variance $\left(\hat{\sigma}_{err}^l\right)^2$ is not. The estimated risk prices and the risk-aversion parameter are all statistically significant at the 95% percent level. As it can be seen from the estimate of the risk-aversion parameter, higher idiosyncratic variance leads to cheaper house prices. Households need to be compensated by one dollar decrease in current house prices for every increase of $300000 in the variance to their lifetime wealth. The R-squared coefficient is fairly high at .6859 meaning that the three factors explain a large share of the cross-sectional variation in prices and wages. Even if we didn't control for amenities, the R-squared would be rather high at .5165. In the next three columns, I repeat the same procedure under different assumptions about which factors are spanned and which aren't. Specification (2) assumes that both HMKT and SMB are spanned by traded assets while HMLH isn't. In specification (3) only the HMKT factor is spanned, and in specification (4) all of the volatility in each metropolitan areas is not spanned by any traded asset. The results are very similar under these different assumptions both in terms of magnitudes and in terms of their significance. The estimated absolute risk-aversion parameter is around $3 \cdot 10^{-6}$, which implies a relative risk-aversion parameter of 1 on average if we were to assume that a household lives in the median city in terms of volatility and earns $20000 in net wages.

Using the estimates in Table 3 together with the factors loadings for each metropolitan area, we can estimate the MSA specific risk premia and the implied amenities from equation (20). Using risk prices from specification (1) of Table3, the appendix Table 8 shows the estimated risk premia

Table 3: Cross-Sectional Estimates of Risk Prices for Traded Factors

	(1)	(2)	(3)	(4)
$\hat{\lambda}_{HMKT}$	-.1166**	-.2093**	-.0473	
	(.0591)	(.0629)	(.0648)	
$\hat{\lambda}_{SMBH}$	-.1285**	-.1223**		
	(.0241)	(.0263)		
$\hat{\lambda}_{HHML}$	.1649**			
	(.0302)			
$\hat{\gamma}/10^5$	-.3462*	-.5932**	-.3163**	-.2618**
	.1944	(.1973)	(.0735)	(.0575)
Amenities	Yes	Yes	Yes	Yes
R^2	0.6859	0.6471	0.5596	0.5559
Groups	177	177	177	177
N. Obs.	4230	4230	4230	4230

Note - The estimated coefficients represent the market price of risk for each of the factors if they are traded. The dependent variable is the part of house prices that is not due to wages and expected growth. The independent variables are the factor loadings estimated from time-series regressions multiplied by $1/\left(r^2\left(r+\lambda\right)\right)$. The absolute risk-aversion parameter $\hat{\gamma}$ is the coefficient on the variable $\hat{\sigma}_{ERR}$, which is the standard deviation of the residual from each time-series regression multiplied by $e^{30r}/\left(4*r^3\left(r+\lambda\right)^2\right)$ as the model predicts. The coefficients in this table are estimated by a panel data between estimator that only uses cross-sectional variation. The amenities included are weather, crime, population, density, and percent water area in MSA. The standard errors are shown in parentheses.

* statistical significance at the 90% level

** statistical significance at the 95% level and the implied amenity value for all the metropolitan areas in alphabetical order. A sample of the fifty most populated MSAs sorted by risk premia is displayed in Table 4. Negative estimates for risk premia are interpreted as the dollar amount a homeowner needs to be compensated for living in a risky MSA. Santa Ana-Anaheim-Irvine, CA is the MSA with the highest risk premium of -$140048. This means that prices there are $140048 cheaper than they would be if all homeowners were risk-neutral. Since homeowners are risk-averse, they are compensated by cheaper home prices for taking the risk of owning. On the bottom of the table we can see that Denver-Aurora-Broomeld, CO experiences home prices than are about $20000 higher than they would be if homeowners were risk-neutral. This is because Denver has almost opposite factor loadings to Santa Anna or San Francisco and lower overall volatility. Overall, coastal cities have large risk premia while the Midwest and the southern metropolitan areas have the lowest. The part of home prices that is not due to wages or risk premia is used as an estimate for amenities in that MSA. For example, after taking out the effects of wages, expected growth and risk premia from home prices in Santa Ana, the implied value of amenities is about $300000. Out of the fifty largest cities in Table 4, the worst MSA in terms of amenities is Detroit-Livonia-Dearborn, MI with a value of about -$32000 and the best metropolitan area is San Francisco-San Mateo-Redwood City, CA with a value of amenities of $316297.

4 Simulations

Using estimates for 216 US metropolitan areas I simulate the model to study the effect of financial innovation on house prices, household sorting across space and on overall productivity in the economy. In order to simulate the model I need estimates of the individual-MSA productivity match ε^l. I take estimates of this distribution from the previous literature.

4.1 The wage equation parameters

Using very detailed confidential migration data on a large set of individuals, Bishop (2008) estimates a wage regression:

$$w_{it}^l = f(age_i) + \omega_i\gamma + \mu_t^l + \theta_i^l + \eta_i + e_t$$

where wages of individual i who works in city l at time t are regressed on age dummies and individual specific characteristics ω_i and city-by-year fixed effects μ_t^l. The parameters of interest that are used here are the standard deviations for the individual-city match θ_i^l and the individual fixed effect η_i.

Table 4: The Fifty Most Populated MSAs Sorted on Risk Premia

	Price	Wages	Amenities	Risk Premia
SantaAna-Anaheim-Irvine,CA	482828	51894	316297	-140048
SanFrancisco-SanMateo-RedwoodCity,CA	714716	76042	382048	-135615
SanJose-Sunnyvale-SantaClara,CA	520378	58531	304677	-130572
LosAngeles-LongBeach-Glendale,CA	340842	42265	200361	-97654
SanDiego-Carlsbad-SanMarcos,CA	361444	46649	208206	-90688
Oakland-Fremont-Hayward,CA	351486	53093	177091	-88480
Sacramento-Arden-Arcade-Roseville,CA	235755	41119	137359	-77007
Riverside-SanBernardino-Ontario,CA	191675	30634	115268	-62994
WestPalmBeach-BocaRaton-BoyntonBeach,FL	243844	58358	49911	-59038
FortLauderdale-PompanoBeach-DeerfieldBeach,FL(MSAD)	239411	41974	94247	-57586
Miami-MiamiBeach-Kendall,FL	251186	35887	89587	-49034
LasVegas-Paradise,NV	178390	39920	60215	-44996
Baltimore-Towson,MD	267441	47881	78048	-43632
Orlando-Kissimmee,FL	192993	35717	70500	-42265
Phoenix-Mesa-Scottsdale,AZ	156698	36156	37786	-34795
Edison-NewBrunswick,NJ	330532	51865	112694	-34683
Wilmington,DE-MD-NJ	244404	43643	86808	-32946
Newark-Union,NJ-PA	390079	56655	140479	-31607
NewYork-WhitePlains-Wayne,NY-NJ	402722	54540	140388	-31317
Nassau-Suffolk,NY	411170	57617	132911	-30188
Portland-Vancouver-Beaverton,OR-WA	276563	39942	85134	-29251
Camden,NJ	195479	42626	44754	-27447
Providence-NewBedford-FallRiver,RI-MA	227287	40887	69347	-22110
Philadelphia,PA	190433	47361	17739	-21036
Chicago-Naperville-Joliet,IL	248207	45510	70747	-15429
Milwaukee-Waukesha-WestAllis,WI	230988	42824	67918	-11209
LakeCounty-KenoshaCounty,IL-WI	218836	51782	33225	-10426
Boston-Quincy,MA	336747	55220	92699	-7879
RockinghamCounty-StraffordCounty,NH	239595	45231	57195	-6151
Minneapolis-St.Paul-Bloomington,MN-WI	186015	47653	7047	-5723
Cambridge-Newton-Framingham,MA	387527	60093	127056	-4981
Peabody,MA	318301	50895	107415	-4670
Gary,IN	117579	35922	330	-3063
St.Louis,MO-IL	137199	41823	-2587	-2983
Nashville-Davidson-Murfreesboro-Franklin,TN	167671	39768	16522	-1207
Warren-Troy-FarmingtonHills,MI	123421	44488	-13735	314
Detroit-Livonia-Dearborn,MI	58617	32094	-32071	533
SanAntonio,TX	151913	34937	33157	1326
Columbus,OH	165190	38741	37593	2023
Charlotte-Gastonia-Concord,NC-SC	197281	39621	41230	2221
Indianapolis-Carmel,IN	122092	39297	-1721	2270
Cincinnati-Middletown,OH-KY-IN	166943	39066	37096	2389
KansasCity,MO-KS	144052	40396	8017	2919
Pittsburgh,PA	126616	42104	-15350	3052
Cleveland-Elyria-Mentor,OH	144077	40118	21777	5183
Atlanta-SandySprings-Marietta,GA	191194	38336	47676	6041
Houston-SugarLand-Baytown,TX	159217	45835	1872	6079
Dallas-Plano-Irving,TX	149710	43458	19554	7398
NewOrleans-Metairie-Kenner,LA	163257	41740	-5321	13291
Denver-Aurora-Broomfield,CO	258493	48010	51834	20281

Note - All the variables are for year 2008 and given in 2008 dollars. The risk premia is estimated by the exposure of a metropolitan area to the three factors and to the idiosyncratic variance. A negative risk premium should be interpreted as a cheaper house price due to the exposure of the MSA to risk. The amenities are calculated as the amount left over from prices after removing the effect of wages, expected growth, and risk premia.

In the model presented here these variables correspond to ε_i^t and ξ_i respectively. The estimated standard deviations that are used in my model are $\hat{\sigma}_\varepsilon = \$15499.21$ and $\hat{\sigma}_\xi = \$7073.40$ in year 2000 dollars.

4.2 Risk-Aversion Parameters

While there is some consensus in the literature about the magnitude of the relative risk-aversion (RRA) parameter at least for Constant Relative Risk Aversion utility functions, there is no agreement on the absolute risk-aversion (ARA) coefficient.[30] For these simulations I experiment with a range of absolute risk-aversion parameters anchored to be not to far from the estimates of Table 3. I simulate the model for ARA coefficients from $3 \cdot 10^{-6}$ to $3 \cdot 10^{-5}$, which in the simulations imply average RRA coefficients from .41 to 4.15. These values for the RRA are in line with estimates of the previous literature that find that the RRA coefficient is between 1 and 5. In the case where I allow for heterogeneity in risk-aversion I draw these parameters from a uniform distribution. The average ARA and RRA parameters are the same as above except for that I allow for bounds around the values given.

4.3 Results

I first simulate the model under the assumption that there is no heterogeneity in risk-aversion. All the results in the homogeneous agents case will be driven by the fact that completing the market will lead to better risk-sharing. I start from the baseline where there is no correlation between stocks and and factors that drive the local economy. I then consider the cases when new financial instruments are created that correlate with three factors proposed above and the case when the financial instrument allows for perfect insurance. The results from a series of simulations are presented in Table 5. In the first panel I study the effects of market completeness on house prices. In the first row the model is simulated for ARA of $.3 \cdot 10^{-5}$, which on average implies a RRA of .41 for the simulated wealth levels. Starting from the case where all variance is noninsurable, if we create an asset that spans the HMKT factor, prices will increase by 2.4%.[31] If in addition we create another asset that also spans the HMLH factor, prices will increase by a total of 3.14%

[30] See for example Vigna (2009)

[31] We have reason to believe that households can hedge against the HMKT risk factor by using existing financial assets as shown from the high R-squared estimates when we regress HMKT on traded financial assets in the previous section. If we agree that the HMKT factor is already spanned, we can alternatively interpret the negative of the magnitudes from these simulations as the effect of prohibiting the households from using financial assets to hedge the HMKT risk.

in relation to what they where when no factors were spanned. If we span all the three factors, prices will go up by 4.24%. If we create assets that span not only the three factors but all of the remaining variation in prices and wages, prices would go up by about 6%. In the next few rows I simulate the model again with higher ARA and RRA parameters. Price changes are very sensitive to the risk-aversion parameter. In the extreme case of a RRA of 4.15 prices increase by 40% when we span all sources of variance.

The second panel in Table 5 shows welfare effects of completing the market in terms of the compensating variation. The compensating variation is the amount of dollars a household should be compensated after a policy change in order to reach the initial utility level. Here I interpret the compensating variation as the maximum payment a homeowner is willing to make in order to have access to a financial asset that spans a particular factor. In the first row the model is simulated for ARA of $.3 \cdot 10^{-5}$, which on average implies a RRA of .41 for the simulated wealth levels. Starting form the case where all variance is noninsurable, on average homeowners would be willing to pay \$858 dollars in order to gain access to a financial asset that spans the HMKT factors, \$2680 for a financial asset that spans all three factors, and \$3600 for one that spans all of the volatility in the market. The willingness to pay is again closely tied to the risk-aversion parameter. Homeowners are willing to pay \$7760, \$15097 and \$22125 if their RRA parameters were 1.38, 2.77 and 4.15 respectively. The willingness to pay is as high as \$31342 for an asset that spans all sources of volatility for the case of high RRA of 4.15.

Next I turn to simulating the model by allowing heterogeneity in risk-aversion over the population. The results from a series of simulations are displayed in Table 6. In each row the mean of the risk-aversion parameters is set to be the same as in Table 5. The first panel shows effect of completing markets on prices. In the first row for example, I draw the absolute risk-aversion parameter from a continuous uniform distribution $U(1, 59) \cdot 10^{-7}$, which for the average wealth implies RRA coefficients as if they were drawn from the uniform distribution $U(.01, .8)$. The effects on prices are similar to those previous table. The range of price increases goes from 2.67% when homeowners are given access to assets that span the HMKT factor to 7.12% when all possible sources of risk are spanned. Prices can jump up by 43% when all variance is spanned if the RRA is drawn from $U(.02, 12)$. For a more reasonable range of RRA parameters drawn from $U(.02, 7)$, prices increase from 5.5% when only the HMKT is spanned to 30% when all the sources of volatility are spanned.

The second panel in Table 6 simulates the effects of market completeness on productivity increases. In general the creation of tradable financial instruments that correlate with housing and

Table 5: The Effects of Completing Markets on Prices and Welfare when Households have Identical Risk Preferences

		HMKT	HMKT+ HMLH	HMKT+ SMBH	HMKT+ HMLH+SMB	All Variance
Price %						
ARA	RRA					
$.3 \cdot 10^{-5}$	0.41	2.64	3.14	3.74	4.24	5.89
$1 \cdot 10^{-5}$	1.38	4.08	5.75	7.77	9.43	14.92
$2 \cdot 10^{-5}$	2.77	6.15	9.48	13.52	16.85	27.83
$3 \cdot 10^{-5}$	4.15	8.21	13.21	19.27	24.26	40.73
Willingness to Pay $						
ARA	RRA					
$.33 \cdot 10^{-5}$	0.41	856	1243	2292	2680	3600
$1 \cdot 10^{-5}$	1.38	1622	2915	6467	7760	10895
$2 \cdot 10^{-5}$	2.77	2775	5411	12461	15097	21339
$3 \cdot 10^{-5}$	4.15	3901	7771	18255	22125	31342

Note - The model is simulated for 216 MSAs using parameters from estimated house price and wage processes. Each column displays the effect of creating financial instruments that correlate with the given housing factors. The last column displays the effects of creating financial instruments that span all of the variance in wages and prices. The first panel shows percent changes in average house prices, and the second panel shows the average compensating variation in dollars. The willingness to pay is the average amount a household would be willing to pay for the creation of the financial instrument. ARA stands for the coefficient of absolute risk-aversion. RRA is the coefficient of relative risk-aversion implied by the ARA for the average household.

Table 6: Then Effects of Completing Markets on Prices, Productivity and Welfare with Heterogeneous Risk Preferences

		HMKT	HMKT+ HMLH	HMKT+ SMBH	HMKT+ HMLH+SMB	All Variance
Price %						
ARA	RRA					
$U(1,59) \cdot 10^{-7}$	$U(.01,.8)$	2.67	3.40	4.60	5.36	7.12
$U(1,199) \cdot 10^{-7}$	$U(.01,3)$	3.90	6.15	9.14	11.54	16.87
$U(1,399) \cdot 10^{-7}$	$U(.02,7)$	5.50	9.74	14.95	19.38	29.15
$U(1,599) \cdot 10^{-7}$	$U(.02,12)$	7.27	13.65	21.51	28.15	42.84
Productivity %						
ARA	RRA					
$U(1,59) \cdot 10^{-7}$	$U(.01,.8)$	0.29	0.74	2.20	2.43	2.62
$U(1,199) \cdot 10^{-7}$	$U(.01,3)$	0.78	1.84	6.69	8.04	9.78
$U(1,399) \cdot 10^{-7}$	$U(.02,7)$	0.91	2.29	8.39	10.86	14.59
$U(1,599) \cdot 10^{-7}$	$U(.02,12)$	1.01	2.67	9.16	12.47	18.58
Willingness to Pay $						
ARA	RRA					
$U(1,59) \cdot 10^{-7}$	$U(.01,.8)$	894	1280	2260	2665	3554
$U(1,199) \cdot 10^{-7}$	$U(.01,3)$	1344	2388	4561	5665	8119
$U(1,399) \cdot 10^{-7}$	$U(.02,7)$	2009	3910	7469	9431	13758
$U(1,599) \cdot 10^{-7}$	$U(.02,12)$	2782	5617	10953	13869	20257

Note - The model is simulated for 216 MSAs using parameters from estimated house price and wage processes. Each column displays the effect of creating financial instruments that correlates with the given housing factors. The last column displays the effects of creating financial instruments that span all of the variance in wages and prices. The first panel shows percent changes in average house prices, the second shows percent changes in average wages, and the third panel shows the average compensating variation in dollars. The willingness to pay is the average amount a household would be willing to pay for the creation of the financial instrument. ARA stands for the coefficient of absolute risk-aversion. RRA is the coefficient of relative risk-aversion implied by the ARA for the average household.

income risk improves the households' ability to hedge risk and consequently lowers housing risk premia. In addition when there is heterogeneity in risk preferences, lowering the amount of non-insurable volatility leads to a different sorting of households across space. In the new sorting equilibrium, human capital is allocated more efficiently leading to higher overall productivity or wage level in the economy. The simulated effects on productivity are sensitive to the assumption on the distribution of the risk-aversion parameters. For a reasonable range, when the RRA parameters are distributed according to the uniform distribution $U(.02, 7)$, wages increase by 8.39% when all of the three factors are spanned, and they increase by 10.86% when all of the sources of volatility are spanned.

The last panel of Table 6 shows welfare effects of completing the market in terms of willingness to pay in dollars. The average willingness to pay for access to a tradable asset varies widely with the risk-aversion parameters as well. For the range of RRA parameters that are drawn from the uniform distribution $U(.02, 7)$ on average homeowners would be willing to pay \$2009 dollars in order to gain access to a financial asset that spans the HMKT factors, \$9431 for a financial asset that spans all three factors, and \$13758 for one that spans all of the volatility in the market.

Taken together, results from these simulations can be taken as support to the idea that creating financial instruments that improve the households ability to manage risk is very beneficial in many levels. Particularly, under different assumptions about parameters values, simulations show that on average creating assets correlate with all of the sources of volatility significantly increase productivity, wages and welfare.

5 Conclusion

In order to understand the links between underlying risk factors, house prices and household location decisions I develop a micro-founded equilibrium model. The approach used is unique in that it merges standard methodologies used in urban economics, which study sorting and spatial properties of the problem, with models from continuous-time finance that study financial assets. This flexible and estimable model simultaneously considers risk-aversion, multiple sources of uncertainty, rich agent heterogeneity, sorting, portfolio choice and asset prices in one unified model. One key theoretical result is that home prices are derived to be a closed-form function of the underlying productivity of the economic base of a city minus a city-specific risk premium, which is a function of agent heterogeneity, sorting and risks in the economy. The problem of household sorting across space turns out to be very similar to that studied by Bayer et. al. (2005). Asset portfolio decisions are also found to be a generalized version of classic results in portfolio choice in finance.

The model is then estimated using US price and wage data and is simulated to study the effect of completing markets on house prices, household sorting across space and on overall productivity in the economy. The estimated risk premia imply that on average homes are about $20000 cheaper than they would be if owners were risk-neutral, although there is large heterogeneity across metropolitan areas. Creating financial instruments that can be used for hedging purposes, lowers housing risk premia, increases welfare and increases productivity in the economy through sorting effects. For a reasonable range of risk-aversion parameters, I find that completing markets can increase home prices by about 20 percent, increase productivity in the economy by 10 percent and significantly improve welfare. The average willingness to pay for access to financial instruments that correlate with all of the sources of risk in the economy is between $10000 to $20000 per homeowner, depending on the assumed risk-aversion. This willingness to pay also varies widely across individuals and the sources of risk they are exposed to: more risk averse agents and agents that locate in more volatile cities are willing to pay much more than the average to gain access to complete markets.

Taken together, these findings draw attention to the potential benefits for creating financial instruments that correlate with house prices and income in every metropolitan area. Although these benefits are large, comparably large implementation difficulties may exist in creating a market for instruments to manage housing risk. As in any new market, some difficulties include marketing to and educating homeowners about these products, pricing them correctly, as well as creating and maintaining enough liquidity in these markets. The results in this paper suggest that perhaps the lack of a well-functioning market for hedging housing risk is not because this risk is trivial, but rather because we haven't found a way to implement the idea successfully yet.

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Appendix

A Proofs

A.1 Proof of Proposition 1

The problem at some time t faced by each household who dies at time T is:

$$maxV\left(X_t, w_t, x_t\right) = \sup_{\theta_t, l} - E_t e^{-\gamma_i\left(X_T + \frac{1}{r}\left(e^{r(T-t)}-1\right)\beta_i A^l\right)} \tag{21}$$

For notational simplicity we drop the i and l subscript from now on. Conditional on being in location l we solve for the value function and for the optimal portfolio decisions. The maximization problem is subject to the wealth evolution equation:

$$\begin{aligned} dX_t &= \theta_t' D_{P_t}^{-1} dP_t + \left(X_t - 1'\theta_t\right) rdt + w_t dt \\ &= \theta_t'\left(\mu dt + \Sigma_P dB_t\right) + \left(X_t - 1'\theta_t\right) rdt + w_t dt \end{aligned}$$

where θ_t is nx1.

The Hamilton-Jacobi-Bellman equation associated with this problem is [32]:

$$\begin{aligned} 0 &= \dot{V} + V_X\left(\left(X_t - \theta_t' 1\right) r + w_t + \theta_t'\mu_t\right) + V_y s_t + V_s\phi\left(m - s_t\right) + \frac{1}{2} V_{XX}\theta_t'\Sigma_P\Sigma_P'\theta + \frac{1}{2} V_{ss}\Sigma_s\Sigma_s' \\ &\quad + V_{Xs}\theta_t'\Sigma_P\Sigma_x' \end{aligned}$$

Now we take FOC and solve for θ_t:

$$\theta_t = -\frac{1}{V_{XX}}\Sigma_{PP}^{-1}\left[V_X\left(\mu_t - 1r\right) + V_{Xs}\Sigma_{Ps}\right]$$

where for simplicity we we have defined $\Sigma_{YZ} = \Sigma_Y\Sigma_Z'$. Plugging back in and simplifying we get:

$$\begin{aligned} 0 &= \dot{V} + V_X\left(Xr + w_t\right) + V_y s_t + V_s\phi\left(m - s_t\right) + \frac{1}{2} V_{ss}\Sigma_{ss} \\ &\quad -\frac{1}{2}\frac{V_X^2\left(\mu_t' - 1'r\right)\Sigma_{PP}^{-1}\left(\mu_t - 1r\right)}{V_{XX}} \\ &\quad -\frac{1}{2}\frac{V_X V_{Xs}\left[\left(\mu_t' - 1'r\right)\Sigma_{PP}^{-1}\Sigma_{Ps} + \Sigma_{sP}\Sigma_{PP}^{-1}\left(\mu_t - 1r\right)\right]}{V_{XX}} \\ &\quad -\frac{1}{2}\frac{V_{Xs}^2\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{V_{XX}} \end{aligned}$$

[32] Svensson and Werner (1993) derive a similar HJB for a related problem

Substitute initial wealth out of the value function.

$$
\begin{aligned}
V\left(t, X_t, w_t, s_t\right) &= sup_{\theta_s} - E_t e^{-\gamma_i\left(e^{r(T-t)}X_t+\int_t^T e^{r(T-u)}\left(\theta_u' D_{P_u}^{-1} dP_u - 1'\theta_u r du\right)+\int_t^T e^{r(T-u)} w_u du + \frac{1}{r}\left(e^{r(T-t)}-1\right)\beta_i A^l\right)} \\
&= -e^{-\gamma_i\left[X_t e^{r(T-t)}+\frac{1}{r}\left(e^{r(T-t)}-1\right)\left(\beta_i A^l+\xi_i+\varepsilon_i^l\right)\right]} g(T-t, y_t, s_t)
\end{aligned}
$$

with terminal condition $g(0,w,s) = e^{-\gamma[p_T]}$. Finally in the final simplified form:

$$
\begin{aligned}
0 &= \dot{g} - g_s\left(\phi\left(m-s_t\right) - \frac{1}{2}\left[\left(\mu_t'-1'r\right)\Sigma_{PP}^{-1}\Sigma_{sP}+\Sigma_{Ps}\Sigma_{PP}^{-1}\left(\mu_t-1r\right)\right]\right) \\
&\quad + g\left(\gamma e^{r(T-t)} y_t + \frac{1}{2}\left(\mu_t'-1'r\right)\Sigma_{PP}^{-1}\left(\mu_t-1r\right)\right) \\
&\quad - \frac{1}{2} g_{ss}\Sigma_{ss} - g_y s_t \\
&\quad + \frac{1}{2}\frac{g_s^2 \Sigma_{Ps}\Sigma_{PP}^{-1}\Sigma_{sp}}{g}
\end{aligned}
$$

Since this is nonlinear we need to do a change of variables to get it to be linear. Making the substitution:

$$
g\left(T-t, y, s\right) = \upsilon\left(T-t, y, s\right)^{\frac{\Sigma_{ss}}{\Sigma_{ss}-\Sigma_{Ps}\Sigma_{PP}^{-1}\Sigma_{sP}}} e^{-\frac{1}{2}\left(\mu_t'-1'r\right)\Sigma_{PP}^{-1}\left(\mu_t-1r\right)(T-t)}
$$

we get:

$$
\begin{aligned}
0 &= \dot{\upsilon} + \gamma e^{r(T-t)} y_t \frac{\Sigma_{ss}-\Sigma_{Ps}\Sigma_{PP}^{-1}\Sigma_{sP}}{\Sigma_{ss}}\upsilon \\
&\quad - \upsilon_s\left(\phi\left(m-s_t\right) - \frac{1}{2}\left[\left(\mu_t'-1'r\right)\Sigma_{PP}^{-1}\Sigma_{Ps}+\Sigma_{sP}\Sigma_{PP}^{-1}\left(\mu_t-1r\right)\right]\right) \\
&\quad - \frac{1}{2}\upsilon_{ss}\Sigma_{ss} - \upsilon_y s_t
\end{aligned}
$$

Making a last substitution $G\left(t,y,s\right) = \upsilon\left(T-t,y,s\right)$ we get the linear pde:

$$
\begin{aligned}
0 &= \dot{G} - \gamma e^{r(T-t)} y_t \frac{\Sigma_{ss}-\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}} G + \left(\phi\left(m-s_t\right) - \Sigma_{Ps}\Sigma_{PP}^{-1}\left(\mu_t-1r\right)\right) G_s \\
&\quad + \frac{1}{2}\Sigma_s\Sigma_s^T G_{ss} + G_y s_t
\end{aligned}
$$

with terminal condition:

$$
G(T,y,s) = e^{\frac{1}{2}\left(\mu_t'-1'r\right)\Sigma_{PP}^{-1}\left(\mu_t-1r\right)\frac{\Sigma_{ss}-\Sigma_{Ps}\Sigma_{PP}^{-1}\Sigma_{sP}}{\Sigma_{ss}}(T-t)-\gamma\frac{\Sigma_{ss}-\Sigma_{Ps}\Sigma_{PP}^{-1}\Sigma_{sP}}{\Sigma_{ss}}p_T}
$$

Now we do a change of measure by multiplying the probability density function by:

$$\xi_T = exp\left[-\frac{1}{2}\left[\frac{\Sigma_{sP}\Sigma_{PP}^{-1}(\mu_t - 1r)}{\sqrt{\Sigma_{ss}}}\right]^2 (T-t) - \left[\frac{\Sigma_{sP}\Sigma_{PP}^{-1}(\mu_t - 1r)}{\sqrt{\Sigma_{ss}}}\right]\frac{\Sigma_s}{\sqrt{\Sigma_{ss}}}(B_T - B_t)\right]$$

Then we can use the Feynman Kac theorem to get:[33]

$$G(t,y,s) = E\left[\xi_T G(T,y,s) e^{\left[-\gamma \frac{\Sigma_{ss}-\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}} \int_t^T e^{r(T-u)} y_u du\right]}\right]$$

Now we can finally write the value function in a form that does not involve the portfolio decision variable θ:

$$V(t,y,s) = -e^{-\gamma\left[X_t e^{r(T-t)} + \frac{1}{r}\left(e^{rT-t}-1\right)\left(\beta_i A^l + \xi_i + \varepsilon_i^l\right)\right] - \frac{1}{2}(\mu_t' - 1'r)\Sigma_{PP}^{-1}(\mu_t - 1r)(T-t)} \left[G(t,y,s)\right]^{\frac{\Sigma_{ss}}{\Sigma_{ss}-\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}}$$

In order to save some notation:

$$V(t,y,s) = -e^{\bar{F}}\left[E_t e^{F_T}\right]^{\frac{\Sigma_{ss}}{\Sigma_{ss}-\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}}$$

where:

$$\begin{aligned}\overline{F} &= -\frac{1}{2}\left(\mu_t' - 1'r\right)\Sigma_{PP}^{-1}(\mu_t - 1r)(T-t) - \frac{1}{2}\frac{\left(\Sigma_{sP}\Sigma_{PP}^{-1}(\mu_t - 1r)\right)^2}{\left(\Sigma_{ss} - \Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}\right)}(T-t) \\ &\quad -\gamma\left[X_t e^{r(T-t)} + \frac{1}{r}\left(e^{rT-t} - 1\right)\left(\beta_i A^l + \xi_i + \varepsilon_i^l\right)\right]\end{aligned}$$

$$\begin{aligned}F_T &= -\left[\frac{\Sigma_{sP}\Sigma_{PP}^{-1}(\mu_t - 1r)}{\sqrt{\Sigma_{ss}}}\right]\frac{\Sigma_s}{\sqrt{\Sigma_{ss}}}(B_T - B_t) \\ &\quad -\gamma_i \frac{\Sigma_{ss} - \Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}}\left(\int_t^T e^{r(T-u)} y_u du + Ay_T + Bs_T + C\right)\end{aligned}$$

Notice that $\overline{F}$ is just a constant while the term F_T depends on the states. Using the assumed processes for y_t and s_t we can write F_T in terms of states at time t. First we make a transformation that transforms the m independent Brownian motions $\mathbb{B}_t$ to a single brownian motion Z_t for each

[33] See Duffie (1996), Theorem and Condition 2 on p. 296

location. To do this we redefine:

$$dZ_t = \frac{\Sigma_s}{\sqrt{\Sigma_{ss}}} d\mathbb{B}_t$$

which means that now we can write the evolution of the drift s_t of the income process as:

$$\begin{aligned} ds_t &= \phi\left(m - s_t\right) dt + \Sigma_s dB_t \\ &= \phi\left(m - s_t\right) dt + \kappa dZ_t \end{aligned}$$

with $\kappa = \sqrt{\Sigma_{ss}}$. This means that s_t is just a simple one dimensional Ornstein-Uhlenbeck process.

We can use the solutions to the processes y_t and s_t to write the components of F_T as:

$$s_T = s_t e^{-\phi(T-t)} + m\left(1 - e^{-\phi(T-t)}\right) + \int_t^T \kappa e^{\phi(u-T)} dZ_u$$

$$\begin{aligned} y_T &= y_t + \int_t^T s_u du \\ &= y_t + s_t \frac{1}{\phi}\left(1 - e^{-\phi(T-t)}\right) + m\left((T-t) - \frac{1}{\phi}\left(1 - e^{-\phi(T-t)}\right)\right) + \int_t^T \frac{\kappa}{\phi}\left(1 - e^{-\phi(T-s)}\right) dZ_s \end{aligned}$$

$$\begin{aligned} \int_t^T e^{r(T-u)} y_u du &= -y_t \frac{1}{r}\left(1 - e^{r(T-t)}\right) + s_t \frac{1}{\phi}\left(-\frac{1}{r}\left(1 - e^{r(T-t)}\right) + \frac{1}{r+\phi}\left(e^{-\phi(T-t)} - e^{r(T-t)}\right)\right) \\ &\quad -m\frac{1}{r^2}\left(rT + 1 - e^{r(T-t)}\left(rt+1\right)\right) + mt\frac{1}{r}\left(1 - e^{r(T-t)}\right) \\ &\quad -\frac{m}{\phi}\left(-\frac{1}{r}\left(1 - e^{r(T-t)}\right) + \frac{1}{r+\phi}\left(e^{-\phi(T-t)} - e^{r(T-t)}\right)\right) \\ &\quad \int_t^T \frac{\kappa}{\phi}\left[-\frac{1}{r}\left(1 - e^{r(T-s)}\right) + \frac{1}{r+\phi}\left(e^{-\phi(T-s)} - e^{r(T-s)}\right)\right] dZ_t \end{aligned}$$

Rewriting F_T in terms of states at time t we get:

$$F_T = -\gamma_i\left(\tilde{k}_{1t} y_t + \tilde{k}_{2t} s_t + \tilde{k}_{3t}\right) + \int_t^T \left(\tilde{k}_4 - \gamma_i \sqrt{\Sigma_{ss}} \tilde{k}_{2t}\right) dZ_u$$

where $dZ_t = \Sigma_s / \sqrt{\Sigma_{ss}} dB_t$ and:

$$\tilde{k}_{1t} = \frac{\Sigma_{ss} - \Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}}\left(A - \frac{1 - e^{r(T-t)}}{r}\right)$$

$$\tilde{k}_{2t} = \frac{\Sigma_{ss} - \Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}} \left\{ A\frac{1-e^{-\phi(T-t)}}{\phi} + Be^{-\phi(T-t)} - \frac{1-e^{r(T-t)}}{\phi r} + \frac{e^{-\phi(T-t)} - e^{r(T-t)}}{\phi(r+\phi)} \right\}$$

$$\tilde{k}_{3t} = -m\tilde{k}_2 + \frac{\Sigma_{ss} - \Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}{\Sigma_{ss}} \left\{ Am(T-t) + Bm + \frac{mt\left(1-e^{r(T-t)}\right)}{r} - m\frac{1}{r^2}\left(rT + 1 - e^{r(T-t)}(rt+1)\right) + C \right\}$$

$$\tilde{k}_{4t} = -\frac{\Sigma_{sP}\Sigma_{PP}^{-1}(\mu - 1r)}{\sqrt{\Sigma_{ss}}}$$

The only random part of F_T now $\int_t^T \left(\tilde{k}_4 - \gamma_i\sqrt{\Sigma_{ss}}\tilde{k}_{2t}\right) dZ_t$, which is an Ito integral over a deterministic function and thus a martingale. Therefore, the distribution of F_T is a normal with mean and variance:[34]

$$E_t(F_T) = -\gamma_i\left(\tilde{k}_{1t}y_t + \tilde{k}_{2t}s_t + \tilde{k}_{3t}\right)$$

$$Var(F_T) = \int_t^T \left(\tilde{k}_4 - \gamma_i\sqrt{\Sigma_{ss}}\tilde{k}_{2t}\right)^2 dt$$

Using the fact that F_T is normally distributed we can write the value function as:

$$V(t,y,s) = -e^{\overline{F}}\left[e^{E(F_T)+\frac{1}{2}Var(F_T)}\right]^{\frac{\Sigma_{ss}}{\Sigma_{ss}-\Sigma_{sP}\Sigma_{PP}^{-1}\Sigma_{Ps}}}$$

Simplifying the above equation and including the location superscript l, the value function is:

$$V(t,y,s,l) = -e^{U_t^l}$$

with*:*

$$U_t^l = -\gamma_i\left[e^{r(T-t)}X_t + \frac{1}{r}\left(e^{r(T-t)} - 1\right)\left(\beta_i M^l + \xi_i + \varepsilon_i^l\right) + \hat{k}_1^l y_t^l + \hat{k}_2^l s_t^l + \hat{k}_3^l + \int_t^T \hat{k}_4^l\hat{k}_2^l\sqrt{\Sigma_{ss}^l}du\right] + \frac{1}{2}\gamma_i^2\left(\Sigma_{ss}^l - \Sigma_{sP}^l\Sigma_{PP}^{-1}\Sigma_{Ps}^l\right)\int_t^T\left(\hat{k}_{t2}^l\right)^2 du - \frac{1}{2}\left(\mu_t' - 1'r\right)\Sigma_{PP}^{-1}\left(\mu_t - 1r\right)(T-t)$$

where we define $\hat{k}_{it}^l = \frac{\Sigma_{ss}^l - \Sigma_{sP}^l\Sigma_{PP}^{-1}\Sigma_{Ps}^l}{\Sigma_{ss}^l}\tilde{k}_{it}^l$ for $i = 1...3$ and $\hat{k}_4^l = \tilde{k}_4^l$. ■

[34] See Shreve (2004) chapter 4.

A.2 Proof of Proposition 2

For clarity we suppress the city-specific l superscript. In the proof of Proposition 1, I show that the optimal amount invested in stocks for an individual who is $t - t_0$ years old and lives in location l is:

$$\theta_{ilt} = -\frac{1}{V_{XX}} (\Sigma_{PP})^{-1} \left[V_X (\mu - 1r) + V_{Xs} \Sigma^l_{Ps} \right]$$

Taking the derivatives of the value function equation (4):

$$V(t, y, s, l) = -e^{U^l_t}$$

$$V_X = -\gamma e^{r(T-t)} e^{U^l_t}$$

$$V_{XX} = \gamma^2 e^{2r(T-t)} e^{U^l_t}$$

$$V_{Xs} = \gamma^2 \hat{k}^l_2 e^{r(T-t)} e^{U^l_t}$$

Substituting these derivatives in the equation for θ_{ilt} we get:

$$\theta_{ilt} = (\Sigma_{PP})^{-1} \left[\frac{1}{\gamma e^{r(T-t)}} (\mu_t - 1r) + \frac{\hat{k}^l_{2t}}{e^{r(T-t)}} \Sigma^l_{Ps} \right]$$

where:

$$\begin{aligned} \hat{k}_{2t} = & \left\{ A \frac{1 - e^{-\phi(T-t)}}{\phi} + B e^{-\phi(T-t)} \right. \\ & \left. - \frac{1 - e^{r(T-t)}}{\phi r} + \frac{e^{-\phi(T-t)} - e^{r(T-t)}}{\phi (r + \phi)} \right\} \end{aligned}$$

■

A.3 Proof of Proposition 6

For a given A and B, in each period we look for a C^l that clears the housing markets in some given period t. For the generation of agents born in some period t, the problem in equation 7 is identical to static horizontal sorting models studied in the urban economics literature. Bayer, McMillan and Rueben (2005) prove that under the assumption that ε^l_i has continuous support there exist a unique vector $\mathbf{C} = \{C^1, C^2...C^L\}$ that clears the market and gives the unique sorting of households across space. Their proof applies exactly to this problem so is not reproduced here. The reader is referred to the original article for details.

Given that we now have unique values for C^l, we turn to determining what A and B need to be in order for the markets to be in equilibrium across time. First note that a measure 1 of households is born and looking to buy a home each period. Because the joint distribution of heterogeneity in $Pr\left(\gamma_i, \beta_i, \xi_i, \varepsilon_i^l\right)$ is iid over time, the same set of households will be in the market in every time period. In other words in every period the full support of the joint distribution $Pr\left(\gamma_i, \beta_i, \xi_i, \varepsilon_i^l\right)$ will be realized.

Consider the sorting problem in equation 7 at two different time periods t and u where $y_t \neq y_u$ and $s_t = s_u$. If $k_1 \neq 0$ then the equilibrium $\mathbf{C}$ of the sorting equilibrium in period t will have to be different than that in period u. Because we are looking for a vector $\mathbf{C}$ that is constant across time we conclude that it must be the case that $k_1 = 0$ for every location l. A similar argument leads to the conclusion that in order to have $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ to be constant across time it must be that $k_2^l = 0$ for every location l. Setting $k_1^l = k_2^l = 0$ we find the unique solutions be:

$$A^l = \frac{1}{r}$$

$$B^l = \frac{1}{r\left(r + \phi^l\right)}$$

Given the above solutions $\mathbf{A}$, and $\mathbf{B}$ we can now get the unique equilibrium market returns for stocks in equation 6. We have therefore found the three vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ that give rise to equilibrium both in the asset market and the housing market. ■

A.4 Proof of Proposition 7

The location l superscript will be omitted for the rest of the proof for notational simplicity, and this analysis is the same for any location l. The idea here is to use the solution to the process for y_t to solve for the expectation in closed-form. Recall that the processes that drive income are:

$$\begin{aligned} dy_t &= s_t dt \\ ds_t &= \phi\left(m - s_t\right) dt + \kappa dZ_t \end{aligned}$$

where $dZ_t = \Sigma_s/\sqrt{\Sigma_{ss}}dB_t$ and $\kappa = \sqrt{\Sigma_{ss}}$. This change of variables transforms the s_t process to a simple one dimensional Ornstein-Uhlenbeck process. The solution for this process for some $T > t$ is:

$$s_T = s_t e^{-\phi(T-t)} + m\left(1 - e^{-\phi(T-t)}\right) + \int_t^T \kappa e^{\phi(v-T)} dZ_v$$

For any $T > t$ we can write y_T as:

$$\begin{aligned}
y_T &= y_t + \int_t^T s_u du \\
&= y_t + \int_t^T \left(s_t e^{-\phi(u-t)} + m\left(1 - e^{-\phi(u-t)}\right) + \int_t^u \kappa e^{\phi(v-u)} dZ_v\right) du \\
&= y_t + \int_t^T \left(s_t e^{-\phi(u-t)} + m\left(1 - e^{-\phi(u-t)}\right)\right) du + \int_t^T \left(\int_v^T \kappa e^{\phi(v-u)} du\right) dZ_v \\
&= y_t + s_t \frac{1}{\phi}\left(1 - e^{-\phi(T-t)}\right) + m\left((T-t) - \frac{1}{\phi}\left(1 - e^{-\phi(T-t)}\right)\right) + \int_t^T \frac{\kappa}{\phi}\left(1 - e^{-\phi(T-v)}\right) dZ_v
\end{aligned}$$

where in the second equality we substitute in for the solution to s_u, in the third equality we changed the order of integration for the last term, and in the last equality we solve the deterministic integrals.

We can now substitute the solution to y directly in the price equation:

$$\begin{aligned}
p_t &= E_t \int_t^\infty e^{-r(u-t)} y_u du + \pi \\
&= \int_t^\infty e^{-r(u-t)} E_t(y_u) du + \pi \\
&= \int_t^\infty e^{-r(u-t)} \left(y_t + s_t \frac{1}{\lambda}\left(1 - e^{-\lambda(u-t)}\right) + m\left(u - t - \frac{1}{\lambda}\left(1 - e^{-\lambda(u-t)}\right)\right)\right) du + \pi \\
&= \frac{1}{r} y_t + \frac{1}{r(r+\phi)} s_t + \frac{\phi}{r^2(r+\phi)} m + \pi
\end{aligned}$$

In the second equality we interchange the integral with the expectation.[35] In the third equality we substitute for the expected value of y_u. Here we use the fact that $E\left(\int_t^u \frac{\kappa}{\phi}\left(1 - e^{-\phi(u-v)}\right) dZ_v\right) = 0$ since Ito integrals of deterministic functions are martingales. ■

[35] In order to interchange the integral and the expectation, first notice that the expectation is an integral over some probability measure. We can then use Fubini's theorem that says that the order of integration can be changed as long as $\int_t^\infty e^{-r(u-t)} E_t\left(\left|\int_t^u \frac{\kappa}{\phi}\left(1 - e^{-\phi(u-v)}\right) dZ_v\right|\right) du < \infty$. This condition is satisfied for our problem. The intuition is that the expectation of the absolute value of the martingale grows slower that the discounting term $e^{-r(u-t)}$. Broadly speaking the expectation grows linearly over time while the discounting term grows exponentially.

B Data Appendix

Table 7: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
Akron,OH	3.059	-0.174	0.341	0.107
Albany-Schenectady-Troy,NY	4.554	-18.886	2.885	0.588
Albuquerque,NM	11.580	-1.059	16.847	0.470
Allentown-Bethlehem-Easton,PA-NJ	3.490	-24.722	0.453	0.583
Amarillo,TX	4.884	2.958	0.661	0.208
Anchorage,AK	18.350	3.117	10.947	0.209
AnnArbor,MI	6.879	-8.313	-2.644	0.371
Atlanta-SandySprings-Marietta,GA	7.654	-4.361	-2.651	0.596
Augusta-RichmondCounty,GA-SC	3.913	-0.744	3.810	0.360
Austin-RoundRock,TX	7.783	8.152	3.007	0.131
Bakersfield,CA	11.340	-21.111	22.663	0.872
Baltimore-Towson,MD	7.352	-22.227	16.914	0.846
BarnstableTown,MA	25.493	-41.569	-9.980	0.767
BatonRouge,LA	8.048	6.369	8.294	0.538
Beaumont-PortArthur,TX	6.983	6.474	4.988	0.596
Bellingham,WA	-1.084	-9.358	27.285	0.780
Bethesda-Frederick-Rockville,MD	11.745	-45.196	22.672	0.852
Binghamton,NY	4.753	-7.221	-1.851	0.225
Birmingham-Hoover,AL	4.780	0.441	2.174	0.370
Bloomington-Normal,IL	4.266	0.564	2.004	0.236
BoiseCity-Nampa,ID	3.750	-0.033	14.016	0.506
Boston-Quincy,MA	23.981	-36.247	-13.300	0.695
Boulder,CO	14.178	6.945	-2.888	0.245
Bremerton-Silverdale,WA	4.590	-8.663	22.584	0.672
Bridgeport-Stamford-Norwalk,CT	34.996	-49.136	-8.938	0.600
Buffalo-NiagaraFalls,NY	0.267	-3.779	0.404	0.105
Burlington-SouthBurlington,VT	5.099	-21.748	3.842	0.804
Cambridge-Newton-Framingham,MA	25.008	-37.554	-18.296	0.658
Camden,NJ	4.685	-19.207	6.015	0.718
Canton-Massillon,OH	3.639	2.355	-0.821	0.286
CapeCoral-FortMyers,FL	7.485	-15.013	17.192	0.752
Casper,WY	4.515	9.585	13.116	0.285
CedarRapids,IA	3.024	1.403	1.477	0.213
Charleston-NorthCharleston-Summerville,SC	10.308	-7.175	8.320	0.489
Charlotte-Gastonia-Concord,NC-SC	0.862	0.501	-1.556	0.027
Charlottesville,VA	6.853	-15.302	14.728	0.741
Chattanooga,TN-GA	4.609	-1.685	1.836	0.435
Cheyenne,WY	6.160	0.039	4.876	0.271
Chicago-Naperville-Joliet,IL	5.863	-12.193	4.882	0.783
Chico,CA	12.545	-22.399	25.952	0.785

Table Continued: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
Cincinnati-Middletown,OH-KY-IN	4.171	-2.410	-0.813	0.465
Cleveland-Elyria-Mentor,OH	5.112	0.422	-0.207	0.282
CollegeStation-Bryan,TX	8.952	5.730	4.084	0.392
ColoradoSprings,CO	11.970	1.701	6.523	0.634
Columbia,SC	2.965	-0.983	0.568	0.241
Columbus,OH	3.280	-1.874	-0.808	0.319
CorpusChristi,TX	10.986	2.339	7.058	0.576
Corvallis,OR	1.374	5.700	21.990	0.670
Dallas-Plano-Irving,TX	7.948	2.233	0.370	0.233
Dalton,GA	2.745	-1.786	1.674	0.130
Davenport-Moline-RockIsland,IA-IL	2.765	3.424	3.542	0.292
Dayton,OH	2.790	-0.443	-0.885	0.133
Deltona-DaytonaBeach-OrmondBeach,FL	9.910	-16.977	17.566	0.857
Denver-Aurora-Broomfield,CO	15.812	4.771	-2.136	0.437
DesMoines-WestDesMoines,IA	2.158	0.827	1.303	0.147
Detroit-Livonia-Dearborn,MI	2.960	-2.075	-0.316	0.205
Durham-ChapelHill,NC	3.903	-2.138	-1.892	0.177
EauClaire,WI	2.270	-1.993	-0.148	0.092
Edison-NewBrunswick,NJ	22.317	-43.143	0.507	0.773
ElPaso,TX	2.602	0.270	5.954	0.230
Elkhart-Goshen,IN	1.795	0.807	1.221	0.047
Erie,PA	3.309	-0.228	0.566	0.211
Eugene-Springfield,OR	0.533	-2.695	16.770	0.759
Evansville,IN-KY	4.108	1.509	0.606	0.401
Fayetteville-Springdale-Rogers,AR-MO	3.640	-4.286	6.666	0.488
Flint,MI	6.336	-3.552	-0.652	0.238
FortCollins-Loveland,CO	10.549	2.618	0.364	0.316
FortLauderdale-PompanoBeach-DeerfieldBeach,FL(MSAD)	17.287	-28.214	27.252	0.837
FortWayne,IN	2.076	-1.141	-0.763	0.124
Fresno,CA	12.855	-23.551	25.696	0.805
Gary,IN	1.152	1.936	4.186	0.416
GrandJunction,CO	7.009	6.067	13.060	0.384
GrandRapids-Wyoming,MI	1.467	-1.924	-1.448	0.289
Greeley,CO	12.939	1.786	-0.790	0.282
Greensboro-HighPoint,NC	1.948	-1.273	-3.363	0.311
Harrisburg-Carlisle,PA	3.728	-3.242	5.761	0.506
Hartford-WestHartford-EastHartford,CT	9.397	-32.412	-13.102	0.667
Holland-GrandHaven,MI	1.718	-2.755	-3.192	0.172
Honolulu,HI	-18.653	-51.363	60.250	0.738
Houston-SugarLand-Baytown,TX	8.153	5.778	4.499	0.391
Huntsville,AL	4.965	-0.057	0.752	0.187
Indianapolis-Carmel,IN	1.691	-0.066	-0.672	0.102
Jackson,MS	4.559	-0.186	2.523	0.244

Table Continued: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
Jacksonville,FL	9.029	-11.038	11.407	0.698
Janesville,WI	1.128	1.005	4.419	0.146
Kalamazoo-Portage,MI	4.265	0.214	1.585	0.204
KansasCity,MO-KS	5.302	-3.242	-1.073	0.548
Kennewick-Pasco-Richland,WA	0.068	2.742	7.110	0.199
Knoxville,TN	4.363	-0.468	4.608	0.515
LaCrosse,WI-MN	4.909	0.216	3.743	0.375
Lafayette,LA	5.760	9.518	9.392	0.690
LakeCounty-KenoshaCounty,IL-WI	2.766	-9.366	0.953	0.458
Lakeland-WinterHaven,FL	7.457	-9.498	16.365	0.564
Lancaster,PA	0.167	-7.425	6.818	0.549
Lansing-EastLansing,MI	4.135	-3.783	-0.958	0.338
LasCruces,NM	5.886	-2.965	8.585	0.641
LasVegas-Paradise,NV	11.720	-20.360	22.298	0.865
Lexington-Fayette,KY	4.485	-2.457	0.756	0.343
Lima,OH	2.825	-1.018	0.448	0.140
Lincoln,NE	2.833	0.498	1.926	0.260
LittleRock-NorthLittleRock-Conway,AR	4.994	0.834	1.884	0.388
Longview,TX	5.003	3.210	4.121	0.539
Longview,WA	2.226	4.943	16.713	0.660
LosAngeles-LongBeach-Glendale,CA	14.561	-54.850	29.291	0.848
Louisville-JeffersonCounty,KY-IN	1.889	0.305	0.683	0.187
Lubbock,TX	5.479	2.047	2.525	0.396
Macon,GA	3.188	-1.321	0.838	0.227
Madera-Chowchilla,CA	11.664	-29.035	35.194	0.813
Madison,WI	3.246	-2.106	5.315	0.210
Manchester-Nashua,NH	15.200	-28.583	-5.439	0.665
Mansfield,OH	3.021	1.412	1.429	0.171
Medford,OR	11.435	-19.949	26.830	0.849
Memphis,TN-MS-AR	0.303	-3.176	-2.724	0.196
Merced,CA	10.050	-27.243	21.825	0.737
Miami-MiamiBeach-Kendall,FL	13.639	-18.152	26.455	0.765
Midland,TX	9.405	13.731	8.258	0.433
Milwaukee-Waukesha-WestAllis,WI	5.457	-5.180	6.919	0.558
Minneapolis-St.Paul-Bloomington,MN-WI	9.183	-8.632	2.726	0.575
Mobile,AL	6.034	3.625	4.347	0.185
Modesto,CA	13.173	-34.490	24.795	0.804
Monroe,LA	5.386	3.338	2.515	0.328
Monroe,MI	3.564	-3.929	0.131	0.164
Napa,CA	20.727	-52.064	29.391	0.738
Naples-MarcoIsland,FL	16.584	-33.255	32.508	0.760
Nashville-Davidson-Murfreesboro-Franklin,TN	4.070	-1.678	1.498	0.164
Nassau-Suffolk,NY	25.344	-47.593	-3.155	0.830

Table Continued: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
NewHaven-Milford,CT	17.303	-31.329	-5.593	0.657
NewOrleans-Metairie-Kenner,LA	9.893	6.286	1.545	0.507
NewYork-WhitePlains-Wayne,NY-NJ	22.818	-45.541	-2.468	0.815
Newark-Union,NJ-PA	24.421	-44.044	-2.095	0.750
Niles-BentonHarbor,MI	1.460	-3.758	0.369	0.310
Oakland-Fremont-Hayward,CA	17.355	-53.726	23.873	0.796
Odessa,TX	2.770	6.388	4.490	0.372
Ogden-Clearfield,UT	6.929	11.549	8.165	0.512
OklahomaCity,OK	8.301	7.904	8.067	0.721
Olympia,WA	2.077	-3.902	22.664	0.741
Omaha-CouncilBluffs,NE-IA	3.561	1.334	0.542	0.394
Orlando-Kissimmee,FL	10.175	-17.405	21.053	0.786
Oxnard-ThousandOaks-Ventura,CA	24.815	-76.045	22.819	0.819
PalmBay-Melbourne-Titusville,FL	8.127	-13.584	14.058	0.864
Peabody,MA	24.809	-33.914	-13.787	0.665
Pensacola-FerryPass-Brent,FL	9.372	-11.313	13.351	0.795
Peoria,IL	1.594	2.570	0.530	0.149
Philadelphia,PA	3.385	-16.903	2.694	0.690
Phoenix-Mesa-Scottsdale,AZ	7.923	-14.578	16.295	0.680
Pittsburgh,PA	3.354	0.944	0.569	0.197
PortSt.Lucie,FL	12.800	-20.766	19.446	0.844
Portland-SouthPortland-Biddeford,ME	9.980	-21.924	-3.937	0.810
Portland-Vancouver-Beaverton,OR-WA	6.510	-1.059	22.181	0.700
Poughkeepsie-Newburgh-Middletown,NY	20.442	-39.836	-4.061	0.747
Providence-NewBedford-FallRiver,RI-MA	13.802	-31.162	-1.481	0.806
Provo-Orem,UT	6.015	11.759	12.309	0.439
Pueblo,CO	8.399	0.135	1.133	0.377
Racine,WI	4.853	-6.015	6.549	0.606
Raleigh-Cary,NC	3.576	0.891	0.982	0.070
Reading,PA	0.805	-10.277	4.342	0.514
Redding,CA	8.762	-21.875	23.831	0.734
Reno-Sparks,NV	12.459	-23.911	23.915	0.774
Richmond,VA	4.555	-9.201	9.631	0.644
Riverside-SanBernardino-Ontario,CA	12.173	-31.933	25.135	0.865
Roanoke,VA	2.537	-2.519	4.227	0.403
Rochester,MN	4.972	-3.118	-1.308	0.238
Rochester,NY	3.017	-4.286	-3.217	0.282
Rockford,IL	1.174	-0.526	4.274	0.394
RockinghamCounty-StraffordCounty,NH	17.781	-27.745	-8.645	0.712
Sacramento-Arden-Arcade-Roseville,CA	11.827	-38.269	23.842	0.688
Saginaw-SaginawTownshipNorth,MI	3.127	-1.026	1.013	0.264
Salem,OR	4.198	3.304	14.215	0.697
Salinas,CA	17.464	-47.843	23.966	0.783

Table Continued: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
SaltLakeCity,UT	5.335	10.671	13.153	0.530
SanAntonio,TX	10.301	5.155	8.380	0.456
SanDiego-Carlsbad-SanMarcos,CA	21.814	-55.869	22.163	0.727
SanFrancisco-SanMateo-RedwoodCity,CA	19.466	-90.385	12.650	0.744
SanJose-Sunnyvale-SantaClara,CA	12.929	-63.083	18.547	0.536
SanLuisObispo-PasoRobles,CA	14.438	-56.354	28.803	0.709
SantaAna-Anaheim-Irvine,CA	28.367	-82.096	38.745	0.837
SantaBarbara-SantaMaria-Goleta,CA	20.126	-50.303	17.341	0.748
SantaCruz-Watsonville,CA	13.819	-65.713	18.778	0.687
SantaFe,NM	10.337	1.302	17.763	0.606
SantaRosa-Petaluma,CA	14.441	-51.070	20.683	0.693
Savannah,GA	7.408	-3.703	7.944	0.648
Scranton-Wilkes-Barre,PA	3.243	-1.967	2.545	0.106
Seattle-Bellevue-Everett,WA	0.932	-14.413	26.672	0.566
Sebastian-VeroBeach,FL	15.148	-17.709	18.043	0.737
Shreveport-BossierCity,LA	7.884	2.726	5.785	0.382
SouthBend-Mishawaka,IN-MI	-0.108	-0.415	0.439	0.026
Spokane,WA	3.372	0.469	15.803	0.639
Springfield,IL	0.849	-0.990	3.784	0.168
Springfield,MA	7.316	-25.214	-4.894	0.772
Springfield,MO	3.873	0.007	3.867	0.394
Springfield,OH	2.514	0.330	-1.375	0.157
St.Louis,MO-IL	4.136	-5.360	0.628	0.666
Stockton,CA	16.603	-41.185	25.143	0.792
Syracuse,NY	2.235	-8.397	-1.345	0.392
Tacoma,WA	5.135	-6.659	19.555	0.628
Tallahassee,FL	4.799	-5.175	11.619	0.714
Tampa-St.Petersburg-Clearwater,FL	11.015	-16.470	16.055	0.853
Toledo,OH	3.403	-1.024	-0.393	0.239
Topeka,KS	4.245	-0.497	3.057	0.477
Trenton-Ewing,NJ	15.577	-33.589	-2.730	0.714
Tucson,AZ	8.111	-12.010	15.598	0.709
Tulsa,OK	8.037	7.470	1.141	0.700
Tyler,TX	5.397	2.634	0.398	0.132
Vallejo-Fairfield,CA	17.277	-37.947	23.233	0.780
VirginiaBeach-Norfolk-NewportNews,VA-NC	5.923	-16.568	14.717	0.788
Visalia-Porterville,CA	7.391	-22.658	24.832	0.825
Warren-Troy-FarmingtonHills,MI	5.288	-4.561	-1.140	0.272
Washington-Arlington-Alexandria,DC-VA-MD-WV	16.022	-47.490	23.720	0.904
Waterloo-CedarFalls,IA	2.769	3.148	5.393	0.181
Wenatchee-EastWenatchee,WA	-1.048	6.139	15.622	0.327
WestPalmBeach-BocaRaton-BoyntonBeach,FL	19.420	-32.849	26.344	0.865
Wilmington,DE-MD-NJ	4.371	-20.335	9.114	0.810

Table Continued: The Factor Loadings and the R-squared

Metropolitan Area	HMKT	SMBH	HMLH	R-squared
Wilmington,NC	6.809	-5.212	14.656	0.567
Winston-Salem,NC	4.749	-0.863	0.574	0.311
Worcester,MA	17.265	-28.101	-7.750	0.720
York-Hanover,PA	2.545	-9.258	8.270	0.525

Table 8: The Estimated Risk Premia and Implied Amenities for 2008

Metropolitan Area	Price	Wage	Amenities	Risk Premia
Akron,OH	133978	37893	17114	1488
Albany-Schenectady-Troy,NY	191780	42523	49034	-23928
Albuquerque,NM	292153	35415	144544	-19299
Allentown-Bethlehem-Easton,PA-NJ	229955	38208	100250	-30157
Amarillo,TX	128082	34729	18884	5841
AnnArbor,MI	165966	39107	43073	-1008
Atlanta-SandySprings-Marietta,GA	191194	38336	47676	6041
Augusta-RichmondCounty,GA-SC	120856	33056	12285	-2980
Austin-RoundRock,TX	198370	37362	49877	1987
Bakersfield,CA	143663	30047	75981	-46690
Baltimore-Towson,MD	267441	47881	78048	-43632
BarnstableTown,MA	317123	51194	88890	-15598
BatonRouge,LA	168206	36346	31159	1614
Beaumont-PortArthur,TX	140564	35507	22388	6259
Bellingham,WA	284456	35592	127464	-53727
Binghamton,NY	120595	34367	19784	-3927
Birmingham-Hoover,AL	153568	39886	5495	1669
Bloomington-Normal,IL	152073	38865	29517	1277
BoiseCity-Nampa,ID	163398	35615	24438	-18861
Boston-Quincy,MA	336747	55220	92699	-7879
Boulder,CO	351691	50058	96189	16956
Bridgeport-Stamford-Norwalk,CT	436013	79108	122523	-36123
Buffalo-NiagaraFalls,NY	115580	37647	10385	-6018
Burlington-SouthBurlington,VT	251974	41139	90307	-26295
Cambridge-Newton-Framingham,MA	387527	60093	127056	-4981
Camden,NJ	195479	42626	44754	-27447
Canton-Massillon,OH	122679	32763	16914	6955
CapeCoral-FortMyers,FL	118378	40898	1526	-36267
Casper,WY	194152	52185	-15784	-12044
CedarRapids,IA	142387	38811	18218	1944
Charleston-NorthCharleston-Summerville,SC	212297	35447	48345	-12361
Charlotte-Gastonia-Concord,NC-SC	197281	39621	41230	2221
Charlottesville,VA	282797	43344	88028	-33482

Table continued: The Estimated Risk Premia and Implied Amenities for 2008

Metropolitan Area	Price	Wage	Amenities	Risk Premia
Chattanooga,TN-GA	126886	34784	1780	-265
Cheyenne,WY	176181	44613	17075	-2685
Chicago-Naperville-Joliet,IL	248207	45510	70747	-15429
Chico,CA	229208	32349	119236	-54105
Cincinnati-Middletown,OH-KY-IN	166943	39066	37096	2389
Cleveland-Elyria-Mentor,OH	144077	40118	21777	5183
CollegeStation-Bryan,TX	139690	28176	44535	7269
ColoradoSprings,CO	208343	38221	60144	3406
Columbia,SC	139798	35328	15821	721
Columbus,OH	165190	38741	37593	2023
Corvallis,OR	277148	37755	99086	-26832
Dallas-Plano-Irving,TX	149710	43458	19554	7398
Dalton,GA	118068	28675	18833	-2710
Davenport-Moline-RockIsland,IA-IL	114984	38571	-10157	634
Dayton,OH	122437	35526	18824	2942
Deltona-DaytonaBeach-OrmondBeach,FL	156630	32098	52917	-35650
Denver-Aurora-Broomfield,CO	258493	48010	51834	20281
DesMoines-WestDesMoines,IA	152250	42506	9953	764
Detroit-Livonia-Dearborn,MI	58617	32094	-32071	533
Durham-ChapelHill,NC	188378	40927	35107	3012
EauClaire,WI	153334	33193	25137	-784
Edison-NewBrunswick,NJ	330532	51865	112694	-34683
ElPaso,TX	135003	28071	39736	-7314
Elkhart-Goshen,IN	135147	32263	35436	-204
Erie,PA	95974	32294	-3583	1804
Eugene-Springfield,OR	207351	33522	63215	-27738
Evansville,IN-KY	110415	36329	-4238	4658
Fayetteville-Springdale-Rogers,AR-MO	149133	32537	39790	-11657
Flint,MI	113815	29488	33532	1381
FortCollins-Loveland,CO	223706	38848	58087	10117
FortLauderdale-PompanoBeach-DeerfieldBeach,FL(MSAD)	239411	41974	94247	-57586
FortWayne,IN	93630	34176	-4478	1411
Fresno,CA	184897	30997	99964	-54344
Gary,IN	117579	35922	330	-3063
GrandJunction,CO	231281	36665	44430	-9774
GrandRapids-Wyoming,MI	88277	33582	-16069	1189
Greeley,CO	195624	28402	88787	11137
Greensboro-HighPoint,NC	143837	35405	28144	4960
Harrisburg-Carlisle,PA	167891	39106	31027	-8813
Hartford-WestHartford-EastHartford,CT	240972	50755	72715	-15708
Holland-GrandHaven,MI	160533	33009	50988	2065
Houston-SugarLand-Baytown,TX	159217	45835	1872	6079
Huntsville,AL	157478	38259	31236	2638

Table continued: The Estimated Risk Premia and Implied Amenities for 2008

Metropolitan Area	Price	Wage	Amenities	Risk Premia
Indianapolis-Carmel,IN	122092	39297	-1721	2270
Jackson,MS	140458	36054	21424	-203
Jacksonville,FL	196673	40028	36462	-21340
Janesville,WI	144590	31826	36699	-5532
Kalamazoo-Portage,MI	147363	33685	33436	1337
KansasCity,MO-KS	144052	40396	8017	2919
Kennewick-Pasco-Richland,WA	165756	33040	66283	-9875
Knoxville,TN	149041	34696	21846	-3172
LaCrosse,WI-MN	149199	35263	20591	-861
Lafayette,LA	135445	40182	-2779	1883
LakeCounty-KenoshaCounty,IL-WI	218836	51782	33225	-10426
Lakeland-WinterHaven,FL	156411	32572	44635	-30456
Lansing-EastLansing,MI	115284	33844	10074	594
LasCruces,NM	150517	27855	52528	-10469
LasVegas-Paradise,NV	178390	39920	60215	-44996
Lexington-Fayette,KY	144070	36413	22959	113
Lima,OH	107168	30351	16977	417
LittleRock-NorthLittleRock-Conway,AR	134756	39012	5170	2762
Longview,TX	95354	36046	-20838	2229
Longview,WA	203250	29703	77843	-18158
LosAngeles-LongBeach-Glendale,CA	340842	42265	200361	-97654
Louisville-JeffersonCounty,KY-IN	133968	37995	784	1069
Lubbock,TX	103485	32447	6290	3503
Macon,GA	123826	34147	13047	46
Madera-Chowchilla,CA	233514	26524	160921	-77485
Madison,WI	216103	44172	44371	-8648
Manchester-Nashua,NH	225464	45432	63246	-15225
Mansfield,OH	110216	29719	24949	1903
Medford,OR	257085	34506	111794	-52034
Memphis,TN-MS-AR	118286	38577	-3265	-189
Merced,CA	120258	27871	79226	-57403
Miami-MiamiBeach-Kendall,FL	251186	35887	89587	-49034
Midland,TX	139240	53968	-49126	7712
Milwaukee-Waukesha-WestAllis,WI	230988	42824	67918	-11209
Minneapolis-St.Paul-Bloomington,MN-WI	186015	47653	7047	-5723
Mobile,AL	130544	30567	13332	371
Modesto,CA	159318	31485	105736	-66758
Monroe,LA	118373	32204	5546	4566
Monroe,MI	117573	33397	12661	-2682
Napa,CA	393989	52169	201297	-96628
Naples-MarcoIsland,FL	232223	62559	32878	-76764
Nashville-Davidson-Murfreesboro-Franklin,TN	167671	39768	16522	-1207
Nassau-Suffolk,NY	411170	57617	132911	-30188

Table continued: The Estimated Risk Premia and Implied Amenities for 2008

Metropolitan Area	Price	Wage	Amenities	Risk Premia
NewHaven-Milford,CT	229016	46918	64867	-17634
NewOrleans-Metairie-Kenner,LA	163257	41740	-5321	13291
NewYork-WhitePlains-Wayne,NY-NJ	402722	54540	140388	-31317
Newark-Union,NJ-PA	390079	56655	140479	-31607
Niles-BentonHarbor,MI	144779	33669	19300	-3714
Oakland-Fremont-Hayward,CA	351486	53093	177091	-88480
Odessa,TX	84268	34622	-24432	2141
Ogden-Clearfield,UT	209648	32799	60305	5439
OklahomaCity,OK	128285	38882	6263	4799
Olympia,WA	254566	39988	82811	-37145
Omaha-CouncilBluffs,NE-IA	135631	43012	-7787	4075
Orlando-Kissimmee,FL	192993	35717	70500	-42265
Oxnard-ThousandOaks-Ventura,CA	405815	46787	259377	-111207
PalmBay-Melbourne-Titusville,FL	100877	37035	1245	-28061
Peabody,MA	318301	50895	107415	-4670
Pensacola-FerryPass-Brent,FL	161147	33338	51831	-23593
Peoria,IL	126718	40787	-12718	3105
Philadelphia,PA	190433	47361	17739	-21036
Phoenix-Mesa-Scottsdale,AZ	156698	36156	37786	-34795
Pittsburgh,PA	126616	42104	-15350	3052
PortSt.Lucie,FL	149184	39777	32702	-40431
Portland-SouthPortland-Biddeford,ME	211700	41522	41508	-10423
Portland-Vancouver-Beaverton,OR-WA	276563	39942	85134	-29251
Poughkeepsie-Newburgh-Middletown,NY	303410	40119	138304	-26197
Providence-NewBedford-FallRiver,RI-MA	227287	40887	69347	-22110
Provo-Orem,UT	241458	23814	102756	-3380
Pueblo,CO	133986	30564	25416	5662
Racine,WI	177849	37012	49031	-12039
Raleigh-Cary,NC	213951	39602	65866	921
Reading,PA	153884	36256	45164	-18381
Redding,CA	219005	34527	105379	-54805
Reno-Sparks,NV	194248	46929	62649	-53131
Richmond,VA	214436	42309	52788	-20864
Riverside-SanBernardino-Ontario,CA	191675	30634	115268	-62994
Roanoke,VA	169746	38727	24979	-6874
Rochester,NY	120462	39812	14085	1750
Rockford,IL	114019	32955	17320	-5917
RockinghamCounty-StraffordCounty,NH	239595	45231	57195	-6151
Sacramento-Arden-Arcade-Roseville,CA	235755	41119	137359	-77007
Saginaw-SaginawTownshipNorth,MI	55675	30143	-27700	180
Salem,OR	194581	32016	52384	-13689
Salinas,CA	247201	42857	133215	-80905
SaltLakeCity,UT	221381	38237	43451	-4870

Table continued: The Estimated Risk Premia and Implied Amenities for 2008

Metropolitan Area	Price	Wage	Amenities	Risk Premia
SanAntonio,TX	151913	34937	33157	1326
SanDiego-Carlsbad-SanMarcos,CA	361444	46649	208206	-90688
SanFrancisco-SanMateo-RedwoodCity,CA	714716	76042	382048	-135615
SanJose-Sunnyvale-SantaClara,CA	520378	58531	304677	-130572
SanLuisObispo-PasoRobles,CA	389682	40635	256355	-110656
SantaAna-Anaheim-Irvine,CA	482828	51894	316297	-140048
SantaBarbara-SantaMaria-Goleta,CA	277409	47957	141104	-73889
SantaCruz-Watsonville,CA	495718	51140	291999	-113154
SantaFe,NM	310957	44927	102140	-16706
SantaRosa-Petaluma,CA	354335	47755	202088	-89487
Savannah,GA	176647	39183	19891	-8893
St.Louis,MO-IL	137199	41823	-2587	-2983
Warren-Troy-FarmingtonHills,MI	123421	44488	-13735	314
WestPalmBeach-BocaRaton-BoyntonBeach,FL	243844	58358	49911	-59038
Wilmington,DE-MD-NJ	244404	43643	86808	-32946

Note - All the variables are for year 2008. The risk premia is estimated by the exposure of a metropolitan area to the three factors and to the idiosyncratic variance. A negative risk premium should be interpreted as a cheaper house price due to the exposure of the MSA to risk. The amenities are calculated as the amount left over from prices after removing the effect of wages, expected growth, and risk premia.