

Examining the Relationship of Awareness and Seller Pricing in Online Markets*

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Abstract

Online markets play an increasingly important role in facilitating economic transactions. In this paper, we study the link between individual seller pricing behavior and macro-level phenomena in online markets, by employing the notion of population awareness. We develop a model of seller pricing where (i) consumers must first become aware of the seller before they consider adopting the seller's product or service, (ii) past adoptions increase the probability that consumers will become aware of the seller in the future, and (iii) awareness decreases over time. For sellers with limited life spans we identify pricing dynamics consistent with penetration pricing. For sellers with longer life spans, the relationship between awareness and price is non-monotonic, and consistent with a tipping point in adoption dynamics. Sellers set higher prices when they have either achieved high population awareness (mass market/superstars) or have been positioned in a market niche, while prices are lower for intermediate awareness levels where competition is most intense. We exhibit the importance of awareness for online market design by examining its relationship with market concentration and platform fee decisions.

Keywords: *Economics of IS, Electronic Commerce, Analytical Modeling*

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1 Introduction

A large fraction of the world’s economic activity has moved online over the past two decades. Recent advances, including the increasingly prevalent mobile internet applications and the burgeoning sharing economy, promise to make this trend more pronounced (Ghose and Han, 2011; Sundararajan, 2016; Horton and Zeckhauser, 2016). Online markets play a key role in this ongoing shift, necessitating that their underlying mechanisms are clearly understood.

A core element of online markets is the process through which sellers set the price of their products or services. Early work recognized digital platforms’ potential to reduce search and transaction costs, resulting in stronger price competition and to increases in consumer welfare (Bakos, 1991, 1997). However, the pursue of the elusive frictionless market has produced mixed results, with previous work identifying evidence of higher, lower, or identical price disparity in online markets when compared to their traditional counterparts (Brynjolfsson and Smith, 2000; Campbell et al., 2005; Ghose and Yao, 2011; Li et al., 2013; Parker et al., 2016).

The apparent contradiction partially stems from the fact that prices in online markets are not static. Digital platforms engender sophisticated pricing schemes: sellers are able to seamlessly adjust prices with higher frequency than in bricks-and-mortar businesses (Oh and Lucas Jr, 2006). In addition, online platforms are in constant flux, with non-negligible dropout rates in both the demand and the supply side of the market, as well as frequent changes to market design. Crucially, agents on online markets

rarely satisfy canonical rationality assumptions ([Arthur, 1999](#); [Daskalakis et al., 2009](#); [Galla and Farmer, 2013](#)). While standard rationality assumptions may be somewhat reasonable for experienced sellers participating in high-stakes transactions, such as in B2B online markets, they are unlikely to hold for other important applications, such as sharing economy markets.

This paper attempts to bring together insights from theoretical and behavioral economics, as well as recent empirical work, to show how the dynamic, boundedly rational seller behavior explains important online market phenomena. We first develop a model of dynamic seller pricing in online markets. The seller optimally adjusts her price in response to a measure of her popularity that we call population awareness. Awareness functions as the seller’s proxy for demand, in that higher values translate to higher probability that she will be discovered by buyers. The determinants of awareness emerge from two intuitive properties that characterize the demand side of the market. First, past purchases increase awareness, resulting in future consumers being more likely to discover the seller. Second, awareness diminishes over time: the seller risks “fading into obscurity” in the absence of new sales.

In assuming that awareness increases in past purchases, we introduce path dependency to our model’s dynamics. This allows us to capture the pronounced degree of uncertainty characterizing sales in online markets. Furthermore, the same assumption nests the fact that awareness is mainly generated by other consumers rather than by the seller herself. We choose to abstract away from the exact processes through which awareness may diffuse; some examples include direct channels such as peer effects and posts on

social media, or indirect ones such as recommender systems, rankings, and search engine results (Acquisti, 2014; Ghose et al., 2014; Ursu, 2016; Wang et al., 2017). The diminishing awareness assumption captures the competitive dynamics that underlie the platform, as a higher degree of competition between platform sellers translates into more rapid decreases in awareness. Intuitively, more intense competition means that a higher number of purchases is needed for a seller to stay on the top of the platform rankings or come up first in related searches. This assumption also harkens back to the operational principles of recommender systems, which typically weigh recent events more heavily (Adomavicius and Tuzhilin, 2005).

Sellers respond to an individual-specific popularity metric instead of explicitly reacting to their competitors' strategies. Our approach is motivated by the increasingly prevalent bounded rationality and out-of-equilibrium behavior literature (Bajari and Hortacsu, 2003; Camerer et al., 2003, 2004). The measure of awareness offers sellers an individual-specific and imperfect way to assess the current state of the market, and provides a modeling approach not burdened by strict informational assumptions such as knowledge of the demand function, the competing sellers' strategies, or even the payoff function of the underlying game. We instead choose to approximate this decision-making process by incorporating important drivers of the sellers' behavior as well as salient features of digital platforms.

The first part of our analysis considers cases where sellers live in a finite horizon setting. Examples that may fall under this category include products characterized by rapid decreases in value due to factors such as seasonality, replacement by new technology, or competition markets where fashion fads

are prevalent. Interestingly, we find that incorporating awareness dynamics results in a price path consistent with penetration pricing (Dean, 1950; Tellis, 1986). Penetration pricing schemes are characterized by a low initial price to raise product awareness through high early adoption rates, and a subsequent increase the price level as the end of the time horizon is reached. Nonetheless, the exact evolution of prices is stochastic and depends on the sales history and the properties of the awareness-generating process, yielding complex price path patterns.

We then turn to the more prevalent case of products of longer lifespans. We exhibit the existence of a tipping point in awareness, that characterizes various degrees of vicious and virtuous cycles. Further, we find that prices cease being monotonically increasing in awareness, instead exhibiting a U-shape. When population awareness is close to the tipping point, we show that sellers set the minimum price in an attempt to stir their level of awareness above the tipping point threshold and toward the more profitable high level. On the other hand, prices attain their maxima when product awareness is either very high or very low. The two extrema of awareness can be interpreted as the seller achieving superstar or niche status correspondingly.

Shifting our focus to the macro-level phenomena that the individual behavior of agents engenders, we develop an agent-based model of sellers operating an online market, and carry out computational experiments. More specifically, we center on market concentration, measured by the fraction of the market captured by the most successful sellers, and the dispersion of sales amongst the same set of top-performing agents. We find that as the platform population size increases, the impact of awareness is likely more

consequential, as the market tips to a state where almost all activity is captured by a small percentage of agents on the seller side. Since this can be a challenge, we proceed to investigate whether increasing the rate at which awareness decays ameliorates this phenomenon. We find that this is the case: increasing the rate at which awareness decays allows transactions to be dispersed across a wider set of sellers. As platform designers can adjust the rate at which the population awareness of sellers decreases, this reveals an important design decision for market operators. We then perform a theoretical analysis of the interplay of awareness and platform fees, and find that the two can substantially affect each other.

Our results reveal the significance of consumer awareness and bounded seller rationality, by showing that they are key drivers of core online platform phenomena. Taken together, these two factors can explain a wide range of commonly observed behaviors, including penetration pricing, tipping points in adoption, price dispersion, the emergence of superstar and niche sellers, and market concentration. Further, the importance of understanding the implications of boundedly rational seller behavior is likely heightened given the recent emergence of sharing economy markets as an important online platform category. Finally, to the best of our knowledge, this is the first paper to exhibit the role of awareness decay as an important online market design decision.

The rest of the paper is organized as follows. Section 2 provides a survey of the previous results and discusses the concepts that are embedded in our model. The finite-horizon model is introduced in Section 3, and is extended to the infinite horizon case in Section 4. Section 5 investigates the market

design implications of awareness. Section 6 concludes.

2 Literature review

2.1 Awareness as a market mechanism

Our model shares features with the class of diffusion models that were developed following the seminal paper of Bass (1969). The Bass model describes a diffusion process by a set of differential equations with consumers belonging to either the innovator or the imitator categories, and was subsequently extended to include pricing, advertising, and product uncertainty (Kalish, 1985; Krishnan and Jain, 2006). Our work departs from Bass-type diffusion models in several aspects. The main channel through which awareness is endogenously generated in our model is past adoptions; when consumers buy the seller's service or product then, *ceteris paribus*, the probability of other consumers becoming aware of the seller increases, an effect that can be traced back to the theory of availability heuristics in cognitive psychology and behavioral economics (Tversky and Kahneman, 1973). While consumers are more likely to adopt a product following a history of many purchases, we do not *ex ante* distinguish between consumers as innovators or imitators. Rather, the order of adoption decisions is random, so that the selection of awareness generating innovators is random; in the Bass family of models the diffusion process is to a large extent predetermined by model parameters. In addition to certain exogenous characteristics of a product, random events play an important part in explaining success or failure. Therefore, the dynamics of a firm's optimal pricing policy are to some extent determined by

the history of random draws.

Consumers who decide to adopt have a disproportionately large effect on potential adopters as compared to consumers who consider the product and decide not to adopt. This assumption is related to concepts developed in the marketing research literature, where word-of-mouth marketing and the role of the consumers as “market mavens” are recognized as important drivers of successful information diffusion (Feick and Price, 1987; Kozinets et al., 2010). More recently, empirical evidence in support of peer-induced adoption is provided by Banerjee et al. (2013) who estimate that a microfinance participant is seven times more likely to inform other households about microfinance loans than a knowledgeable non-participant. Similar results on peer-induced decisions are obtained in the case of enrollment in retirement plans (Duflo and Saez, 2003) and stock purchases (Ivković and Weisbenner, 2007). The time-decaying nature of consumer awareness has been a long recognized phenomenon in marketing (Hutchinson and Moore, 1984).

In online markets, population awareness for a seller is mainly generated through product rankings. Choudhary and Zhang (2016) investigate the welfare implications of algorithmic product recommendations. Ursu (2016) provides experimental evidence that a high search ranking for hotels on Expedia increases sales unconditional on click, but has no effect on sales conditional on a click. This suggests that a high ranking increases sales due to costs associated with ordered search: if more sales result in higher ranking, we would expect increased demand even when the rankings do not convey any information about the product’s attributes. The same results are found in previous research that provides evidence of the effect of sales

on product ratings (Ghose et al., 2014), though the magnitude of the effect can vary between different types of media (Zheng et al., 2016).

2.2 Pricing

Changes in population awareness in our model result in significant price fluctuation. We are not the first to notice such pricing behavior. Board (2008) and Garrett (2016) consider durable-good monopolists who face varying influx of demand, and find mechanisms through which a seller can take advantage of these fluctuations to price-discriminate over time. Mixed pricing strategies due to consumer inertia in the model of sales of Varian (1980) explain fluctuating prices in competitive environments. Campbell (2015) studies a situation where customers inform their friends about a product. Our model differs from this work in that we explicitly model awareness decay as well as differences in impact of previous adoptions.

More adoptions and hence higher awareness lead to increased demand in our model. In that regard, the underlying mechanism generates demand-side externalities similar to network effects. However, this increased demand is due to increased probability of consumers becoming aware, and is therefore distinct from network effects as the number of adopters do not affect the consumers' utilities, directly nor indirectly. Despite this difference, our results resemble some of those found for network goods. For example, low introductory pricing as an equilibrium pricing strategy has been identified by Cabral et al. (1999), and our results regarding tipping in adoption are similar to the notion of a critical mass in relation to a multiplicity of equilibria in adoption of network goods (Katz and Shapiro, 1985; Economides,

1996).

Price dispersion in online markets has traditionally been used to estimate their efficiency. Ghose and Yao (2011) provide a summary of the contrasting results found in the literature, and show that price dispersion overestimated when the posted prices are used instead of the transaction prices. Li et al. (2013) find evidence from Amazon data that price dispersion increases when competitors have higher cross-selling capabilities and adopt loss leader pricing schemes. Parker et al. (2016) employ a natural experiment to show that better price information decreases price dispersion. Our work adds to this stream of literature by showing how awareness can also explain increases in price dispersion.

3 A model of seller awareness

In this section we introduce our model and focus on the finite horizon case. Sellers respond to changes in awareness by solving a dynamic program and adjusting their price. The measure of awareness offers a simple way to capture the information about the state of the market, as well as the competitive dynamics that underlie it (see also our discussion in Section 1). We characterize our model by obtaining core structural results.

3.1 The finite horizon model

At each time period $t \in \{0, 1, \dots, T - 1\}$ a one-period lived consumer is born. The consumer, whom we refer to as consumer t to economize on notation, has a one-time chance to discover the seller and potentially buy

the product or service. Consumer t arrives (or becomes aware of/discovers the seller) with probability $Q(a_t) \in [0, 1]$, $Q' \geq 0$, where a_t is the seller’s population awareness at time t . We model population awareness as a stock that follows the law of motion

$$a_{t+1} = \delta(a_t + kb_t), \tag{1}$$

where $b_t \in \{0, 1\}$ is a binary variable indicating whether a sale was made in period t , $\delta \in (0, 1]$ is a discount factor which captures awareness decay in the population, and $k > 0$ is the awareness impact of a positive adoption decision. If a consumer becomes aware of the seller, she learns her privately observed valuation $\theta_t \in [0, \theta_h]$, drawn from a publicly known distribution $F(\theta)$, i.i.d. across consumers. For a given price p_t , the consumer makes a positive adoption decision ($b_t = 1$) if $\theta_t \geq p_t$, or in other words if her valuation exceeds the seller’s price.

3.1.1 Awareness

Awareness has two intuitive properties. First, past adoptions increase population awareness. To translate this property in online market terms, as more buyers buy the seller’s product or service, the seller climbs in the platform rankings. Subsequently, the seller is more likely to be recommended by the algorithms the platform employs, as well as amassing more reviews. All of these factors make subsequent users more likely to discover the product. Further, variations in quality and “virality” are captured by the awareness impact parameter k ; products that generate more “buzz” and higher quality

products are characterized by higher impact parameter values.

The second important property of awareness is that it decreases over time. If buyers stop buying the seller's product or service, then the seller will drop in the platform rankings, and new buyers will be less likely to discover her. A more subtle property of the parameter decay δ is that it captures the intensity of the competition on the platform. In the presence of many competing buyers δ is low and awareness decreases fast, as more buys are needed for the seller to stay ahead of his peers. In the case of a seller that faces no competition, δ is close to one, as the seller is always considered by potential buyers.

3.1.2 The seller's problem

The seller faces a constant marginal cost c , such that $c < \theta_h$. At time t the seller decides on a price that maximizes the future-discounted sum of her expected profits:

$$E \left[\sum_{s=t}^{T-1} \beta^{s-t} Q(a_s) (1 - F(p_s)) (p_s - c) \right].$$

The seller's optimization problem is cast in terms of the dynamic programming framework through the following Bellman equation:

$$\begin{aligned} V_t(a_t) = \max_{p_t \geq 0} Q(a_t) & \left[(1 - F(p_t)) (p_t - c + \beta V_{t+1}(a_{t+1}^1)) + F(p_t) \beta V_{t+1}(a_{t+1}^0) \right] \\ & + [1 - Q(a_t)] \beta V_{t+1}(a_{t+1}^0), \end{aligned} \tag{2}$$

where $a_{t+1}^1 = \delta(a_t + k)$ is the population awareness following a positive adoption decision ($b_t = 1$), and $a_{t+1}^0 = \delta a_t$ is the population awareness in the opposite case ($b_t = 0$).

The Bellman equation admits a simple interpretation. At time t and with population awareness a_t , a consumer becomes aware of the product with probability $Q(a_t)$. If the seller's price is p_t , then the consumer's valuation θ_t exceeds p_t with probability $1 - F(p_t)$, in which case the seller earns period t profits of $p_t - c$ and a continuation value of $V_{t+1}(a_{t+1}^1)$, since the number of adopters increase by one unit. The consumer will not make a purchase if (i) she becomes aware with probability $Q(a_t)$, and realizes a valuation $\theta_t < p_t$, or (ii) she does not become aware of the seller. In either case, no new users adopt, and the seller's continuation is $V(a_{t+1}^0)$.

The first-order condition of the maximization problem of Equation (2) is:

$$Q(a_t) \left[(1 - F(p_t)) - f(p_t)(p_t - c + \beta V_{t+1}(a_{t+1}^1)) + f(p_t)\beta V_{t+1}(a_{t+1}^0) \right] = 0.$$

Conditional on a consumer arriving, a marginal increase in p_t has the following effects on expected revenue: a positive price effect $1 - F(p_t)$, and a negative expected output effect $-f(p_t)p_t$. In addition, higher prices decrease the probability of receiving a higher continuation value $V_{t+1}(a_{t+1}^1)$. As a result, this creates an incentive for the seller to decrease the price compared to the case where past adoptions and population awareness are unrelated. This illustrates the tradeoff that product sellers face when making pricing

decisions.

In conducting our analysis we will use the virtual valuation formulation of the first order condition, which is

$$p_t - \frac{1 - F(p_t)}{f(p_t)} = c - \beta[V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)], \quad (3)$$

where the fraction $[1 - F(p_t)]/f(p_t)$ is the reciprocal of the hazard ratio of the valuation distribution, and the left hand side can be interpreted as the (present) expected marginal revenue conditional on the consumer becoming aware of a product. Since $V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)$ is clearly non-negative, the price is lower compared to the case of zero interaction between awareness and sales ($k = 0$).

Finally, we will assume that virtual valuations are increasing in price, that is $1 - \frac{d(1-F(p))/f(p)}{dp} > 0$. This assumption guarantees that a unique global maximizer exists, is a weaker version of the increasing hazard rate assumption, and is satisfied for most unimodal distributions including the uniform and normal distributions.

3.2 Structural results

3.2.1 The two-period model

The two-period model is the simplest finite horizon case that captures the dynamics of awareness generating adoption while admitting analytic solutions. At period 1 the price is given by

$$p_1^* - \frac{1 - F(p_1^*)}{f(p_1^*)} = c.$$

Note that the price of the last period is independent of population awareness (end of the world effect). The corresponding value function is:

$$V_1(a_1) = Q(a_1)[1 - F(p_1^*)](p_1^* - c),$$

where $a_1 = \delta(a_0 + kb_0)$. The difference in the probability that the consumer of period 1 will discover the seller (equivalently, arrive) if one more consumer has adopted in the past is denoted by:

$$\Delta Q(a_1) = Q(\delta[a_0 + k]) - Q(\delta a_0).$$

Since p_1^* is independent of whether an adoption takes place in $t = 0$, p_0^* only depends on $\Delta Q(a_1)$. Equation 3 then gives us:

$$p_0^* - \frac{1 - F(p_0^*)}{f(p_0^*)} = c - \Delta Q(a_1)\beta[1 - F(p_1^*)](p_1^* - c).$$

The following result formalizes the intuition that when adoptions increase arrival probabilities more, prices are lower.

Proposition 1. *Optimal price is decreasing in $\Delta Q(a_1)$.*

We can also explicitly relate population awareness with the functional form of Q .

Proposition 2. *The price is increasing (resp. decreasing) in population awareness a_0 , if Q is concave (resp. convex) at δa_0 and $\delta[a_0 + k]$.*

Proposition 2 reveals that the exact effect of population awareness hence depends on the shape of Q . The intuition behind this result is straightfor-

ward. When Q is convex, we have increasing returns to awareness in the sense that positive adoption decisions lead to larger changes in future arrival probabilities, which in turn incentivizes the seller to set a lower price. On the other hand, when Q is concave there are diminishing returns to awareness, and the seller puts relatively higher weight on current profits; no adoptions today have little negative effect on future adoption probabilities.

We derive an additional result in the case where consumers valuations are drawn from the standard uniform distribution, i.e. $F(\theta) \sim U[0, 1]$.

Proposition 3. *Suppose consumer valuations follows a standard uniform distribution, $\theta \sim U[0, 1]$, and $T = 2$. Then the price never drops below marginal cost.*

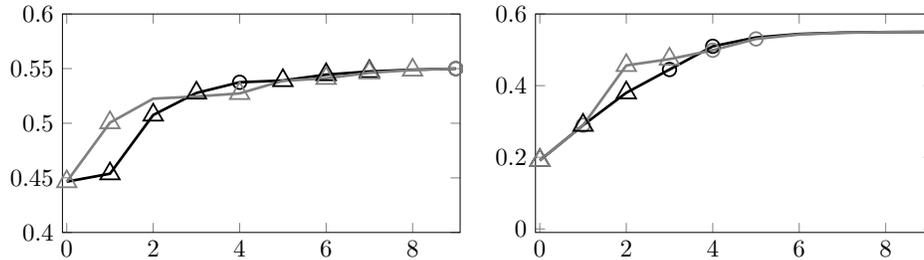
Proposition 3 establishes the existence of cases where increasing future profit is never sufficiently valuable to incur losses today. In Section 3.2.2 we numerically show that this does not extend to the T -period case, where pricing below marginal cost in early periods may occur for sufficiently large T .

3.2.2 T periods

For more than two periods analytic solutions are hard to obtain, even under very specific restrictions on the form of Q . This difficulty is common in dynamic programming; in our case its source is the complexity caused by the dependence of optimal prices on continuation values. Nevertheless, it is important to examine whether our results for the two-period model hold in the $T > 2$ case. Toward that end, we employ numerical methods in this section to test the robustness of our results.

An insight from the two-period analysis is that the functional form of Q is an important determinant of the relationship between population awareness and prices. We numerically show here that the results of Proposition 2 extend to the T -period case. If Q is convex (concave), and it is not possible to attain the upper (lower) bounds of 1 (resp. 0), then prices are lower (higher) following an adoption compared to no adoption. The intuition is the same as in Proposition 2. Adoption increases population awareness relative to non-adoption. If Q is convex (concave), then higher awareness leads to a larger (smaller) difference in future arrival probabilities. The convexity (concavity) of Q results in lower (higher) prices for increased population awareness at a given time.

Figure 1: Example adoption paths for the finite horizon model



Notes: \triangle : adoption, \circ : arrival without adoption. The x-axis and y-axis indicate time periods and seller price respectively. Higher awareness leads to higher prices when there are diminishing returns to awareness (concave), and lower prices when there are increasing returns to awareness (convex). $\theta_t \sim U[0, 1], T = 10, c = .1, \beta = 1$. The first simulation (left) utilizes a Pareto distribution ($Q(a) = 1 - 1/(a + 1)$), and the second simulation (right) a convex distribution ($Q(a) = \min\{a^2, 1\}$)

Price simulations are shown in Figure 1. In Figure 1 (a), the arrival probability $Q(a)$ follows a Pareto distribution (concave over its domain). At any period, higher awareness corresponds to higher prices. For example, in

period $t = 0$, a sale occurs in the simulation corresponding to the gray line, while there is no consumer arrival in the simulation corresponding to the black line. As a consequence, the former's price is higher in $t = 1$ because of higher awareness. The price in the black simulation eventually exceeds the one in the gray one when the population awareness exceeds that of the gray simulation. Furthermore, the price difference between the two eventually gets smaller over time after they both achieve to generate high awareness and as the end-of-the-world effect becomes stronger as time approaches $T-1$.

Figure 1 (b) plots simulations with $Q(a) = \min\{a^2, 1\}$ (convex for all feasible a_t). In this case, higher awareness leads to lower prices, as states in which awareness is high have more potential to attract consumers. This provides an incentive for sellers to set lower prices today in order to increase the probability of getting awareness generating sales. The results of Proposition 2 appear to be robust for the strict concavity and convexity cases.

4 Infinite horizon analysis

The finite-horizon model we developed in Section 3 aided us in studying the relation between population awareness and prices. A shortcoming of our analysis is that the impact of the positive time trend inherent in finite-horizon formulations is hard to disentangle from the effect of population awareness. While end-of-the-world effects are present in a considerable number of real-life applications (e.g. products and services for which seasonality effects are prevalent), it is also important to study what happens in their

absence, or in cases where there is uncertainty about the lifespan of a product.¹ For example, Amazon sellers seldom list their products for only a few days.

Toward that end, in this section we extend our model to the infinite horizon case. Our analysis exhibits the emergence of interesting phenomena such as niche and superstar providers, as well as the existence of tipping points in the awareness process.

4.1 The infinite horizon model

Assume that awareness can attain only a finite number of states and, consequently, that the underlying system can only be in a finite set of states, each characterized by an arrival probability. Formally, let the arrival probability in period t be denoted by $q_t \in \{q_0, q_1, \dots, q_{n-1}\}$, with $0 \leq q_0 < \dots < q_{n-1} \leq 1$. The population awareness law of motion is then replaced by the following state transition rule:

$$\text{If } q_t = q_i, \text{ then } q_{t+1} = \begin{cases} q_{i+1} & \text{if } b_t = 1 \text{ and } i < n - 1, \\ q_i & \text{if } b_t = 1 \text{ and } i = n - 1, \\ q_{i-1} & \text{if } b_t = 0 \text{ and } i > 0, \\ q_i & \text{if } b_t = 0 \text{ and } i = 0. \end{cases} \quad (4)$$

Equation 4 can be interpreted as the discrete state analogue of the law of motion of Equation 1. Arrival probabilities increase following an adoption,

¹ The discount factor $\beta \in (0, 1)$ could alternatively be interpreted in terms of a ‘‘Poisson death model.’’ To see how, we may write $\beta = \rho/(1+r)$, where ρ is the probability of being alive in the next period (in our case, the probability of the provider being in the market in the next period), and r is a discount rate.

and otherwise decrease. For clarity of exposition, we assume in what follows that $\Delta q_i = q_{i+1} - q_i = (q_{n-1} - q_0)/(n - 1)$ is constant for $0 \leq i \leq n - 2$. Our results do not depend on these assumptions, and our analysis can be straightforwardly carried out for other choices of parameters. Observe that, if we keep $q_{n-1} - q_0$ constant, smaller n larger jumps in future arrival probabilities. A small n therefore corresponds to a large awareness impact k in the finite time version of the model.

The value function of the seller is no longer a function of time, and the Bellman equation becomes:

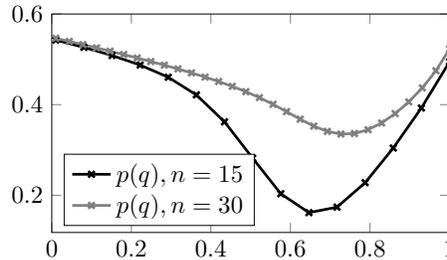
$$V(q_i) = \max_{p_i} q_i \left[(1 - F(p_i))(p_t - c + \beta V(q_{i+1}) + F(p_i)\beta V(q_{i-1})) \right] + (1 - q_i)\beta V(q_{i-1}). \quad (5)$$

At the boundary states ($i = 0$ and $i = n - 1$) the value function is naturally handled by the transition rule of Equation 4 (i.e., $V(q_{-1}) := V(q_0)$ and $V(q_n) := V(q_{n-1})$). We can now solve the Bellman equation numerically. More details about our numerical methods are provided in the Appendix.

Figure 2 depicts the seller's optimal price function for the cases of $n = 15$ and $n = 30$. We first observe that price is clearly non-monotonic in Q . When Q approaches either of its extrema (0 and 1) and as n grows larger, the corresponding optimal prices converge to what would be optimal if adoption decisions did not affect future arrival probabilities. Intuitively, a seller would set a lower price when positive adoption decisions substantially increase expected future profits. For low awareness states, i.e. for niche product,

a sale today is not likely to increase future arrival probabilities to a large degree, and sellers set a high price to maximize profit in the rare event that an arrival occurs. On the other hand, providers with high awareness (superstar providers/mass-market products), are not substantially affected by an arrival that does not lead to adoption; subsequent arrivals will occur in the near future with high probability. In these cases, it is optimal for the seller to set a high price. Prices are set at the lowest level when sellers have not yet positioned themselves in neither a niche nor a superstar role. This provides a partial answer to the question posed by [Brynjolfsson et al. \(2010\)](#): in the market dynamics created by awareness, sellers are incentivized to price high in both the market superstar and niche positions.

Figure 2: Optimal seller price for the infinite horizon model



Notes: The x-axis and y-axis indicate arrival probabilities and seller price respectively. The parameters that were used in this figure were $q_0 = 0.01$, $q_{n-1} = 1$, $\beta = .95$, $c = .1$, $\theta \sim U[0, 1]$. Price and value functions are plotted against $Q(\cdot)$ for $n = 15, 30$.

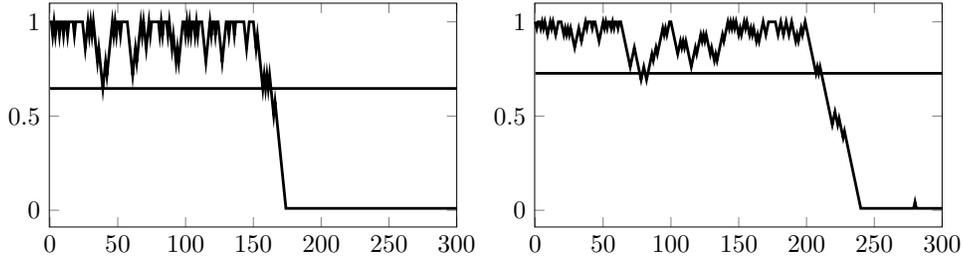
4.2 Tipping points

The results of our experiments reveal that price attains its minimum for intermediate arrival probabilities. The benefit of a low price is more pro-

nounced for these states, since failing to generate awareness may lead to a sequence of non-arrivals, which can in turn push the seller into obscurity. The state that corresponds to the lowest price can be interpreted as a tipping point of the adoption process, above (resp. below) which arrivals are increasingly more (resp. less) frequent.

Figure 3 plots simulations of the arrival probabilities for $n \in \{15, 30\}$. We observe that when awareness decreases to a sufficient extent below the tipping point the product quickly falls into low awareness states. This effect is exacerbated when changes in population awareness are more abrupt, captured here as smaller n .

Figure 3: Tipping points in population awareness



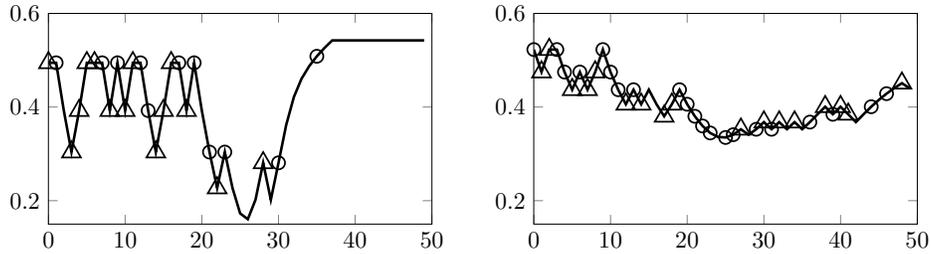
Notes: The x-axis and y-axis indicate time periods and arrival probabilities respectively. The horizontal lines represent the arrival probability $Q(\cdot)$ corresponding to the lowest price. The parameters used for this figure are $q_0 = 0.01, q_{n-1} = 1, \beta = .95, c = .1, \theta \sim U[0, 1]$.

The awareness impact of adoption decisions also has a considerable effect on prices. We observed in Figure 2 that prices are lower across all states for smaller n . This hints at the considerable effect that the awareness impact of adoption decisions have on prices: a bad sequence of non-adopters is more pronounced when the awareness impact of adoptions is high, low-value

consumers are more important, and we expect the adoption process to be more unstable.

Figure 4 illustrates this point. In the left panel, the seller eventually experiences a sequence of non-adoptions and reacts by reducing price to tip the awareness back to the higher state. In the depicted simulation the attempt fails, and price subsequently increases as arrivals become less frequent and the product fades into obscurity; the niche of consumers that will discover the product will be charged a high price to obtain it. In the right panel, the changes in awareness are smaller, and the seller reacts less aggressively; awareness remains relatively high and streaks of unlucky draws have smaller impact on her profit.

Figure 4: Pricing in the infinite horizon model



Notes: \triangle : sale, \circ : arrival without sale. The initial value of $q = q_{n-1}$ was set to 1. Other parameters used in this Figure are $q_0 = 0.01$, $q_{n-1} = 1$, $\beta = .95$, $c = .1$, $\theta \sim U[0, 1]$.

5 Implications of awareness for market design

5.1 An agent-based model of an online market

We utilize the model of Section 4 to develop an agent-based model that simulates decision processes of a platform’s sellers and consumers. Our model allows us to incorporate the notions of consumer awareness and bounded rationality in our analyses.

Analogously to the model of Section 4, each seller has a known population awareness, and chooses a product price based on this measure. Awareness assumes values from a predetermined discrete set, with higher levels translating into higher probability that the seller will be discovered by consumers. Each seller’s awareness decreases in every period if the seller fails to make a sale, and increases if she does. The rates of increase and decrease vary across our simulations. Consistent with our approach in the previous sections, sellers are boundedly rational and do not form expectations about others’ behavior.

A single consumer arrives during every round. We conduct experiments utilizing two different ways through which a consumer may select sellers. Under the sequential selection assumption, the consumer goes through the set of sellers in decreasing awareness order. Each seller is discovered with probability analogous to the associated population awareness. A purchase is made if the seller’s price is lower than the consumer’s valuation, without the consumer considering any other sellers. Under the consideration set assumption, the consumer forms a consideration set by selecting sellers with probability analogous to their population awareness. After the consideration

set is constructed, the consumer chooses to purchase from the seller with the lowest price. If the seller's with the lowest price still exceeds the consumer's valuation of the product or service, then no purchase is made. Consumers make a one-time decision and then leave the platform.

Each instance of our computational model is carried out for a pre-specified number of periods. Seller awareness is randomly initialized; other methods of initialization that we tried do not qualitatively change our results. The order of events in every period is described below.

1. **Sellers adjust their price.** Sellers update their prices with respect to their population awareness. Sellers that have attained the maximum or minimum awareness levels will not upgrade their price, as long as they make or do not make a sale in the previous period respectively.
2. **A consumer arrives and decides.** A consumer arrives, and forms a consideration set following one of the two rules described above (sequential decision rule, or consideration set formation). The consumer makes a one time purchase decision from a seller (if any).
3. **Population awareness is updated for every seller.** The population awareness of the seller that made a sale increases, if any such seller exists. The awareness of all other sellers decreases. The exact amount of awareness increase and decrease is a parameter of our simulation.

5.2 Awareness and market concentration

A long-standing question about online markets is whether the platform dynamics converge to a long tail or to a winner-takes-all market, or whether

the market alternates between the two (Brynjolfsson et al., 2010). More broadly, we are interested in measuring market concentration and identifying its drivers (Brynjolfsson et al., 2011).

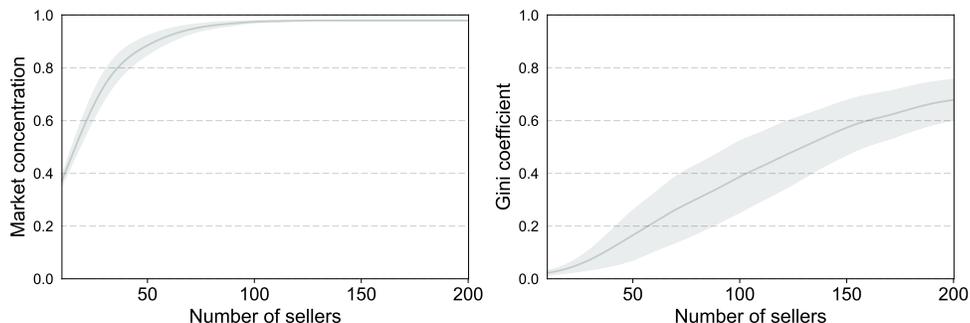
To explore the role of awareness on market concentration we conduct the following experiment. We run many instances of the market for 2000 sequential consumer arrivals, and track the number of sales by each seller on each instance. We then vary the number of sellers in the platform, and compute the average market concentration and Gini coefficient for the top 20 percent of sellers.

Figure 5 plots the results of our computational experiment. Market concentration (left panel) begins at a fairly low level, but rapidly expands as the number of sellers increases. For more than 100 sellers, we notice complete market concentration on the set of top sellers. The Gini coefficient (right panel) allows us to examine the dispersion of the sales within the top sellers. The concentration measured by the Gini coefficient within the set of top sellers is initially lower, increases at a lower rate than overall market concentration, and exhibits higher variance, albeit it also follows the overall market concentration. This result informs us about the importance of the awareness phenomenon: as the market grows, it rapidly becomes extremely concentrated.

5.3 Awareness decay as a platform design decision

The natural mechanism of market awareness pushes the platform into a state wherein market power is concentrated within in a restricted set of users. This situation may be harmful for the platform. On the consumer side, product

Figure 5: Market concentration and Gini coefficient as a function of market size



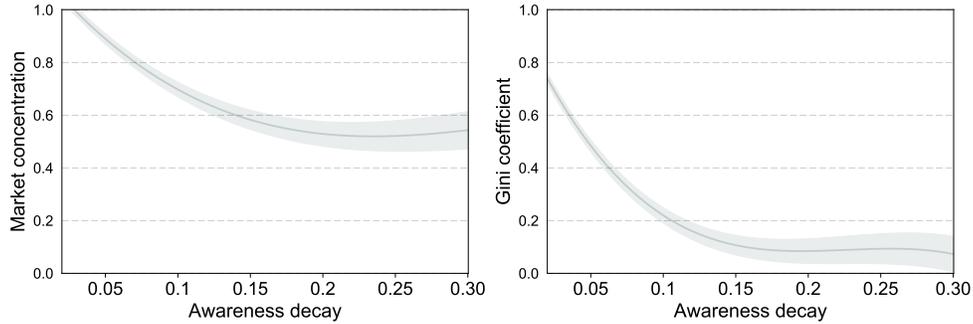
Notes: The left panel plots the concentration of sales computed as the percentage of sales made by the top 20% of sellers over the simulation horizon. The right panel plots the Gini coefficient computed on the distribution of sales of the top 20% of sellers over the simulation horizon. Each point plots the mean for the corresponding number of sellers, and the error bar gives the 95% confidence interval.

variety is reduced as only a smaller set of products is considered. On the seller side, few sellers retain most of the demand, potentially resulting in high dropout degree amongst the sellers that fade into obscurity. As a result, it is important for market designers to consider mechanisms that ameliorate this problem.

We consider the effects of varying awareness decay rates. Figure 6 plots a computation experiment where we increase awareness decay and measure market concentration quantities. Interestingly, we see that increasing awareness decay reduces both the Gini coefficient and the market concentration within the top 20% percent of sellers. The dampening effect is initially linear for both measures, and eventually levels off.

While platform designers may not set awareness decay indirectly, they can set it indirectly. A simple way to increase the degree of awareness decay

Figure 6: Market concentration and Gini coefficient as a function of awareness decay



Notes: Concentration of sales by the top 20% of sellers (left panel) and Gini coefficient (right panel) computed on the same set of sellers over the simulation horizon. The mean for each rate of awareness decay of sellers and the 95% error bar are plotted.

is through mechanisms via which awareness diffuses—posts on social media, recommender systems, product rankings, and search engine results. For example, such mechanisms may be redesigned so that time windows become narrower, and the recency effect is stronger.

5.4 Awareness and platform fees

The business model of many online platforms is by now a familiar one: the platform allows sellers to set the prices of their products, and typically levies a fixed percentage fee resembling a tax-type commission on the final sale price.² On the seller’s side, when the platform imposes a tax on the transaction price, it is straightforward to see that maximizing total profit is equivalent to the seller operating free-of-charge. Appendix C carries out the

²For example, the percentage fee on Etsy is 3.5%, on Ebay it is 10%, and on Google Play 30%. Both Etsy and Ebay combine percentage fees on the sales price with other fees such as listing fees.

analysis for a platform that platform utilizes a dynamic taxation scheme.

Our analysis reveals some interesting insights that we summarize here. First, optimal platform fees are lower because of awareness: sellers aggressively adjust their price responding to different awareness levels, and the platform takes this reaction into account when deciding on the fees. Moreover, we observe an interesting effect on the opposite direction: platform fees decrease the difference in sellers' profits for different awareness states, which translates to a dampening effect on price fluctuation. These results further exhibit that population awareness can be an important factor to take into consideration when designing online markets.

6 Conclusion

In this paper we examined the interplay of boundedly rational price setters, consumer awareness, and macro-level market phenomena. Our analysis reveals that awareness is likely a key underlying mechanism in online platforms, imbuing regular products with properties akin to those of network goods. The common modus operandi of modern online markets utilizes search and product rankings generated by past performance. This allows the visibility of sellers, and hence population awareness, to be subject to those users that have successfully adopted in the past and to the dynamics captured in our model. The mechanism of awareness increases price dispersion, and allows those sellers with high stocks of awareness to extract a premium, making the market less competitive.

Whether awareness is harmful for a platform depends on its current

objectives. While this is likely less consequential for incumbents, platforms that aim for growth, expanded sales, or facing stark competition should increase the rate of awareness decay. Making it easier to transit between superstar/niche states makes firms compete more, and results in lower, non-monopoly prices. As we discussed in Section 5, this can be achieved by narrowing the time windows in the reputation and recommendation systems employed by the platform.

The results of this paper are made more important in light of the burgeoning “sharing economy” markets (Sundararajan, 2016). A key distinguishing aspect of these platform-market hybrids is that suppliers usually own the capital made available in the platform Hall and Krueger (2016). A consequence of this is that on many of these platforms sellers are allowed to set their own prices. Further, suppliers on such platforms are often engaged in lower-stakes transactions; their in-platform activities are not their main occupation, i.e., they are not professional sellers. Consequently, sellers lack both the experience and the incentives to fully optimize their pricing choices. Our model captures the implications of both these factors on the market, both on the micro and the macro levels, and points out the adverse phenomena that these dynamics can give rise to.

This work raises various important questions, but is, of course, not without limitations. First, our model could be extended to shed light on specific facets of the effect of awareness on online platforms. For example, the explicit consideration of a fully competitive setting presents one such opportunity. Incorporating other aspects of online markets such as product reviews could also help obtain insight in more specific contexts. Further, an inter-

esting next step would be the empirical investigation of awareness-related phenomena on online markets. Estimating the awareness footprint of different reputation and recommendation system designs result in, as well as the sensitivity of agents' pricing behavior to these mechanisms, has potential to yield important recommendations on platform design and management.

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A Proofs

Proof of Proposition 1

Proof. The result follows by differentiation and the fact that valuations are increasing in price, since

$$\frac{dp_0^*}{d\Delta Q(a_1)} = \frac{-1}{1 - \frac{d(1-F(p_0^*))/f(p_0^*)}{dp}} \beta(1 - F(p_0^*))p_0^* < 0.$$

□

Proof of Proposition 2

Proof. Consider the effect of a marginal increase in a_0 on the optimal price,

$$\frac{\partial p_0^*}{\partial a_0} = \frac{-1}{1 - \frac{d(1-F(p_0^*))/f(p_0^*)}{dp}} (1 - F(p_1^*))p_1^* \beta \frac{\partial \Delta Q(a_1)}{\partial a_0}$$

where

$$\frac{\partial \Delta Q}{\partial a_0} = \frac{\partial Q(\delta[a_0 + k])}{\partial a_0} - \frac{\partial Q(\delta a_0)}{\partial a_0}.$$

If Q is concave at δa_0 and $\delta[a_0 + k]$ then $\frac{\partial \Delta Q}{\partial a_0} < 0$, which means that p_0^* is increasing in a_0 . For convex Q the inequality is reversed. □

Proof of Proposition 3

Proof. From Equation 3, the price p_0 is given by:

$$p_0 = \max \left\{ \frac{1+c}{2} - \beta \frac{\Delta Q(a_1)}{8} (1-c)^2, 0 \right\}.$$

The inequality $p_0 < c$ is equivalent to

$$\frac{1+c}{2} - \beta \frac{\Delta Q(a_1)}{8} (1-c)^2 < c \Leftrightarrow \beta \Delta Q(a_1) > \frac{4}{1-c}.$$

Since $\beta \Delta Q(a_1) \in [0, 1]$, the inequality $4/(1-c) < 1$ must hold for below-marginal cost pricing to occur. But this implies $c < -3$, which violates the

non-negative marginal cost assumption. \square

B Numerical solution methods

B.1 Finite horizon

Our objective is to compute the value function of the finite-horizon dynamic programming problem. Toward that end we have to take into account all feasible population awareness states, given in the matrix $\mathbf{A} \in \mathbb{R}^{T \times 2^T}$. More specifically, each column corresponds to a time period $t \in \{0, \dots, T-1\}$, and each row $r \in \{0, 2^{T-1} - 1\}$ corresponds to a given sequence of sales $\{b_0, b_1, b_2, \dots\}$. Note that the order of adoption decisions matters due to the time-depreciation of awareness in the model. Since there are two feasible events each period (adoption or no adoption), there exist 2^{t-1} population awareness states at time t with $t = 0$ as the initial period.

$$\mathbf{A} = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & \dots & a_{0,T-2} & a_{0,T-1} \\ 0 & a_{1,1} & a_{1,2} & \dots & a_{1,T-2} & a_{1,T-1} \\ 0 & 0 & a_{2,2} & \dots & \vdots & \vdots \\ 0 & 0 & a_{3,2} & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & a_{2^{T-2}-1, T-2} & \vdots \\ 0 & 0 & 0 & \dots & 0 & \vdots \\ 0 & 0 & 0 & \dots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & a_{2^{T-1}-1, T-1} \end{pmatrix}$$

The elements of the matrix are constructed using the law of motion (equation (1)), which yields the following relation:

$$a_{r,t} = \begin{cases} \delta a_{r,t-1} & \text{if } t < 2^{t-1} - 1 \\ \delta(a_{r+2^{t-1}, t-1} + k) & \text{if } t \geq 2^{t-1} - 1. \end{cases} \quad (\text{A1})$$

For example, the elements a_{12} and a_{22} are given by the equations $a_{12} = \delta a_{11}$

and $a_{22} = \delta(a_{01} + k)$. For any given column, the top half of the non-zero elements represent population awareness when an adoption did not occur the preceding period ($b_{t-1} = 0$), while for the bottom an adoption did occur ($b_{t-1} = 1$). For each time period, awareness values that are not attainable are set to 0.

We can now use \mathbf{A} to construct the matrix $\mathbf{Q} \in \mathbb{R}^{T \times 2^T}$ in which an element $q_{rt} = \mathbf{Q}(a_{rt})$ denotes the arrival probability associated with awareness-period pair. Using \mathbf{Q} we can further derive the value function matrix \mathbf{V} and the optimal price matrix \mathbf{P} . Each pair of elements $v_{r,t}$ and $p_{r,t}$ are given by the Bellman equations:

$$v_{r,t} = \max_{p_{r,t}} q_{r,t} \left[(1-F(p_{r,t}))(p_{r,t}-c+\beta v_{r+2^t,t+1}) + F(p_{r,t})\beta v_{r,t+1} \right] + [1-q_{r,t}]\beta v_{r,t+1}, \quad (\text{A2})$$

$$p_{r,t} = \arg \max_{p_{r,t}} q_{r,t} \left[(1-F(p_{r,t}))(p_{r,t}-c+\beta v_{r+2^t,t+1}) + F(p_{r,t})\beta v_{r,t+1} \right] + [1-q_{r,t}]\beta v_{r,t+1}. \quad (\text{A3})$$

We can compute \mathbf{V} and \mathbf{P} using backward induction starting with $v_{r,T-1}$ for $r \in \{0, 2^{T-1} - 1\}$, where $v_{r,T} = 0$ for all r by construction.

B.1.1 Simulations

Having constructed matrices \mathbf{Q} and \mathbf{P} simulations are conducted as follows. For each period t , we draw $\theta_t \sim F(\theta)$ the valuation of consumer t , and $X_t \sim U[0, 1]$. Exactly one of the following four outcomes occurs:

- i. $X_t \leq Q(a_{r,t})$ and $\theta_t \geq p_{r,t}$: A consumer arrives and adopts.
- ii. $X_t \leq Q(a_{r,t})$ and $\theta_t < p_{r,t}$: A consumer arrives and does not adopt.
- iii. $X_t > Q(a_{r,t})$ and $\theta_t \geq p_{r,t}$: A consumer does not arrive, but would have adopted if he arrived.
- iv. $X_t > Q(a_{r,t})$ and $\theta_t < p_{r,t}$: A consumer does not arrive, and would not have adopted if he arrived.

In the price simulations, the first event means moving from position $p_{r,t}$ to position $p_{r+2^{t-1},t+1}$ of the matrix \mathbf{P} , while the three latter events correspond to a horizontal move from $p_{r,t}$ to $p_{r,t+1}$. In Figure 1 adoptions are represented by a triangle, arrivals and non-adoptions by a circle, and non-arrivals by the absence of marks.

B.2 Infinite horizon

In the infinite horizon case, we will approximate the solution by constructing the matrix $\mathbf{V} \in \mathbb{R}^{n \times T}$ and applying backward induction. Since we know that $\lim_{T \rightarrow \infty} V_t(q_i) = V(q_i)$ for $0 < \beta < 1$, convergence is guaranteed (Stokey and Lucas with Prescott, 1989). More specifically, backward induction here is equivalent to implementing value function iteration with an initial guess of $V(q) = 0$ for all q .

$$\mathbf{V} = \begin{pmatrix} V_0(q_0) & V_1(q_0) & \dots & V_{T-1}(q_0) \\ V_0(q_1) & V_1(q_1) & \dots & V_{T-1}(q_1) \\ \vdots & \vdots & \ddots & \vdots \\ V_0(q_{n-1}) & V_1(q_{n-1}) & \dots & V_{T-1}(q_{n-1}) \end{pmatrix}$$

The above matrix is constructed by solving the following Bellman equation backwards.

$$V_t(q_i) = \max_{p_{it}} q_i \left[(1 - F(p_{it})) (p_{it} - c + \beta V_{t+1}(q_{i+1}) + F(p_{it}) \beta V_{t+1}(q_{i-1})) \right] + (1 - q_i) \beta V_{t+1}(q_{i-1}).$$

At the boundary states ($i = 0$ and $i = n - 1$) the value function is naturally handled by the transition rule in Equation 4 (i.e., $V(q_{-1}) := V(q_0)$ and $V(q_n) := V(q_{n-1})$). Let \mathbf{v}_t denote the $t + 1$ 'th column of \mathbf{V} , which corresponds to time t . Our convergence criterion is then given by the Euclidean norm of the difference of the two first value functions:

$$\| \mathbf{v}_0 - \mathbf{v}_1 \|_2 < \epsilon,$$

where ϵ is a number sufficiently close to zero. In our simulations, we set $\epsilon = 10^{-6}$.

C Platform fees

For a flat per-transaction fee τ_t , the first order condition of Equation 3 changes slightly, and the price set by a representative seller is now given by:

$$p_t(\tau_t) - \frac{1 - F(p_t(\tau_t))}{f(p_t(\tau_t))} = c + \tau_t - \beta[V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)]. \quad (\text{A4})$$

Note that we are using a flat fee instead of a percentage one to make exposition simpler. It can be straightforwardly verified that under dynamic taxing, the results are identical for percentage fees.

The platform's maximizes its expected profit, and the Bellman equation is given by:

$$W_t(a_t) = \max_{\tau_t} Q(a_t) \left[(1 - F(p_t(\tau_t)))(\tau_t + \beta W_{t+1}(a_{t+1}^1)) + F(p_t(\tau_t))\beta W_{t+1}(a_{t+1}^0) \right] + [1 - Q(a_t)]\beta W_{t+1}(a_{t+1}^0), \quad (\text{A5})$$

where the interpretation is equivalent to that of Equation 2. The optimal fee τ_t is then given by the first-order condition:

$$\tau_t - \frac{1 - F(p(\tau_t))}{f(p(\tau_t))} dp/d\tau_t = -\beta[W_{t+1}(a_{t+1}^1) - W_{t+1}(a_{t+1}^0)]. \quad (\text{A6})$$

The above expression conveys the insight that the interaction between future adoptions and awareness lessens platform fees, as the concern for future platform profits is taken into account. The platform takes the reaction of the seller into account through the term $dp/d\tau > 0$.

C.1 Two-period analysis

When consumers valuations are uniformly distributed ($\theta \sim U[0, 1]$) the platform fee is:

$$\tau_t^* = \frac{1}{2}[1 - c + \beta(\Delta V_{t+1} - \Delta W_{t+1})], \quad (\text{A7})$$

where $\Delta V_{t+1} = V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)$ and $\Delta W_{t+1} = W_{t+1}(a_{t+1}^1) - W_{t+1}(a_{t+1}^0)$. The sales fee is, ceteris paribus, increasing in ΔV_{t+1} . This is because the final price is decreasing in ΔV_{t+1} for a given τ , and the short run incentive for the platform is to offset the seller's price reduction by increasing τ . Inserting the platform fee from Equation A7 into the seller's problem yields the price

$$p(\tau^*) = \frac{1}{4}[3 + c - \beta(\Delta V_{t+1} + \Delta W_{t+1})],$$

and a price margin:

$$p(\tau_t^*) - \tau_t^* - c = \frac{1}{4}[1 - c - 3\beta\Delta V_{t+1} + \beta\Delta W_{t+1}].$$

In the two-period case ($t \in \{0, 1\}$) expected platform and seller profits at $t = 1$ are:

$$W_1(a_1) = Q(a_1)(1 - p_1(\tau_1^*))\tau_1^* = Q(a_1)\frac{1}{8}(1 - c)^2,$$

$$V_1(a_1) = Q(a_1)(1 - p_1(\tau_1^*))(p(\tau_1^*) - c - \tau_1^*) = Q(a_1)\frac{(1 - c)^2}{16}.$$

respectively. The fee in $t = 0$ is then:

$$\tau_0^* = \frac{1}{2}(1 - c)\left[1 + \frac{\beta\Delta Q(1 - c)}{16}\right],$$

and price charged by the seller:

$$p_0(\tau_0^*) = \frac{3 + c}{4} - \frac{3}{64}\beta\Delta Q(1 - c)^2. \quad (\text{A8})$$

Note that the price is higher compared to no-fees case, which is due to the inefficiencies caused by the well-known double marginalization effect (Spengler, 1950). Moreover, the price is less sensitive to differences in future arrival probabilities compared to vertical integration, as shown by the following result.

Proposition 4. *For $T = 2$, the initial period price is less sensitive to changes in the difference in future arrival probabilities under vertical sepa-*

ration than under vertical integration.

Proof. The derivatives of the price in with respect to ΔQ are under vertical integration ($\tau = 0$) and under vertical separation ($\tau = \tau^*(\Delta Q)$) are:

$$\frac{dp_0(0)}{d\Delta Q} = -\frac{1}{8}\beta(1-c)^2 \quad \text{and} \quad \frac{dp_0(\tau^*)}{d\Delta Q} = -\frac{3}{64}\beta(1-c)^2$$

respectively. The inequality

$$\left| \frac{dp_0(\tau^*)}{d\Delta Q} \right| < \left| \frac{dp_0(0)}{d\Delta Q} \right|$$

simplifies to $3 < 8$. □

For a given $\tau > 0$ the price is therefore less sensitive to the difference in continuation values since future profits are split between the seller and the platform. The platform's continuation value also depends positively on the seller's continuation value and enters the price indirectly through the fee τ , which makes the price more sensitive to changes in future profits. Nonetheless, the effect of a change in ΔQ becomes smaller under vertical separation. This makes intuitive sense, as vertical separation decreases total profits, which in turn decreases the differences in potential future total profits for a given ΔQ .