CHAPTER 28

The Option to Delay and Valuation Implications

In traditional investment analysis, a project or new investment should be accepted only if the returns on the project exceed the hurdle rate; in the context of cash flows and discount rates, this translates into investing in projects with positive net present values (NPVs). The limitation of this view of the world, which analyzes projects on the basis of expected cash flows and discount rates, is that it fails to consider fully the options that are usually associated with many investments.

This chapter will consider an option that is embedded in many projects, namely the option to wait and take the project in a later period. Why might a firm want to do this? If the present value of the cash flows on the project are volatile and can change over time, a project with a negative net present value today may have a positive net present value in the future. Furthermore, a firm may gain by waiting on a project even after a project has a positive net present value, because the option has a time premium that exceeds the cash flows that can be generated in the next period by accepting the project. This option is most valuable in projects where a firm has the exclusive right to invest in a project and becomes less valuable as the barriers to entry decline.

There are three cases where the option to delay can make a difference when valuing a firm. The first is undeveloped land in the hands of real estate investor or company. The choice of when to develop rests in the hands of the owner and presumably development will occur when real estate values increase. The second is a firm that owns a patent or patents. Since a patent provides a firm with the exclusive rights to produce the patented product or service, it can and should be valued as an option. The third is a natural resource company that has undeveloped reserves that it can choose to develop at a time of its choosing—presumably when the price of the resource is high.

THE OPTION TO DELAY A PROJECT

Projects are typically analyzed based on their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. Expected cash flows and discount rates change over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no
special advantages over their competitors in taking projects, the fact that net present values can be positive in the future may not be significant. In an environment in which a project can be taken by only one firm because of legal restrictions or other barriers to entry to competitors, however, the changes in the project's value over time give it the characteristics of a call option.

**Payoff on the Option to Delay**

Assume that a project requires an initial up-front investment of X, and that the present value of expected cash inflows from investing in the project, computed today, is V. The net present value of this project is the difference between the two:

$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years, and that the present value of the cash inflows may change over that time because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

- If \( V > X \) Invest in the project: Project has positive net present value.
- \( V < X \) Do not invest in the project: Project has negative net present value.

If the firm does not invest in the project over its life, it incurs no additional cash flows, though it will lose what it invested to get exclusive rights to the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 28.1, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project.

![FIGURE 28.1 The Option to Delay a Project](image-url)
Note that this payoff diagram is that of a call option—the underlying asset is the project, the strike price of the option is the investment needed to take the project, and the life of the option is the period for which the firm has rights to the project. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Inputs for Valuing the Option to Delay

The inputs needed to apply option pricing theory to valuing the option to delay are the same as those needed for any option. We need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate, and the equivalent of the dividend yield.

Value of the Underlying Asset

In the case of product options, the underlying asset is the project to which the firm has exclusive rights. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment. This present value can be obtained by doing a standard investment analysis. There is likely to be a substantial amount of error in the cash flow estimates and the present value, however. Rather than being viewed as a problem, this uncertainty should be viewed as the reason the project delay option has value. If the expected cash flows on the project were known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option.

Variance in the Value of the Asset

As noted in the prior section, there is likely to be considerable uncertainty associated with the cash flow estimates and the present value that measures the value of the project now. This is partly because the potential market for the product may be unknown, and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

1. If we have invested in similar projects in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a consumer product company like Gillette might estimate the variance associated with introducing a new blade for its razors.

2. We can assign probabilities to various market scenarios, estimate cash flows and a present value under each scenario, and then calculate the variance across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis—the size of the market, the market share, and the profit margin, for instance—and simulations used to estimate the variance in the present values that emerge. This approach tends to work best when there are only one or two sources of significant uncertainty about future cash flows.

---

1In practical terms, the probability distributions for inputs like market size and market share can often be obtained from market testing.
3. We can use the variance in the value of firms involved in the same business (as the project being considered) as an estimate of the variance. Thus, the average variance in the value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows; the higher the variance, the higher the value of the project delay option. Thus, the value of an option to invest in a project in a stable business will be less than the value of one in an environment where technology, competition, and markets are all changing rapidly.

**Exercise Price on Option**  The option to delay a project is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this initial investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the investment is reflected in the present value of cash flows on the project.

**Expiration of the Option and the Riskless Rate** The project delay option expires when the rights to the project lapse. Investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While expiration dates can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), they become far more difficult to obtain when the right is less clearly defined. If, for instance, a firm has a competitive advantage on a product or project, the option life can be defined as the expected period over which the advantage can be sustained.

**Cost of Delay** Chapter 5 noted that an American option generally will not be exercised prior to expiration. When you have the exclusive rights to a project, though, and the net present value turns positive, you would not expect the owner of the rights to wait until the rights expire to exercise the option (invest in the project). Note that there is a cost to delaying investing in a project, once the net present value turns positive. If you wait an additional period, you may gain if the variance pushes value higher but you also lose one period of protection against competition. You have to consider this cost when analyzing the option and there are two ways of estimating it:

1. Since the project rights expire after a fixed period, and excess profits (which are the source of positive present value) are assumed to disappear after that time as new competitors emerge, each year of delay translates into one less year of value-creating cash flows. If the cash flows are evenly distributed over time, and the life of the patent is \( n \) years, the cost of delay can be written as:

\[
\text{Annual cost of delay} = \frac{1}{n}
\]
Thus, if the project rights are for 20 years, the annual cost of delay works out to $\frac{1}{20}$ or 5% a year. Note, though, that this cost of delay rises each year, to $\frac{1}{19}$ in year 2, $\frac{1}{18}$ in year 3, and so on, making the cost of delaying exercise larger over time.

2. If the cash flows are uneven, the cost of delay can be more generally defined in terms of the cash flow that can be expected to occur over the next period as a percent of the present value today:

$$\text{Cost of delay} = \frac{\text{Cash flow next period}}{\text{Present value now}}$$

In either case, the likelihood that a firm will delay investing in a project is higher early in the exclusive rights period rather than later and will increase as the loss in present value from waiting a period increases.

ILLUSTRATION 28.1: Valuing the Option to Delay a Project

Assume that you are interested in acquiring the exclusive rights to market a new product that will make it easier for people to access their e-mail on the road. If you do acquire the rights to the product, you estimate that it will cost you $50 million up-front to set up the infrastructure needed to provide the service. Based on your current projections, you believe that the service will generate only $10 million in after-tax cash flows each year. In addition, you expect to operate without serious competition for the next five years.

From a static standpoint, the net present value of this project can be computed by taking the present value of the expected cash flows over the next five years. Assuming a discount rate of 15% (based on the riskiness of this project), we obtain the following net present value for the project:

$$\text{NPV of project} = -50,000,000 + 10,000,000 \times (\text{PV of annuity, 15\%, 5 years})$$
$$= -50,000,000 + 33,500,000 = -16,500,000$$

This project has a negative net present value.

The biggest source of uncertainty about this project is the number of people who will be interested in the product. While current market tests indicate that you will capture a relatively small number of business travelers as your customers, they also indicate the possibility that the potential market could be much larger. In fact, a simulation of the project’s cash flows yields a standard deviation of 42% in the present value of the cash flows, with an expected value of $33.5 million.

To value the exclusive rights to this project, we first define the inputs to the option pricing model:

- Value of underlying asset (S) = PV of cash flows from product if introduced now = $33.5 million
- Strike price (K) = Initial investment needed to introduce the product = $50 million
- Variance in underlying asset’s value = 0.42^2 = 0.1764
- Time to expiration = Period of exclusive rights to product = 5 years
- Dividend yield = 1/Life of the patent = 1/5 = 0.20

Assume that the five-year riskless rate is 5%. The value of the option can be estimated as follows:

$$\text{Call value} = 33.5 \exp(-0.05(5)) \times 0.2250 - 50.0 \exp(-0.05(5)) \times 0.0451 = 1.019 \text{ million}$$

The rights to this product, which has a negative net present value if introduced today, is $1.019 million. Note, though, as measured by N(d1) and N(d2), the likelihood is low that this project will become viable before expiration.
delay.xls: This spreadsheet allows you to estimate the value of an option to delay an investment.

ARBITRAGE POSSIBILITIES AND OPTION PRICING MODELS

The discussion of option pricing models in Chapter 5 noted that they are based on two powerful constructs—the idea of replicating portfolios and arbitrage. Models such as the Black-Scholes and binomial assume that you can create a replicating portfolio, using the underlying asset and riskless borrowing or lending, that has cash flows identical to those on an option. Furthermore, these models assume that since investors can then create riskless positions by buying the option and selling the replicating portfolio, they have to sell for the same price. If they do not, investors should be able to create riskless positions and walk away with guaranteed profits—the essence of arbitrage. This is why the interest rate used in option pricing models is the riskless rate.

With listed options on traded stocks or assets, arbitrage is clearly feasible, at least for some investors. With options on nontraded assets, it is almost impossible to trade the replicating portfolio, although you can create it on paper. In Illustration 28.1, for instance, you would need to buy 0.225 units (the option delta) of the underlying project (a nontraded asset) to create a portfolio that replicates the call option.

There are some who argue that the impossibility of arbitrage makes it inappropriate to use option pricing models to value real options, whereas others try to adjust for this limitation by using an interest rate higher than the riskless rate in the option pricing model. We do not think that either of these responses is appropriate. Note that while you cannot trade on the replicating portfolios in many real options, you still can create them on paper (as we did in Illustration 28.1) and value the options. The difficulties in creating arbitrage positions may result in prices that deviate by a large amounts from this value, but that is an argument for using real option pricing models and not for avoiding them. Increasing the riskless rate to reflect the higher risk associated with real options may seem like an obvious fix, but doing this will only make call options (such as the one valued in Illustration 28.1) more valuable, not less.

If you want to be more conservative in your estimate of value for real options to reflect the difficulty of arbitrage, you have two choices. One is to use a higher discount rate in computing the present value of the cash flows that you would expect to make from investing in the project today, thus lowering the value of the underlying asset (S) in the model. In Illustration 28.1, using a 20 percent discount rate rather than a 15 percent rate would result in a present value of $29.1 million, which would replace the $33.5 million as S in the model. The other choice is to value the option and then apply an illiquidity discount to it (similar to the one we used in valuing private companies) because you cannot trade it easily.
Problems in Valuing the Option to Delay

While it is quite clear that the option to delay is embedded in many projects, several problems are associated with the use of option pricing models to value these options. First, the underlying asset in this option, which is the project, is not traded, making it difficult to estimate its value and variance. The value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since we are attempting the estimate a variance in project value over time.

Second, the behavior of prices over time may not conform to the price path assumed by the option pricing models. In particular, the assumption that value follows a diffusion process, and that the variance in value remains unchanged over time, may be difficult to justify in the context of a project. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, there may be no specific period for which the firm has rights to the project. Unlike the case of a patent, for instance, in which the firm has exclusive rights to produce the patented product for a specified period, the firm’s rights often are less clearly defined, in terms of both exclusivity and time. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a project for a period of time. An example would be a company with strong brand name recognition in retailing or consumer products. The rights are not legal restrictions, however, and will erode over time. In such cases, the expected life of the project itself is uncertain and only an estimate. In the valuation of the rights to the product in the previous section a life of five years for the option was used, but competitors could in fact enter sooner than anticipated. Alternatively, the barriers to entry may turn out to be greater than expected, and allow the firm to earn excess returns for longer than five years. Ironically, uncertainty about the expected life of the option can increase the variance in present value, and through it, the expected value of the rights to the project.

Implications and Extensions of Delay Options

Several interesting implications emerge from the analysis of the option to delay a project as an option. First, a project may have a negative net present value currently based on expected cash flows, but the rights to it may still be valuable because of the option characteristics.

Second, a project may have a positive net present value but still not be accepted right away. This can happen because the firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option that is in the money. A firm is more likely to wait if it has the rights to the project for a long time, and the variance in project inflows is high. To illustrate, assume a firm has the patent rights to produce a new type of disk drive for computer systems and building a new plant will yield a positive net present value today. If the technology for manufacturing the disk drive is in flux, however, the firm may delay investing in the project in the hopes that the improved technology will increase the expected cash flows and consequently the value of the project. It has to weigh this benefit against the cost of delaying the project, which will be the cash flows that will be forsaken by not investing in it.
Third, factors that can make a project less attractive in a static analysis can actually make the rights to the project more valuable. As an example, consider the effect of uncertainty about the size of the potential market and the magnitude of excess returns. In a static analysis, increasing this uncertainty increases the riskiness of the project and may make it less attractive. When the project is viewed as an option, an increase in the uncertainty may actually make the option more valuable, not less. The chapter will consider two cases, product patents and natural resource reserves, where the project delay option allows value to be estimated more precisely.

**Option Pricing Models**

Once you have identified the option to delay a project as a call option and identified the inputs needed to value the option, it may seem like a trivial task to actually value the option. There are, however, some serious estimation issues that we have to deal with in valuing these options. Chapter 5 noted that while the more general model for valuing options is the binomial model, many practitioners use the Black-Scholes model, which makes far more restrictive assumptions about price processes and early exercise to value options. With listed options on traded assets, you can do this at fairly low cost. With real options, there can be a substantial cost to this practice for the following reasons:

- Unlike listed options, real options tend to be exercised early, if they are in the money. While there are ways in which the Black-Scholes model can be adjusted to allow for this early exercise, the binomial model allows for much more flexibility.
- The binomial option pricing model allows for a much wider range of price processes for the underlying asset than the Black-Scholes model, which assumes that prices are not only continuous but log-normally distributed. With real options, where the present value of the cash flows is often equivalent to the price, the assumptions of nonnormality and continuous distributions may be difficult to sustain.

The biggest problem with the binomial model is that the prices at each node of the binomial tree have to be estimated. As the number of periods expands, this will become more and more difficult to do. You can, however, use the variance estimate in the Black-Scholes to come up with measures of the magnitude of the up and down movements, which can be used to obtain the binomial tree.

Having made a case for the binomial model, you may find it surprising that we use the Black-Scholes model to value any real options. We do so not only because the model is more compact and elegant to present, but because we believe that it will provide a lower bound on the value in most cases. To provide a frame of reference, we will present the values that we would have obtained using a binomial model in each case.

**From Black-Scholes to Binomial**

It is a fairly simple exercise to convert the inputs to the Black-Scholes model into a binomial model. To make the adjustment, you have to assume a multiplicative binomial process, where the magnitude of the jumps, in percent terms, remains unchanged from period to period. If you assume symmetric
probabilities, the up (u) and down (d) movements can be estimated as a function of the annualized variance in the price process and how many periods you decide to break each year into (t).

\[
\begin{align*}
    u &= \exp \left( \sigma \sqrt{dt} + \left( r + \frac{\sigma^2}{2} \right) dt \right) \\
    d &= \exp \left( -\sigma \sqrt{dt} + \left( r + \frac{\sigma^2}{2} \right) dt \right)
\end{align*}
\]

where \( dt = 1/\text{Number of periods each year} \)

To illustrate, consider the project delay option valued in Illustration 28.1. The standard deviation in the value was assumed to be 42 percent, the risk-free rate was 5\%, and the dividend yield was 20 \%. To convert the inputs into a binomial model, assume that each year is a time period and estimate the up and down movements as follows:

\[
\begin{align*}
    u &= \exp \left( 0.42 \sqrt{0.05} \left( 0.05 - 0.20 \right) \sqrt{t} \right) = 1.1994 \\
    d &= \exp \left( -0.42 \sqrt{0.05} \left( 0.05 - 0.20 \right) \sqrt{t} \right) = 0.5178
\end{align*}
\]

The value today is $33.5 million. To estimate the end values for the first branch:

Value with up movement = $33.5(1.1994) = $40.179 million
Value with down movement = $33.5(0.5178) = $17.345 million

You could use these values then to get the three potential values at the second branch. Note that the value of $17.345 million growing at 19.94 percent is exactly equal to the value of $40.179 million dropping by 48.22 percent. The binomial tree for the five periods is shown in Figure 28.2.

You could estimate the value of the option from this binomial tree to be $1.02 million, slightly higher than the estimate obtained from the Black-Scholes model of $1.019 million. The differences will narrow as the option becomes more in-the-money and you shorten the time periods you use in the binomial model.

**VALUING A PATENT**

A number of firms, especially in the technology and pharmaceutical sectors, can patent products or services. A product patent provides a firm with the right to develop and market a product, and thus can be viewed as an option.

**Patents as Call Options**

The firm will develop a patent only if the present value of the expected cash flows from the product sales exceed the cost of development, as shown in Figure 28.3.
this does not occur, the firm can shelve the patent and not incur any further costs. If $I$ is the present value of the costs of commercially developing the patent and $V$ is the present value of the expected cash flows from development, then:

$$\text{Payoff from owning a product patent} = \begin{cases} V - I & \text{if } V > I \\ 0 & \text{if } V \leq I \end{cases}$$

Thus a product patent can be viewed as a call option, where the product is the underlying asset.

Biogen is a biotechnology firm with a patent on a drug called Avonex, which has received FDA approval for use in treating multiple sclerosis (MS). Assume you are trying to value the patent and that you have the following estimates for use in the option pricing model:

- An internal analysis of the financial viability of the drug today, based on the potential market and the price that the firm can expect to charge for the drug, yields a present value of cash flows of $3.422 billion prior to considering the initial development cost.
- The initial cost of developing the drug for commercial use is estimated to be $2.875 billion, if the drug is introduced today.
- The firm has the patent on the drug for the next 17 years, and the current long-term Treasury bond rate is 6.7%.
- The average variance in firm value for publicly traded biotechnology firms is 0.224.

We assume that the potential for excess returns exists only during the patent life, and that competition will eliminate excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-protected returns. (For the initial analysis, the cost of delay will be \( \frac{1}{17} \), next year it will be \( \frac{1}{16} \), the year after \( \frac{1}{15} \), and so on.)

Based on these assumptions, we obtain the following inputs to the option pricing model.

- Present value of cash flows from introducing the drug now = \( S = $3.422 \) billion
- Initial cost of developing drug for commercial use (today) = \( K = $2.875 \) billion
- Patent life = \( t = 17 \) years
- Riskless rate = \( r = 6.7\% \) (17-year Treasury bond rate)
- Variance in expected present values = \( \sigma^2 = 0.224 \)
- Expected cost of delay = \( y = \frac{1}{17} = 5.89\% \)

These yield the following estimates for \( d \) and \( N(d) \):

\[
\begin{align*}
  d_1 &= 1.1362 \quad N(d_1) = 0.8720 \\
  d_2 &= -0.8512 \quad N(d_2) = 0.2076
\end{align*}
\]
Plugging back into the dividend-adjusted Black-Scholes option pricing model, we get:

\[
\text{Value of the patent} = 3,422 \exp(-0.0589)(17)(0.8720) - 2,875 \exp(-0.067)(17)(0.2076) = $907 \text{ million}
\]

To provide a contrast, the net present value of this project is only $547 million:

\[
\text{NPV} = 3,422 \text{ million} - 2,875 \text{ million} = $547 \text{ million}
\]

The time premium of $360 million on this option ($907 - $547) suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time, and make exercise (development) more likely in future years.

To illustrate, we will value the call option, assuming that all of the inputs, other than the patent life, remain unchanged and changing the patent life. For instance, assume that there are 16 years left on the patent. Holding all else constant, the cost of delay increases as a result of the shorter patent life:

\[
\text{Cost of delay} = 1/16
\]

The decline in the present value of cash flows (which is S) and increase in the cost of delay (y) reduce the expected value of the patent. Figure 28.4 graphs the option value and the net present value of the project each year.

Based on this analysis, if nothing changes, you would expect Avonex to be worth more as a commercial product than as a patent if there were less than eight years left on the patent, which would also then be the optimal time to commercially develop the product.

\[
\text{FIGURE 28.4} \quad \text{Patent Value versus Net Present Value}
\]

\[
\text{product.xls: This spreadsheet allows you to estimate the value of a patent.}
\]

\[
\text{Exercise the option here: Convert patent to commercial product.}
\]

\[
\text{Value}
\]

\[
\text{Number of Years Left on Patent}
\]

1With a binomial model, we estimate a value of $915 million for the same option.
Valuing a Firm with Patents

If the patents owned by a firm can be valued as options, how can this estimate be incorporated into firm value? The value of a firm that derives its value primarily from commercial products that emerge from its patents can be written as a function of three variables:

1. The cash flows it derives from patents that it has already converted into commercial products.
2. The value of the patents that it already possesses that have not been commercially developed.
3. The expected value of any patents that the firm can be expected to generate in future periods from new patents that it might obtain as a result of its research.

\[
\text{Value of firm} = \text{Value of commercial products} + \text{Value of existing patents} + (\text{Value of new patents that will be obtained in the future} - \text{Cost of obtaining these patents})
\]

The value of the first component can be estimated using traditional cash flow models. The expected cash flows from existing products can be estimated for their commercial lives and discounted back to the present at the appropriate cost of capital to arrive at the value of these products. The value of the second component can be obtained using the option pricing model described earlier to value each patent.

COMPETITIVE PRESSURES AND OPTION VALUES

The preceding section has taken the view that a firm is protected from competition for the life of the patent. This is generally true only for the patented product or process, but the firm may still face competition from other firms that come up with their own products to serve the same market. More specifically, Biogen can patent Avonex, but Merck or Pfizer can come up with their own drugs to treat multiple sclerosis and compete with Biogen.

What are the implications for the value of the patent as an option? First, the life of the option will no longer be the life of the patent but the lead time that the firm has until a competing product is developed. For instance, if Biogen knows that another pharmaceutical firm is working on a drug to treat MS and where this drug is in the research pipeline (early research or stage in the FDA approval process), it can use its estimate of how long it will take before the drug is approved for use as the life of the option. This will reduce the value of the option and make it more likely that the drug will be commercially developed earlier rather than later.

The presence of these competitive pressures may explain why commercial development is much quicker with some drugs than with others, and why the value of patents is not always going to be greater than a discounted cash flow valuation. Generally speaking, the greater the number of competing products in the research pipeline, the less likely it is that the option pricing model will generate a value that is greater than the traditional discounted cash flow model.
The value of the third component will be based on perceptions of a firm’s research capabilities. In the special case where the expected cost of research and development in future periods is equal to the value of the patents that will be generated by this research, the third component will become zero. In the more general case, firms such as Merck and Pfizer that have a history of generating value from research will derive positive value from this component as well.

How would the estimate of value obtained using this approach contrast with the estimate obtained in a traditional discounted cash flow model? In traditional discounted cash flow valuation, the second and the third components of value are captured in the expected growth rate in cash flows. Firms such as Pfizer are allowed to grow at much higher rates for longer periods because of the technological edge they possess and their research prowess. In contrast, the approach described in this section looks at each patent separately and allows for the option component of value explicitly.

The biggest limitation of the option-based approach is the information that is needed to put it in practice. To value each patent separately, you need access to proprietary information that is usually available only to managers of the firm. In fact, some of the information, such as the expected variance to use in option pricing, may not even be available to insiders and will have to be estimated for each patent separately.

Given these limitations, the real option approach should be used to value small firms with one or two patents and little in terms of established assets. A good example would be Biogen in 1997, which was valued in the preceding section. For firms such as Merck and Pfizer, that have significant assets in place and dozens of patents, discounted cash flow valuation is a more pragmatic choice. Viewing new technology as options provides insight into Cisco’s successful growth strategy over the previous decade. Cisco has been successful at buying firms with nascent and promising technologies (options) and converting them into commercial success (exercising these options).

ILLUSTRATION 28.3: Valuing Biogen as a Firm

In illustration 28.2, the patent that Biogen owns on Avonex was valued as a call option and the estimated value was $907 million. To value Biogen as a firm two other components of value would have to be considered:

1. Biogen had two commercial products (a drug to treat hepatitis B and a drug called Intron) at the time of this valuation that it had licensed to other pharmaceutical firms. The license fees on these products were expected to generate $50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the pretax cost of debt of the licensing firms (7%) was used:

   \[ \text{Present value of license fees} = \frac{50 \times (1 - 1.07^{-12})}{.07} = 397.13 \text{ million} \]

2. Biogen continued to fund research into new products, spending about $100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years
and 5% thereafter. While it was difficult to forecast the specific patents that would emerge from this research, it was assumed that every dollar invested in research would create $1.25 in value in patents (valued using the option pricing model described earlier) for the next 10 years, and break even after that (i.e., generate $1 in patent value for every $1 invested in R&D). There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%. The value of this component was then estimated as follows:

\[
\text{Value of future research} = \sum_{t=1}^{\infty} \frac{(\text{Value of patents}_t - \text{R&D}_t)}{(1 + r)^t}
\]

The following table summarizes the value of patents generated each period and the R&D costs in that period. Note that there is no surplus value created after the tenth year:

<table>
<thead>
<tr>
<th>Year</th>
<th>Value of Patents Generated</th>
<th>R&amp;D Cost</th>
<th>Excess Value</th>
<th>Present Value at 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$150.00</td>
<td>$120.00</td>
<td>$ 30.00</td>
<td>$ 26.09</td>
</tr>
<tr>
<td>2</td>
<td>$180.00</td>
<td>$144.00</td>
<td>$ 36.00</td>
<td>$ 27.22</td>
</tr>
<tr>
<td>3</td>
<td>$216.00</td>
<td>$172.80</td>
<td>$ 43.20</td>
<td>$ 28.40</td>
</tr>
<tr>
<td>4</td>
<td>$259.20</td>
<td>$207.36</td>
<td>$ 51.84</td>
<td>$ 29.64</td>
</tr>
<tr>
<td>5</td>
<td>$311.04</td>
<td>$248.83</td>
<td>$ 62.21</td>
<td>$ 30.93</td>
</tr>
<tr>
<td>6</td>
<td>$373.25</td>
<td>$298.60</td>
<td>$ 74.65</td>
<td>$ 32.27</td>
</tr>
<tr>
<td>7</td>
<td>$447.90</td>
<td>$358.32</td>
<td>$ 89.58</td>
<td>$ 33.68</td>
</tr>
<tr>
<td>8</td>
<td>$537.48</td>
<td>$429.98</td>
<td>$107.50</td>
<td>$ 35.14</td>
</tr>
<tr>
<td>9</td>
<td>$644.97</td>
<td>$515.98</td>
<td>$128.99</td>
<td>$ 36.67</td>
</tr>
<tr>
<td>10</td>
<td>$773.97</td>
<td>$619.17</td>
<td>$154.79</td>
<td>$ 38.26</td>
</tr>
</tbody>
</table>

The total value created by new research is $318.3 million.

The value of Biogen as a firm is the sum of all three components—the present value of cash flows from existing products, the value of Avonex (as an option), and the value created by new research:

\[
\text{Value} = \text{CF: commercial products} + \text{Value: undeveloped patents} + \text{Value: future R&D}
\]

\[
= $397.13 \text{ million} + $907 \text{ million} + $318.30 \text{ million} = $1,622.43 \text{ million}
\]

Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.5 million) to arrive at a value per share:

\[
\text{Value per share} = \frac{$1,622.43 \text{ million}}{35.5} = $45.70
\]

\[\text{To be honest, this is not an estimate based on any significant facts other than Biogen's history of success in coming up with new products. You can obtain an estimate of this number from the return and cost of capital. For instance, if you assume a return on capital of 15 percent and cost of capital of 10 percent in perpetuity, $1 invested would yield the following:}\]

\[
\text{Value of created} = 1 + \frac{(\text{ROC} - \text{Cost of capital})}{\text{Capital invested}} = 1 + \frac{(0.15 - 0.10)}{0.10} = 1.50
\]

\[\text{This discount rate was estimated by looking at the costs of equity of young publicly traded biotechnology firms with little or no revenue from commercial products.}\]
Undeveloped Reserves as Options

In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based on the estimated quantity and the price of the resource. Thus, in a gold mine, the underlying asset is the value of the estimated gold reserves in the mine, based on the price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the estimated reserves and the cost of the development is the profit to the owner of the resource (see Figure 28.5). Defining the cost of development as X, and the estimated value of the resource as V makes the potential payoffs on a natural resource option the following:

\[
\text{Payoff on natural resource investment} = \begin{cases} 
V - X & \text{if } V > X \\ 
0 & \text{if } V \leq X 
\end{cases}
\]

Thus the investment in a natural resource option has a payoff function that resembles a call option.
Inputs for Valuing a Natural Resource Option

To value a natural resource investment as an option, we need to make assumptions about a number of variables:

- **Available reserves of the resource and estimated value if extracted today.** Since the quantity of the reserve is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract. The value of the reserves is then the product of the estimated reserves and the contribution (price of the resource minus variable cost of extraction) per unit of reserve.

- **Estimated cost of developing the resource.** The estimated cost of developing the resource reserve is the exercise price of the option. In an oil reserve, this would be the fixed cost of installing the rigs to extract oil from the reserve. With a mine, it would be the cost associated with making the mine operational. Since oil and mining companies have done this before in a variety of settings, they can use their experience to come up with a reasonable measure of development cost.

- **Time to expiration of the option.** The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that period will be the life of the option. In many offshore oil leases, for instance, the oil tracts are leased to the oil company for a fixed period. The second approach is based on the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a mine inventory of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.

- **Variance in value of the underlying asset.** The variance in the value of the underlying asset is determined by the variability in the price of the resource and the variability in the estimate of available reserves. In the special case where the quantity of the reserve is known with certainty, the variance in the underlying asset’s value will depend entirely on the variance in the price of the natural resource.

- **Cost of delay.** The net production revenue is the annual cash flow that will be generated, once a resource reserve has been developed, as a percentage of the
market value of the reserve. This is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in-the-money (value of the reserves is greater than the cost of developing these reserves), by not developing the reserve the firm is costing itself the net production revenue it could have generated by doing so.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since oil or gold or any other natural resource reserve cannot be developed instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to reduce the value of the developed reserve for the loss of cash flows during the development period. Thus, if there is a one-year lag in development, you can estimate the cash flow you would make over the year as a percent of your reserve value, and discount the current value of the developed reserve at that rate. This is the equivalent of removing the first year's cash flow from your investment analysis and lowering the present value of your cash flows.

ILLUSTRATION 28.4: Valuing an Oil Reserve

Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be $600 million, and the development lag is two years. Exxon has the rights to exploit this reserve for the next 20 years, and the marginal value (price per barrel minus marginal cost per barrel) per barrel of oil is currently $12. Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in oil prices is 0.03.

Given this information, the inputs to the Black-Scholes model can be estimated as follows:

- Current value of the asset \( S \) = Value of the developed reserve discounted back the length of the development lag at the dividend yield = \( 12 \times 50/(1.05)^2 = 544.22 \)
- Exercise price = Cost of developing reserve = $600 million
- Time to expiration on the option = 20 years
- Variance in the value of the underlying asset \( \sigma \) = 0.03
- Riskless rate = 8%
- Dividend yield = Net production revenue/Value of reserve = 5%

Based on these inputs, the Black-Scholes model provides the following call value:

\[
\begin{align*}
    d_1 &= 1.0359 \quad N(d_1) = 0.8498 \\
    d_2 &= 0.2613 \quad N(d_2) = 0.6030 \\
    \text{Call value} &= 544.22 \exp(-0.05/20)(0.8498) - 600 \exp(-0.08/20)(0.6030) = 97.08 \text{ million}
\end{align*}
\]

This oil reserve, though not viable at current prices, is still valuable because of its potential to create value if oil prices go up.9

---

9The following is a simplified version of the illustration provided by Siegel, Smith, and Paddock to value an offshore oil property.

9For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at $12 per barrel. If we do not make this assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

9In this example, we assume that the only uncertainty is in the price of oil, and the variance therefore becomes the variance in ln(oil prices).

9With a binomial model, we arrive at an estimate of value of $99.15 million.
Valuing a Firm with Undeveloped Reserves

The examples provided earlier illustrate the use of option pricing theory in valuing individual mines and oil tracts. Since the assets owned by a natural resource firm can be viewed primarily as options, the firm itself can be valued using option pricing models.

**Individual Reserves versus Aggregate Reserves**

The preferred approach would be to consider each option separately, value it and cumulate the values of the options.

---

10This is the variance of a product of two variables.
to get the value of the firm. Since this information is likely to be difficult to obtain for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant of this approach is to value the entire firm's undeveloped reserves as one option. A purist would probably disagree, arguing that valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio of options (which is what the natural resource firm really owns). Nevertheless, the value obtained from the model still provides a reasonable estimate of the value of undeveloped reserves.

Inputs to Option Valuation If you decide to apply the option pricing approach to estimate the value of aggregate undeveloped reserves, you have to estimate the inputs to the model. In general terms, while the process resembles the one used to value an individual reserve, there are a few differences.

- **Value of underlying asset.** You should cumulate all of the undeveloped reserves owned by a company and estimate the value of these reserves, based on the price of the resource today and the average variable cost of extracting these reserves today. The variable costs are likely to be higher for some reserves and lower for others, and weighting the variable costs at each reserve by the quantity of the resource of that reserve should give you a reasonable approximation of this value. At least hypothetically, we are assuming that the company can decide to extract all of its undeveloped reserves at one time and not affect the price of the resource.

- **Exercise price.** For this input, you should consider what it would cost the company today to develop all of its undeveloped reserves. Again, the costs might be higher for some reserves than for others, and you can use a weighted average cost.

- **Life of the option.** A firm will probably have different lives for each of its reserves. As a consequence, you will have to use a weighted average of the lives of the different reserves.\(^1\)

- **Variance in the value of the asset.** Here, there is a strong argument for looking at only the oil price as the source of variance, since a firm should have a much more precise estimate of its total reserves than it does of any one of its reserves.

- **Dividend yield (cost of delay).** As with an individual reserve, a firm with viable reserves will be giving up the cash flows it could receive in the next period from developing these reserves if it delays exercise. This cash flow, stated as a percent of the value of the reserves, becomes the equivalent of the dividend yield. The development lag reduces the value of this option just as it reduces the value of an individual reserve. The logical implication is that undeveloped reserves will be worth more at oil companies that can develop their reserves quicker than at less efficient companies.

\(^1\)If you own some reserves in perpetuity, you should cap the life of the reserve at a large value—say, 30 years—in making this estimate.
ILLUSTRATION 28.5: Valuing an Oil Company: Gulf Oil in 1984

Gulf Oil was the target of a takeover in early 1984 at $70 per share (it had 165.30 million shares outstanding and total debt of $9.9 billion). It had estimated reserves of 3,038 million barrels of oil and the total cost of developing these reserves at that time was estimated to be $30.38 billion dollars (the development lag is approximately two years). The average relinquishment life of the reserves is 12 years. The price of oil was $22.38 per barrel, and the production costs, taxes, and royalties were estimated at $7 per barrel. The bond rate at the time of the analysis was 9.00%. If Gulf were to choose to develop these reserves, it was expected to have cash flows next year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Value of underlying asset = Value of estimated reserves discounted back for period of development lag

\[= 3,038 \times \frac{($22.38 - $7)}{1.05^2} = $42,380.44\]

Note that you could have used forecasted oil prices and estimated cash flows over the production period and estimated the value of the underlying asset to be the present value of all of these cash flows. We have used a shortcut of assuming that the current contribution margin of $15.38 a barrel will remain unchanged in present value terms over the production period.

Exercise price = Estimated cost of developing reserves today = $30,380 million
Time to expiration = Average length of relinquishment option = 12 years
Variance in value of asset = Variance in oil prices = 0.03
Riskless interest rate = 9%
Dividend yield = Net production revenue/Value of developed reserves = 5%

Based on these inputs, the Black-Scholes model provides the following value for the call:

\[
d_1 = 1.6548 \quad N(d_1) = 0.9510 \\
d_2 = 1.0548 \quad N(d_2) = 0.8542
\]

\[
\text{Call value} = 42,380.44 \exp^{-0.09(12)}(0.9510) - 30,380 \exp^{-0.09(12)}(0.8542) = $13,306 million
\]

This stands in contrast to the discounted cash flow value of $12 billion that you obtain by taking the difference between the present value of the cash flows of developing the reserve today ($42.38 billion) and the cost of development ($30.38 billion). The difference can be attributed to the option possessed by Gulf to choose when to develop its reserves.

The option value ($13.3 billion) represents the value of the undeveloped reserves of oil owned by Gulf Oil. In addition, Gulf Oil had free cash flows to the firm from its oil and gas production of $915 million from already developed reserves and assume that these cashflows are likely to be constant and continue for 10 years (the remaining lifetime of developed reserves). The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:

\[
\text{Value of already developed reserves} = 915(1 - 1.125^{-10})/.125 = $5,065.83
\]

Adding the value of the developed and undeveloped reserves of Gulf Oil provides the value of the firm.

\[
\begin{align*}
\text{Value of undeveloped reserves} &= $13,306 million \\
\text{Value of production in place} &= $ 5,066 million \\
\text{Total value of firm} &= $18,372 million \\
\text{Less outstanding debt} &= $ 9,900 million \\
\text{Value of equity} &= $ 8,472 million \\
\text{Value per share} &= $ 8,472/165.3 = $51.25
\end{align*}
\]

This analysis would suggest that Gulf Oil was overvalued at $70 per share.

\[^{12}\text{With a binomial model, we estimate the value of the reserves to be $13.73 billion.}\]
OTHER APPLICATIONS

While patents and undeveloped reserves of natural resource companies lend themselves best to applying option pricing, there are other assets referenced in earlier chapters that can also be valued as options.

- Chapter 26, in the context of real estate valuation, noted that vacant land could be viewed as an option on commercial development.
- Chapter 27 presented an argument that copyrights and licenses could be viewed as options, even if they are not commercially viable today.

Table 28.1 presents the inputs you would use to value each of these options in an option pricing model. Much of what we have said about the other option applications apply here as well. The value is derived from the exclusivity that you have to commercially develop the asset. That exclusivity is obtained by legal sanction in the case of licenses and copyrights, and from the scarcity of land in the case of undeveloped reserves.

CONCLUSION

In traditional investment analysis, we compute the net present value of a project’s cash flows and conclude that firms should not invest in a project with a negative net present value. This is generally good advice, but it does not imply that the rights to this project are not valuable. Projects that have negative net present values today may have positive net present values in the future, and the likelihood of this occurring is directly a function of the volatility in the present value of the cash flows from the project.
This chapter valued the option to delay an investment and considered the implications of this option for three valuation scenarios—the value of a firm that derives all or a significant portion of value from patents that have not been commercially exploited yet, the value of a natural resource company with undeveloped reserves of the resource, and the value of a real estate firm with undeveloped land. In each case, we showed that using discounted cash flow valuation would result in an understatement of the values of these firms.

### Questions and Short Problems

In the problems following, use an equity risk premium of 5.5 percent if none is specified.

1. A company is considering delaying a project with after-tax cash flows of $25 million but that costs $300 million to take on (the life of the project is 20 years, and the cost of capital is 16%). A simulation of the cash flows leads you to conclude that the standard deviation in the present value of cash inflows is 20%. If you can acquire the rights to the project for the next 10 years, what is the value of the rights? (The six-month T-bill rate is 8%, the 10-year bond rate is 12%, and the 20-year bond rate is 14%.)

2. You are examining the financial viability of investing in some abandoned copper mines in Chile, which still have significant copper deposits in them. A geologist survey suggests that there might be 10 million pounds of copper in the mines still, and that the cost of opening up the mines will be $3 million (in present
value dollars). The capacity output rate is 400,000 pounds a year, and the price of copper is expected to increase 4% a year. The Chilean government is willing to grant a 25-year lease on the mine. The average production cost is expected to be 40 cents a pound, and the current price per pound of copper is 85 cents. (The production cost is expected to grow 3% a year, once initiated.) The annualized standard deviation in copper prices is 25%, and the 25-year bond rate is 7%.

a. Estimate the value of the mine using traditional capital budgeting techniques.
b. Estimate the value of the mine based on an option pricing model.
c. How would you explain the difference between the two values?

3. You have been asked to analyze the value of an oil company with substantial oil reserves. The estimated reserves amount to 10 million barrels, and the estimated cost of developing these reserves today is $120 million. The current price of oil is $20 per barrel, and the average production cost is estimated to be $6 per barrel. The company has the rights to these reserves for the next 20 years, and the 20-year bond rate is 7%. The company also proposes to extract 4% of its reserves each year to meet cash flow needs. The annualized standard deviation in the price of the oil is 20%. What is the value of this oil company?

4. You are analyzing a capital budgeting project. The project is expected to have a PV of cash inflows of $250 million and will cost $200 million today to take on. You have done a simulation of the project cash flows, and the simulation yields a variance in present value of cash inflows of 0.04. You have the rights to this project for the next 20 years. The 20-year Treasury bond rate is 8%.

a. What is the value of the project based on traditional NPV?
b. What is the value of the project as an option?
c. Why are the two values different? What factor or factors determine the magnitude of this difference?

5. Cyclops Inc., a high technology company specializing in state-of-the-art visual technology, is considering going public. While the company has no revenues or profits yet on its products, it has a 10-year patent to a product that will enable contact lens users to get no-maintenance lenses that will last for years. While the product is technically viable, it is exorbitantly expensive to manufacture, and the potential market for it will be relatively small currently. (A cash flow analysis of the project suggests that the present value of the cash inflows on the project, if adopted now, would be $250 million, while the cost of the project will be $500 million.) The technology is rapidly evolving, and a simulation of alternative scenarios yields a wide range of present values, with an annualized standard deviation of 60%. The 10-year bond rate is 6%.

a. Estimate the value of this company.
b. How sensitive is this value estimate to the variance in project cash flows?

What broader lessons would you draw from this analysis?