An ERP for the Sensex

- Inputs for the computation
  - Sensex on 9/5/07 = 15446
  - Dividend yield on index = 3.05%
  - Expected growth rate - next 5 years = 14%
  - Growth rate beyond year 5 = 6.76% (set equal to riskfree rate)

- Solving for the expected return:

\[
15446 = \frac{537.06}{(1 + r)} + \frac{612.25}{(1 + r)^2} + \frac{697.86}{(1 + r)^3} + \frac{795.67}{(1 + r)^4} + \frac{907.07}{(1 + r)^5} + \frac{907.07(1.0676)}{(r - .0676)(1 + r)^5}
\]

- Expected return on stocks = 11.18%
- Implied equity risk premium for India = 11.18% - 6.76% = 4.42%

Aswath Damodaran
Changing Country Risk: Brazil CRP & Total ERP from 2000 to 2020

Figure 15: US ERP and Brazil Implied CRP
The evolution of Emerging Market Risk

Aswath Damodaran

<table>
<thead>
<tr>
<th>Start of year</th>
<th>PBV (Developed)</th>
<th>PBV (Emerging)</th>
<th>ROE (Developed)</th>
<th>ROE (Emerging)</th>
<th>US T.Bond Rate</th>
<th>Growth Rate (Developed)</th>
<th>Growth Rate (Emerging)</th>
<th>Cost of Equity (Developed)</th>
<th>Cost of Equity (Emerging)</th>
<th>Differential</th>
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<tr>
<td>2004</td>
<td>2.00</td>
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<td>4.25%</td>
<td>3.75%</td>
<td>4.75%</td>
<td>7.28%</td>
<td>10.55%</td>
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<td>11.93%</td>
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<td>3.72%</td>
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<td>3.89%</td>
<td>4.89%</td>
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<td>1.67</td>
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<td>12.88%</td>
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<td>4.20%</td>
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<td>4.02%</td>
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<td>4.52%</td>
<td>10.30%</td>
<td>12.47%</td>
<td>2.17%</td>
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<tr>
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<td>1.20</td>
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<td>8.48%</td>
<td>11.02%</td>
<td>2.21%</td>
<td>1.71%</td>
<td>2.71%</td>
<td>7.35%</td>
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<td>1.56%</td>
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<td>3.34%</td>
<td>4.34%</td>
<td>7.51%</td>
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<td>1.76%</td>
<td>1.26%</td>
<td>2.26%</td>
<td>6.01%</td>
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<tr>
<td>2014</td>
<td>1.95</td>
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<td>9.64%</td>
<td>3.04%</td>
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<td>3.54%</td>
<td>5.99%</td>
<td>7.61%</td>
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<tr>
<td>2015</td>
<td>1.88</td>
<td>1.56</td>
<td>9.69%</td>
<td>9.75%</td>
<td>2.17%</td>
<td>1.67%</td>
<td>2.67%</td>
<td>5.94%</td>
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<tr>
<td>2016</td>
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<td>9.24%</td>
<td>10.16%</td>
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<td>1.77%</td>
<td>2.77%</td>
<td>5.52%</td>
<td>7.42%</td>
<td>1.89%</td>
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<td>2017</td>
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<td>8.71%</td>
<td>9.53%</td>
<td>2.68%</td>
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<tr>
<td>2018</td>
<td>1.98</td>
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<td>11.23%</td>
<td>11.36%</td>
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<td>3.18%</td>
<td>6.75%</td>
<td>8.11%</td>
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<tr>
<td>2019</td>
<td>1.64</td>
<td>1.31</td>
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<td>11.35%</td>
<td>2.68%</td>
<td>2.18%</td>
<td>3.18%</td>
<td>8.22%</td>
<td>9.42%</td>
<td>1.19%</td>
</tr>
</tbody>
</table>
Discount Rates: III

Relative Risk Measures

Aswath Damodaran
The CAPM Beta: The Most Used (and Misused) Risk Measure

- The standard procedure for estimating betas is to regress stock returns \( R_j \) against market returns \( R_m \) -
  \[ R_j = a + b R_m \]
  where \( a \) is the intercept and \( b \) is the slope of the regression.

- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.

- This beta has three problems:
  - It has high standard error
  - It reflects the firm’s business mix over the period of the regression, not the current mix
  - It reflects the firm’s average financial leverage over the period rather than the current leverage.
Unreliable, when it looks bad..
Or when it looks good..
During 2019 and 2020, GME was an extraordinarily volatile stock, as short sellers and long only investors fought out a battle.

Aswath Damodaran
And subject to game playing
Measuring Relative Risk: You don’t like betas or modern portfolio theory? No problem.

- Do you believe that the marginal investors who price risk are diversified?
  - Yes: Do you believe in price-based risk measures?
    - Yes: The CAPM
    - No: Accounting Betas
  - No: Cost of Debt based models

- Do you believe in price-based risk measures?
  - Yes: Relative Price Volatility
    - Yes: Proxy Models
      - Yes: The CAPM Plus
        - Yes: Implied Cost of Capital
        - No: Accounting Ratio based models
    - No: Relative Earnings volatility
  - No: Multi-factor Models
Don’t like the diversified investor focus, but okay with price-based measures

1. Relative Standard Deviation
   - Relative Volatility = Std dev of Stock / Average Std dev across all stocks
   - Captures all risk, rather than just market risk

2. Proxy Models
   - Look at historical returns on all stocks and look for variables that explain differences in returns.
   - You are, in effect, running multiple regressions with returns on individual stocks as the dependent variable and fundamentals about these stocks as independent variables.
   - This approach started with market cap (the small cap effect) and over the last two decades has added other variables (momentum, liquidity etc.)

3. CAPM Plus Models
   - Start with the traditional CAPM (Rf + Beta (ERP)) and then add other premiums for proxies.
Don’t like the price-based approach..

1. **Accounting risk measures**: To the extent that you don’t trust market-priced based measures of risk, you could compute relative risk measures based on
   - **Accounting earnings volatility**: Compute an accounting beta or relative volatility
   - **Balance sheet ratios**: You could compute a risk score based upon accounting ratios like debt ratios or cash holdings (akin to default risk scores like the Z score)

2. **Qualitative Risk Models**: In these models, risk assessments are based at least partially on qualitative factors (quality of management).

3. **Debt based measures**: You can estimate a cost of equity, based upon an observable costs of debt for the company.
   - Cost of equity = Cost of debt * Scaling factor
   - The scaling factor can be computed from implied volatilities.

Aswath Damodaran
Determinants of Betas & Relative Risk

Beta of Firm (Unlevered Beta)

Nature of product or service offered by company:
Other things remaining equal, the more discretionary the product or service, the higher the beta.

Implications
1. Cyclical companies should have higher betas than non-cyclical companies.
2. Luxury goods firms should have higher betas than basic goods.
3. High priced goods/service firms should have higher betas than low prices goods/services firms.
4. Growth firms should have higher betas.

Operating Leverage (Fixed Costs as percent of total costs):
Other things remaining equal the greater the proportion of the costs that are fixed, the higher the beta of the company.

Implications
1. Firms with high infrastructure needs and rigid cost structures should have higher betas than firms with flexible cost structures.
2. Smaller firms should have higher betas than larger firms.
3. Young firms should have higher betas than more mature firms.

Financial Leverage:
Other things remaining equal, the greater the proportion of capital that a firm raises from debt, the higher its equity beta will be.

Implications
Highly levered firms should have higher betas than firms with less debt.
Equity Beta (Levered beta) = Unlever Beta (1 + (1- t) (Debt/Equity Ratio))
In a perfect world... we would estimate the beta of a firm by doing the following:

1. Start with the beta of the business that the firm is in.
2. Adjust the business beta for the operating leverage of the firm to arrive at the unlevered beta for the firm.
3. Use the financial leverage of the firm to estimate the equity beta for the firm:
   \[ \text{Levered Beta} = \text{Unlevered Beta} \times (1 + (1 - \text{tax rate}) \times \frac{\text{Debt}}{\text{Equity}}) \]
Adjusting for operating leverage...

- Within any business, firms with lower fixed costs (as a percentage of total costs) should have lower unlevered betas. If you can compute fixed and variable costs for each firm in a sector, you can break down the unlevered beta into business and operating leverage components.
  - Unlevered beta = Pure business beta * (1 + (Fixed costs/ Variable costs))
- The biggest problem with doing this is informational. It is difficult to get information on fixed and variable costs for individual firms.
- In practice, we tend to assume that the operating leverage of firms within a business are similar and use the same unlevered beta for every firm.
Adjusting for financial leverage...

- **Conventional approach:** If we assume that debt carries no market risk (has a beta of zero), the beta of equity alone can be written as a function of the unlevered beta and the debt-equity ratio

\[ \beta_L = \beta_u (1+ ((1-t)D/E)) \]

In some versions, the tax effect is ignored and there is no (1-t) in the equation.

- **Debt Adjusted Approach:** If beta carries market risk and you can estimate the beta of debt, you can estimate the levered beta as follows:

\[ \beta_L = \beta_u (1+ ((1-t)D/E)) - \beta_{debt} (1-t) (D/E) \]

While the latter is more realistic, estimating betas for debt can be difficult to do.

Asthath Damodaran
**Bottom-up Betas**

**Step 1:** Find the business or businesses that your firm operates in.

**Step 2:** Find publicly traded firms in each of these businesses and obtain their regression betas. Compute the simple average across these regression betas to arrive at an average beta for these publicly traded firms. Unlever this average beta using the average debt to equity ratio across the publicly traded firms in the sample. Unlevered beta for business = Average beta across publicly traded firms/ (1 + (1- t) (Average D/E ratio across firms))

**Possible Refinements**

If you can, adjust this beta for differences between your firm and the comparable firms on operating leverage and product characteristics.

While revenues or operating income are often used as weights, it is better to try to estimate the value of each business.

If you expect the business mix of your firm to change over time, you can change the weights on a year-to-year basis.

If you expect your debt to equity ratio to change over time, the levered beta will change over time.

**Step 3:** Estimate how much value your firm derives from each of the different businesses it is in.

**Step 4:** Compute a weighted average of the unlevered betas of the different businesses (from step 2) using the weights from step 3. Bottom-up Unlevered beta for your firm = Weighted average of the unlevered betas of the individual business

**Step 5:** Compute a levered beta (equity beta) for your firm, using the market debt to equity ratio for your firm. Levered bottom-up beta = Unlevered beta (1+ (1-t) (Debt/Equity))

Aswath Damodaran
Why bottom-up betas?

- The standard error in a bottom-up beta will be significantly lower than the standard error in a single regression beta. Roughly speaking, the standard error of a bottom-up beta estimate can be written as follows:

\[
\text{Std error of bottom-up beta} = \frac{\text{Average Std Error across Betas}}{\sqrt{\text{Number of firms in sample}}}
\]

- The bottom-up beta can be adjusted to reflect changes in the firm’s business mix and financial leverage. Regression betas reflect the past.

- You can estimate bottom-up betas even when you do not have historical stock prices. This is the case with initial public offerings, private businesses or divisions of companies.
### Estimating Bottom Up Betas & Costs of Equity: Vale

<table>
<thead>
<tr>
<th>Business</th>
<th>Sample</th>
<th>Sample size</th>
<th>Unlevered beta of business</th>
<th>Revenues</th>
<th>Peer Group EV/Sales</th>
<th>Value of Business</th>
<th>Proportion of Vale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metals &amp; Mining</td>
<td>Global firms in metals &amp; mining, Market cap&gt;$1 billion</td>
<td>48</td>
<td>0.86</td>
<td>$9,013</td>
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<tr>
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<tr>
<td><strong>Vale Operations</strong></td>
<td></td>
<td><strong>0.8440</strong></td>
<td></td>
<td><strong>$47,151</strong></td>
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<td><strong>$106,543</strong></td>
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<table>
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<tr>
<th>Business</th>
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<th>D/E ratio</th>
<th>Levered beta</th>
<th>Risk free rate</th>
<th>ERP</th>
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<td>Metals &amp; Mining</td>
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<td>1.1503</td>
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<td>11.23%</td>
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### Embraer’s Bottom-up Beta

<table>
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<th>Business</th>
<th>Unlevered Beta</th>
<th>D/E Ratio</th>
<th>Levered beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerospace</td>
<td>0.95</td>
<td>18.95%</td>
<td>1.07</td>
</tr>
</tbody>
</table>

- Levered Beta = Unlevered Beta \( \times (1 + (1 - \text{tax rate}) \times \text{D/E Ratio}) \)
  
  \[
  = 0.95 \times (1 + (1 - 0.34) \times 0.1895) = 1.07
  \]

- Can an unlevered beta estimated using U.S. and European aerospace companies be used to estimate the beta for a Brazilian aerospace company?
  
  a. Yes
  
  b. No

  What concerns would you have in making this assumption?
Gross Debt versus Net Debt Approaches

- Analysts in Europe and Latin America often take the difference between debt and cash (net debt) when computing debt ratios and arrive at very different values.

- For Embraer, using the gross debt ratio
  - Gross D/E Ratio for Embraer = 1953/11,042 = 18.95%
  - Levered Beta using Gross Debt ratio = 1.07

- Using the net debt ratio, we get
  - Net Debt Ratio for Embraer = (Debt - Cash)/ Market value of Equity
    = (1953-2320)/ 11,042 = -3.32%
  - Levered Beta using Net Debt Ratio = 0.95 (1 + (1-.34) (.0332)) = 0.93

- The cost of Equity using net debt levered beta for Embraer will be much lower than with the gross debt approach. The cost of capital for Embraer will even out since the debt ratio used in the cost of capital equation will now be a net debt ratio rather than a gross debt ratio.
The Cost of Equity: A Recap

Cost of Equity = Riskfree Rate + Beta * (Risk Premium)

Preferably, a bottom-up beta, based upon other firms in the business, and firm’s own financial leverage.

Has to be in the same currency as cash flows, and defined in same terms (real or nominal) as the cash flows.

**Historical Premium**
1. Mature Equity Market Premium: Average premium earned by stocks over T.Bonds in U.S.
2. Country risk premium = Country Default Spread* (σEquity/σCountry bond)

**Implied Premium**
Based on how equity market is priced today and a simple valuation model.

Aswath Damodaran