So far this book has concentrated on the valuation of publicly traded firms. In this chapter, we turn our attention to the thousands of firms that are private businesses. These businesses range in size from small family businesses to some that rival large publicly traded firms. The principles of valuation remain the same, but there are estimation problems that are unique to private businesses. The information available for valuation tends to be much more limited in terms of both history and depth, since private firms are often not governed by the strict accounting and reporting standards of publicly traded firms. In addition, the standard techniques for estimating risk parameters such as beta and standard deviation require market prices for equity, an input that is lacking for private firms.

When valuing private firms, the motive for the valuation matters and can affect the value. In particular, the value that is attached to a publicly traded firm may be different when it is being valued for sale to an individual, for sale to a publicly traded firm, or for an initial public offering. In particular, whether there should be a discount on value for illiquidity and nondiversifiable risk or a premium for control will depend on the motive for the valuation. Each of these components will be considered over the course of this chapter.

WHAT MAKES PRIVATE FIRMS DIFFERENT?

There are a number of common characteristics shared by private firms with publicly traded firms, but there are four significant differences that can affect how we estimate inputs for valuation.

1. Publicly traded firms are governed by a set of accounting standards that allow us not only to identify what each item in a financial statement includes but also to compare earnings across firms. Private firms, especially if they are not incorporated, operate under far looser standards, and there can be wide differences between firms on how items are accounted for.

2. There is far less information about private firms in terms of both the number of years of data that is typically available and, more importantly, the amount of information available each year. For instance, publicly traded firms have to break down operations by business segments in their filings with the SEC and provide information on revenues and earnings by segment. Private firms do not have to provide this information, and usually do not.
3. A constantly updated price for equity and historical data on this price are very useful pieces of information that we can obtain easily for publicly traded firms but not for private firms. In addition, the absence of a ready market for private firm equity also means that liquidating an equity position in a private business can be far more difficult (and expensive) than liquidating a position in a publicly traded firm.

4. In publicly traded firms, the stockholders tend to hire managers to run the firms, and most stockholders hold equity in several firms in their portfolios. The owner of a private firm tends to be intimately involved with management, and often has all of his or her wealth invested in the firm. The absence of separation between the owner and management can result in an intermingling of personal expenses with business expenses, and a failure to differentiate between management salary and dividends (or their equivalent). The absence of diversification can affect our measurement of risk.

Each of the differences cited can change value by affecting discount rates, cash flows, and expected growth rates.

To examine the issues that arise in the context of valuing private firms, we will consider two firms. The first firm is Chez Pierre, an upscale French restaurant in New York city, and the second is a private software firm called InfoSoft. We will value Chez Pierre for sale in a private transaction, whereas we will value InfoSoft for sale in an initial public offering (IPO).

**ESTIMATING VALUATION INPUTS AT PRIVATE FIRMS**

The value of a private firm is the present value of expected cash flows discounted back at an appropriate discount rate. Since this construct is not different from the one we used to value publicly traded firms, the differences between private firms and publicly traded firms have to show up in how we estimate these inputs to the discounted cash flow model.

**Discount Rates**

If we choose to value equity, we discount cash flows to equity at the cost of equity, whereas if we choose to value the firm, we discount cash flows at the cost of capital. While the fundamental definitions of these costs have not changed, the process of estimating them may have to be changed given the special circumstances surrounding private firms.

**Cost of Equity**  In assessing the cost of equity for publicly traded firms, we looked at the risk of investments through the eyes of the marginal investors in these firms. With the added assumption that these investors were well diversified, we were able to define risk in terms of risk added on to a diversified portfolio or market risk. The beta in the capital asset pricing model (CAPM) and betas (in the multifactor models) that measure this risk are usually estimated using historical stock prices. The absence of historical price information for private firm equity and the failure on the part of many private firm owners to diversify can create serious problems with estimating and using betas for these firms.
Approaches to Estimating Market Betas  

The standard process of estimating the beta in the capital asset pricing model involves running a regression of stock returns against market returns. Multifactor models use other statistical techniques, but they also require historical price information. In the absence of such information, as is the case with private firms, there are three ways in which we can estimate betas: accounting betas, fundamental betas, and bottom-up betas.

Accounting Betas  

While price information is not available for private firms, accounting earnings information is. We could regress changes in a private firm’s accounting earnings against changes in earnings for an equity index (such as the S&P 500) to estimate an accounting beta:

\[
\Delta \text{Earnings}_{\text{private firm}} = a + b \Delta \text{Earnings}_{\text{S&P 500}}
\]

The slope of the regression (b) is the accounting beta for the firm. Using operating earnings would yield an unlevered beta, whereas using net income would yield a levered or equity beta.

There are two significant limitations with this approach. The first is that private firms usually measure earnings only once a year, leading to regressions with few observations and limited statistical power. The second is that earnings are often smoothed out and subject to accounting judgments, leading to mismeasurement of accounting betas.

ILLUSTRATION 24.1: Estimating an Accounting Beta—Infosoft  

InfoSoft, even though it is a private business, has been in existence since 1992 and has accounting earnings going back to that year. This table summarizes the annual accounting earnings changes at InfoSoft and for the S&P 500 for each year between 1992 and 2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>Earnings: S&amp;P 500</th>
<th>Change in Earnings</th>
<th>Earnings (in '000s): Infosoft</th>
<th>Change in Earnings: Infosoft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>20.87</td>
<td>-28.89%</td>
<td>$ 25</td>
<td></td>
</tr>
<tr>
<td>1993</td>
<td>26.90</td>
<td>18.03%</td>
<td>$ 45</td>
<td>80.00%</td>
</tr>
<tr>
<td>1994</td>
<td>31.75</td>
<td>18.74%</td>
<td>$ 80</td>
<td>77.78%</td>
</tr>
<tr>
<td>1995</td>
<td>37.70</td>
<td>7.77%</td>
<td>$125</td>
<td>56.25%</td>
</tr>
<tr>
<td>1996</td>
<td>40.63</td>
<td>8.52%</td>
<td>$135</td>
<td>8.00%</td>
</tr>
<tr>
<td>1997</td>
<td>44.09</td>
<td>0.41%</td>
<td>$160</td>
<td>18.52%</td>
</tr>
<tr>
<td>1998</td>
<td>44.27</td>
<td>16.74%</td>
<td>$165</td>
<td>3.13%</td>
</tr>
<tr>
<td>1999</td>
<td>51.68</td>
<td>8.61%</td>
<td>$200</td>
<td>21.21%</td>
</tr>
<tr>
<td>2000</td>
<td>56.13</td>
<td>18.51%</td>
<td>$220</td>
<td>10.00%</td>
</tr>
<tr>
<td>2001</td>
<td>38.85</td>
<td>18.79%</td>
<td>$150</td>
<td>-31.82%</td>
</tr>
<tr>
<td>2002</td>
<td>46.04</td>
<td>18.79%</td>
<td>$280</td>
<td>86.67%</td>
</tr>
<tr>
<td>2003</td>
<td>54.69</td>
<td>18.79%</td>
<td>$420</td>
<td>50.00%</td>
</tr>
<tr>
<td>2004</td>
<td>67.68</td>
<td>23.75%</td>
<td>$600</td>
<td>42.86%</td>
</tr>
<tr>
<td>2005</td>
<td>76.45</td>
<td>12.96%</td>
<td>$750</td>
<td>25.00%</td>
</tr>
<tr>
<td>2006</td>
<td>87.72</td>
<td>14.75%</td>
<td>$900</td>
<td>20.00%</td>
</tr>
<tr>
<td>2007</td>
<td>82.54</td>
<td>-5.91%</td>
<td>$800</td>
<td>-11.11%</td>
</tr>
<tr>
<td>2008</td>
<td>65.39</td>
<td>-20.78%</td>
<td>$600</td>
<td>-25.00%</td>
</tr>
<tr>
<td>2009</td>
<td>60.8</td>
<td>-7.02%</td>
<td>$550</td>
<td>8.33%</td>
</tr>
<tr>
<td>2010</td>
<td>83.66</td>
<td>37.60%</td>
<td>$900</td>
<td>63.64%</td>
</tr>
</tbody>
</table>
Regressing the changes in earnings at InfoSoft against changes in profits for the S&P 500 yields the following.

\[
\text{InfoSoft earnings change} = 0.10 + 1.84 (\text{S&P 500 earnings change})
\]

Based upon this regression, the beta for InfoSoft is 1.83. In calculating this beta, we used net income to arrive at an equity beta. Using operating earnings for both the firm and the S&P 500 should yield the equivalent of an unlevered beta.

**Fundamental Betas**

There have been attempts made by researchers to relate the betas of publicly traded firms to observable variables such as earnings growth, debt ratios, and variance in earnings. Beaver, Kettler, and Scholes (1970) examined the relationship between betas and seven variables: dividend payout, asset growth, leverage, liquidity, asset size, earnings variability, and the accounting beta. Rosenberg and Guy (1976) also attempted a similar analysis.

Updating this regression using data for 2239 U.S. companies from January 2011, we obtained the following:

\[
\text{Beta} = 0.93 - 0.04 \text{ ROE} + 0.167 \text{ FA/TA} + 0.17 \text{ DC} + 0.74 \text{ g} - 0.31t
\]

\[
(42.37) (5.65) (6.50) (5.85) (9.57) (6.31) \text{ R}^2 = 9.3\%
\]

where,

- ROE = Return on equity
- FA/TA = Fixed assets/ Total assets
- DC = BV of debt/ (BV of debt + BV of equity)
- g = Expected annual growth rate in net income over next five years
- t = Effective tax rate

Thus, firms that have higher debt-to-capital ratios and expected growth have higher betas, whereas firms that deliver higher returns on equity and face higher effective tax rates have lower betas. Since all of the independent variables can be obtained for a private business, you could estimate a fundamental beta for the business. A caveat on using this regression is that the R-squared of the regression is only 9.3%, suggesting that the predictions will come with large standard errors.

**ILLUSTRATION 24.2: Estimating a Fundamental Beta: InfoSoft**

To use the cross-sectional regression that we reported earlier to estimate a beta for InfoSoft, we have to estimate the values for each of the independent variables for the firm.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on equity</td>
<td>18%</td>
</tr>
<tr>
<td>Fixed Assets/ Total Assets</td>
<td>40%</td>
</tr>
<tr>
<td>BV of Debt/ (BV of Debt + BV of Equity)</td>
<td>0%</td>
</tr>
<tr>
<td>Expected annual growth rate in net income</td>
<td>50%</td>
</tr>
<tr>
<td>Effective tax rate</td>
<td>20%</td>
</tr>
</tbody>
</table>

Inputting these values into the regression, we obtain a predicted value for the beta:

\[
\text{Beta} = 0.93 - 0.04 (0.18) + 0.167 (0.40) + 0.17 (0.00) + 0.74 (0.50) - 0.31 (0.20) = 1.30
\]

This would yield an estimate of 1.30 for InfoSoft's beta. The standard error on this estimate is 0.21, resulting in a range of 1.09 to 1.51 for the beta, with 67% probability.
**Bottom-Up Betas**  When valuing publicly traded firms, we used the unlevered betas of the businesses that the firms operated in to estimate bottom-up betas—the costs of equity were based on these betas. We did so because of the low standard errors on these estimates (due to the averaging across large numbers of firms) and the forward-looking nature of the estimates (because the business mix used to weight betas can be changed). We can estimate bottom-up betas for private firms, and these betas have the same advantages that they do for publicly traded firms. Thus, the beta for a private steel firm can be estimated by looking at the average betas for publicly traded steel companies. Any differences in financial or even operating leverage can be adjusted for in the final estimate.

In making the adjustment of unlevered betas for financial leverage, we do run into a problem with private firms, since the debt-to-equity ratio that should be used is a market value ratio. While many analysts use the book value debt-to-equity ratio to substitute for the market ratio for private firms, we would suggest one of the following alternatives:

- Assume that the private firm’s market leverage will resemble the average for the industry. If this is the case, the levered beta for the private firm can be written as:

\[
\beta_{\text{private firm}} = \beta_{\text{unlevered}} \left[ 1 + (1 - \text{Tax rate})(\text{Industry average debt/Equity}) \right]
\]

- Use the private firm’s target debt-to-equity ratio (if management is willing to specify such a target) or its optimal debt ratio (if one can be estimated) to estimate the beta:

\[
\beta_{\text{private firm}} = \beta_{\text{unlevered}} \left[ 1 + (1 - \text{Tax rate})(\text{Optimal debt/Equity}) \right]
\]

The adjustment for operating leverage is simpler and is based on the proportion of the private firm’s costs that are fixed. If this proportion is greater than is typical in the industry, the beta used for the private firm should be higher than the average for the industry.

---

**ILLUSTRATION 24.3: Estimating Bottom-Up Betas—Chez Pierre and InfoSoft**

To estimate a bottom-up beta for Chez Pierre, we looked at publicly traded restaurants in the United States. In January 2011, these firms had an average unlevered beta of 1.21 and an average market debt to equity ratio of 22.08%. We will assume that Chez Pierre will have the same unlevered beta and...
Maintain a debt-to-equity ratio, similar to that of publicly traded firms. Using a 40% tax rate, we get a levered beta for Chez Pierre of 1.37.

\[
\text{Levered beta for Chez Pierre} = 1.21 \left[1 + (1 - .40)(.2208)\right] = 1.37
\]

To estimate a beta for InfoSoft, we obtained the betas and market debt-equity ratios for publicly traded software firms. Since there are 264 software firms in the sample, with wide variations in market capitalization and growth prospects, we also look at sub-classes of these firms that might be more comparable to InfoSoft.

### Betas and Leverage of Publicly Traded Software Firms

<table>
<thead>
<tr>
<th>Grouping</th>
<th>Number of Firms</th>
<th>D/E Ratio</th>
<th>Unlevered Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>All software firms</td>
<td>333</td>
<td>5.61%</td>
<td>1.08</td>
</tr>
<tr>
<td>Small-cap Software Firms</td>
<td>108</td>
<td>6.35%</td>
<td>1.60</td>
</tr>
<tr>
<td>(Market cap &lt;$1 billion)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entertainment Software Firms</td>
<td>26</td>
<td>4.55%</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Note that the debt-equity ratios are market value debt-equity ratios. Note also that the difference in the size of the firms should not affect the betas directly, but it might have an indirect effect, since smaller firms tend to have higher operating leverage. We will use an unlevered beta of 1.60 for InfoSoft, based upon the average beta of small cap software firms. To estimate a levered beta, we recognize that InfoSoft has no debt outstanding and no plans to borrow money. Its levered beta is therefore equal to its unlevered beta of 1.60.

### Adjusting for Nondiversification

Betas measure the risk added by an investment to a diversified portfolio. Consequently, they are best suited for firms where the marginal investor is diversified. With private firms, the owner is often the only investor and thus can be viewed as the marginal investor. Furthermore, in most private firms, the owner tends to have much of his or her wealth invested in the private business and does not have an opportunity to diversify. Consequently, it can be argued that betas will understate the exposure to market risk in these firms.

At the limit, if the owner has all of his or her wealth invested in the private business and is completely undiversified, the owner is exposed to all risk in the firm and not just the market risk (which is what the beta measures). There is a fairly simple adjustment that can allow us to bring in this nondiversifiable risk into the beta computation. To arrive at this adjustment, assume that the standard deviation in the private firm’s equity value (which measures total risk) is \( \sigma \), and that the standard deviation in the market index is \( \sigma_m \). If the correlation between the stock and the index is defined to be \( \rho \), the market beta can be written as:

\[
\text{Market beta} = \rho \frac{\sigma}{\sigma_m}
\]

To measure exposure to total risk (\( \sigma \)), we could divide the market beta by \( \rho \). This would yield the following:

\[
\text{Market beta}/\rho = \frac{\sigma}{\sigma_m}
\]
This is a relative standard deviation measure, where the standard deviation of the private firm’s equity value is scaled against the market index’s standard deviation to yield what we will call a total beta.

\[
\text{Total beta} = \frac{\text{Market beta}}{\rho_{jm}}
\]

The total beta will be higher than the market beta, and will depend on the correlation between the firm and the market—the lower the correlation, the higher the total beta.

You might wonder how a total beta can be estimated for a private firm, where the absence of market prices seems to rule out the calculation of either a market beta or a correlation coefficient. Note, though, that we were able to estimate the market beta of the sector by looking at publicly traded firms in the business. We can obtain the correlation coefficient by looking at the same sample and use it to estimate a total beta for a private firm.

The question of whether the total beta adjustment should be made cannot be answered without examining why the valuation of the private firm is being done in the first place. If the private firm is being valued for sale, whether and how much the market beta should be adjusted will depend on the potential buyer or buyers. If the valuation is for an initial public offering, there should be no adjustment for nondiversification, since the potential buyers are stock market investors. If the valuation is for sale to another individual or private business, the extent of the adjustment will depend on the degree to which the buyer’s portfolio is diversified; the more diversified the buyer, the higher the correlation with the market and the smaller the total beta adjustment.

**ILLUSTRATION 24.4: Estimating a Bottom-Up Beta—Chez Pierre**

Consider the estimate of market beta that we obtained for Chez Pierre in the previous illustration. Using publicly traded restaurants as our comparable firms, we obtained an unlevered beta of 1.21 for Chez Pierre. The average correlation coefficient for these publicly traded firms with the markets is 48.41% The total unlevered beta for the Yankees can be estimated as follows:

\[
\text{Total unlevered beta} = \frac{1.21}{0.4841} = 2.50
\]

Using Chez Pierre’s tax rate of 40% and a debt-to-equity ratio of 22.08% (the restaurant sector’s average) yields a total levered beta of 2.07.

\[
\text{Total levered beta} = 2.50 \left[ 1 + (1 - .40)(.2208) \right] = 2.83
\]

This total beta estimate, in a sense, takes the limiting view that the potential buyer will own only Chez Pierre. To the extent that the buyer has some diversification, the correlation coefficient will be adjusted upward; if the buyer has a diversified portfolio, the correlation coefficient will approach 1 and the total beta will converge on the market beta.
To get from the cost of equity to the cost of capital, we need two additional inputs—the cost of debt, which measures the rate at which firms can borrow, and the debt ratio that determines the weights in the cost of capital computation. This section considers how best to estimate each of these inputs for a private firm.

**Cost of Debt**

The cost of debt represents the rate at which a firm can borrow money. To estimate it for publicly traded firms, we generally use either the yields on long-term government bonds or the yields on corporate bonds. For private firms, we need to find an equivalent rate. One approach is to use the cost of debt for a similar publicly traded firm. Another approach is to use the expected return on the firm's debt, which can be estimated using a risk-adjusted discount rate model. This model takes into account the riskiness of the firm's debt, as measured by its volatility and the volatility of its earnings and cash flows.

**ALTERNATIVE ADJUSTMENTS FOR PRIVATE FIRM RISK**

If you are leery about using the total beta approach to estimate the cost of equity for a private firm, there are three alternatives:

1. **Venture capital returns:** Look at the actual returns earned by investors who invest in private companies over long periods of history, relative to the risk-free rate and returns on publicly traded firms. Thus, if venture capital investors have earned 5 percent more than the S&P 500 after adjusting for risk, you could view this as a premium for investing in private businesses and add the number to your cost of equity, computed using a conventional risk and return model:

   \[
   \text{Adjusted cost of equity} = \text{Risk-free rate} + \text{Market beta} \times \text{Equity risk premium} + \text{Venture capital premium}
   \]

   The counter is that venture capitalists cannot really be compared to private business owners, since they are not only more diversified but are also pursuing exit strategies.

2. **Build up approach:** In this approach, you again start with the expected return from a conventional risk and return model, and add premiums to reflect the special risks associated with investing in small, private businesses. Two commonly used premiums are the small cap premium, reflecting the actual premium earned by very small, publicly traded companies over and above the market return (about 4 percent to 5 percent between 1928 and 2010) and the illiquidity premium, reflecting the higher returns earned by less liquid, public investments (with liquidity measured in trading volume and bid-ask spreads).

   \[
   \text{Adjusted cost of equity} = \text{Risk-free rate} + \text{Market beta} \times \text{Equity risk premium} + \text{Small cap premium} + \text{Illiquidity premium}
   \]

   The biggest peril with this approach is the risk of double counting. After all, the small cap premium may just reflect the fact small companies tend to be less liquid.

3. **Implied private costs of equity:** As data on private transactions gets richer, it is possible that we could use transactions prices to back out internal rates of return, given the expected cash flows. These internal rates of return will be the implied costs of equity to buyers.

**From Cost of Equity to Cost of Capital**

To get from the cost of equity to the cost of capital, we need two additional inputs—the cost of debt, which measures the rate at which firms can borrow, and the debt ratio that determines the weights in the cost of capital computation. This section considers how best to estimate each of these inputs for a private firm.
bonds issued by these firms or the ratings for these bonds to get default spreads. Private firms generally are not rated and do not have bonds outstanding. Consequently, we have to use one of the following alternative approaches:

- If the private firm has borrowed money recently (in the past few weeks or months), we can use the interest rate on the borrowing as a cost of debt. Since the cost of debt has to be current, the book interest rate\(^1\) on debt issued in the past is generally not a good measure of the cost of debt.
- If the private firm is being valued for an initial public offering, we can assume that the cost of debt for the private firm will move toward the average cost of debt for the industry to which the firm belongs. We are essentially assuming that the private firm, once public, will structure its debt policy to resemble those of comparable firms.
- When estimating the cost of debt for publicly traded firms in Chapter 8, we used the interest coverage ratios of these firms to estimate synthetic ratings, and then used the default spreads on these ratings to arrive at the costs of debt. To allow for the fact that private firms tend to be smaller and riskier than most publicly traded firms, we would use the relationship between interest coverage ratios and ratings for a subset of smaller, publicly traded firms, summarized in Table 24.1.

To estimate the cost of debt for a private firm with an interest coverage ratio of 5.1, for instance, we would use a synthetic rating of A− and the default spread associated with that rating. Thus, if firms that are rated A− typically pay 1.25 percent above the riskless rate to borrow, we would add that default spread to the riskless rate to estimate the cost of debt for the private firm.

<table>
<thead>
<tr>
<th>Interest Coverage Ratio</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 12.50</td>
<td>AAA</td>
</tr>
<tr>
<td>9.50–12.50</td>
<td>AA</td>
</tr>
<tr>
<td>7.50–9.50</td>
<td>A+</td>
</tr>
<tr>
<td>6.00–7.50</td>
<td>A</td>
</tr>
<tr>
<td>4.50–6.00</td>
<td>A−</td>
</tr>
<tr>
<td>3.50–4.50</td>
<td>BBB</td>
</tr>
<tr>
<td>3.00–3.50</td>
<td>BB</td>
</tr>
<tr>
<td>2.50–3.00</td>
<td>B+</td>
</tr>
<tr>
<td>2.00–2.50</td>
<td>B</td>
</tr>
<tr>
<td>1.50–2.00</td>
<td>B+</td>
</tr>
<tr>
<td>1.25–1.50</td>
<td>CCC</td>
</tr>
<tr>
<td>0.80–1.25</td>
<td>CC</td>
</tr>
<tr>
<td>0.50–0.80</td>
<td>C</td>
</tr>
<tr>
<td>&lt;0.50</td>
<td>D</td>
</tr>
</tbody>
</table>

\(^1\) Book interest rate = Interest expenses/Book value of debt.
This approach may underestimate the cost of debt if banks charge higher interest rates for private firms than for otherwise similar publicly traded firms. In that case, you would add an additional spread to reflect this difference, if you were valuing the firm for sale in a private transaction, but not if you were valuing it for sale to a publicly traded firm or an initial public offering.

**Debt Ratios** The debt ratio represents the proportion of the market value of a firm that comes from debt financing. For publicly traded firms, we use the market prices of publicly traded stocks and bonds to arrive at this ratio. Since neither input will be available for private firms, we have to consider one of the following options:

- In estimating levered betas, we suggested that the industry-average or target debt ratios could be used in the computation. Consistency demands that we use the same debt ratio for computing the cost of capital. Thus, if the industry-average debt-to-equity ratio is used to estimate the levered beta, the industry-average debt-to-capital ratio should be used to estimate the cost of capital. If the target debt-to-equity ratio is used for the levered beta computation, the target debt-to-capital ratio should be used in the cost of capital calculation.

- While market values of equity and debt are not available for private firms, we can use our estimated values of equity and debt from the valuation, though this creates circular reasoning in the analysis. You need the cost of capital (and the debt ratio) to estimate firm and equity value, and you need the equity value to estimate the cost of capital. You could overcome this problem by iterating toward a value—you could start with the book-debt ratio and cost of capital, estimate a firm and equity value, use these values to arrive at a new debt ratio and cost of capital, and reestimate firm and equity value. You would continue until the debt and equity values in the cost of capital computation converge on the estimated values.  

**ILLUSTRATION 24.5: Estimating Cost of Debt**

Infosoft has no debt, and we did not estimate a cost of debt for the firm. For Chez Pierre, we estimated an interest-coverage ratio based upon the operating income of $400,000 and its annual lease expenses of $120,000:

\[
\text{Interest coverage ratio} = \frac{400,000}{120,000} = 3.33
\]

That interest-coverage ratio yielded a synthetic rating of B+. Adding the default spread of 4%, for a B+ rated bond, to the risk-free rate of 3.5% provides a pretax cost of debt of 7.5% for the firm.

\[
\text{Pretax cost of debt} = \text{Risk-free rate} + \text{Default spread} = 3.5\% + 5\% = 7.5\%
\]

\[
\text{Aftertax cost of debt} = 7.5\% \times (1 - 0.40) = 4.5\%
\]

\[\text{Author: There is no Table 24.4.}\]
ILLUSTRATION 24.6: Estimating Cost of Capital

To estimate the cost of capital for Chez Pierre and InfoSoft, we will stay consistent with the assumptions we have made about leverage so far in this chapter. Chez Pierre, we assumed, would stay close to industry average debt-to-equity ratio of 22.08%, which translates into a market debt to capital ratio of 18.09%. For InfoSoft, we stayed with the assumption that the firm has a debt ratio of 0%.

For Chez Pierre, given that we are valuing the firm for sale to an undiversified individual, we estimated a total beta of 2.83. Using the Treasury bond rate of 3.5% prevalent at the time of this valuation and a market risk premium of 5%, we estimate a cost of equity of 17.65%.

\[
\text{Cost of Equity} = 3.5\% + 2.83(5\%) = 17.65\%
\]

Using the after-tax cost of debt of 4.5% estimated in Illustration 24.5, we can estimate the cost of capital.

\[
\text{Cost of capital} = 17.65\%(0.8191 + 4.5\%(0.1809)) = 15.27\%
\]

For InfoSoft, where we are pricing an initial public offering, we use the market beta estimate of 1.60. Using the Treasury bond rate of 3.5% and a risk premium of 5% yields a cost of equity of 11.50%.

\[
\text{Cost of equity} = 3.5\% + 1.60(5\%) = 11.50\%
\]

Since the firm has no debt, the cost of capital is also 11.50%.

---

**Cash Flows**

The definitions of the cash flow to equity and cash flow to the firm are identical for both private and publicly traded firms. The cash flow to equity is the cash flow after taxes, debt payments and issues, and reinvestment needs. The cash flow to the firm is the cash flow after taxes and reinvestment needs, but before debt payments.

There are three issues that do affect estimation of cash flows with private firms. The first is that many private firms do not adequately consider the salaries for owner-managers, since many owners do not distinguish between income that they receive as dividends and income they receive as salaries. The second is the intermingling of personal and business expenses that often occurs at small private businesses that can cause income to be mismeasured. The third is the effect of taxes on value, since individual tax status and tax rates vary much more widely than corporate tax rates.

**Owner Salaries and Equity Cash Flows** In valuing firms, we draw a simple distinction between salaries and dividends. Salaries are compensation for professional services rendered to the firm and should be treated as operating expenses.
Dividends or other equity cash withdrawals from the firm are returns on equity capital invested and determine the value of equity. The separation between managers and stockholders in publicly traded firms results in a distinction between salaries (which are paid to managers) and dividends (which are paid to stockholders) that is clear. In a private business, the owner is often the firm’s manager and its only equity investor. If the private firm is not incorporated, the income earned by the owner is taxed at the same rate, whether it is categorized as a salary or as a dividend. Consequently, an owner will be indifferent between receiving a salary of $10,000 and a dividend of $90,000 and a salary of $90,000 and a dividend of $10,000. As a consequence, owners do not pay themselves a salary in many small private firms, or even if they do, the salary does not reflect the services they render to the firm.

When valuing a private firm, we generally make forecasts based on the operating income reported by the firm. If that operating income does not reflect a salary adjustment for the owner, it will be overstated and result in a value that is too high. To get a more precise estimate of operating income, we have to estimate the appropriate compensation for the owner-managers, based on the role they play in the firm and the cost of hiring replacements for them. Thus, the owner of a private business might play several roles—cashier, accountant, stockperson, and salesperson, and the management salary would have to include the cost of hiring a person or two to provide the same services.

**Intermixing Business and Personal Expenses** The intermingling of business and personal expenses is a particular problem in small private businesses, since owners often have absolute power over many aspects of the business. Many private business owners maintain offices in their residences, have vehicles that they maintain for personal and business use, and share other services between work and home. In some cases, family members are hired to fill phantom positions in order to distribute income or to reduce taxes.

If personal expenses are consolidated with business expenses or are otherwise a part of business expenses, the operating income for a private firm has to be estimated prior to these expenses. The problem with making these adjustments, however, is that private firm owners are usually not forthcoming about the extent of these expenses, and there may be tax consequences.

**Tax Effects** When valuing publicly traded firms, the tax rate that we use in valuation is defined to be the marginal corporate tax rate. While different firms may face different marginal tax rates, the differences in tax rates across potential buyers of a private firm can be much larger. In fact, the tax rate can vary from the corporate tax rate (if the potential buyer is a corporation) to the highest marginal tax rate for individuals (if the potential buyer is a wealthy individual) to a lower marginal tax rate if the potential buyer is an individual with lower income. The tax rate will affect both the cash flows (through the after-tax operating income) and the cost of capital (through the cost of debt). As a consequence, the value of a private firm can vary across different buyers.
Private business can be organized in many different ways, with very different tax consequences. In its simplest form, a private business can be a sole proprietorship, where the line between business and the individual is blurred and the income from the business is reported on the owner’s tax returns. Here, the solution is simple. Compute the cash flows, after taking into account the owner’s tax liability, and discount those cash flows back at a rate of return that the owner would need on an after-tax basis, to compensate for the risk taken. It is possible, however, that this business could be worth a different amount to a potential buyer who faces a different tax rate.

Things become more interesting when you have a partnership, where income is split among the partners based on the proportion of the partnership that they own and is shown as income (and taxed) on their individual tax returns. To the extent that they face similar tax rates, you can use a consensus tax rate that reflects the partners’ standing, and compute after-tax cash flows and discount back at a risk-adjusted post-tax rate of return.

Finally, consider a subchapter S corporation. The entity is not taxed on its income, but the stockholders in the corporation are taxed on their share of the income, even if it is not paid out as dividend. (A stockholder in a publicly traded company is taxed only on the dividend paid, and can defer paying capital gains taxes until he or she sells the stock.) You can value this firm, using one of two approaches:

1. You can use the same logic that we used for partnerships and find a tax rate that reflects what the stockholders pay. You can then after-tax cash flows and discount back at a rate of return that is risk-adjusted and after-taxes.
2. You can hone in on pretax cash flows and discount them back at a risk-adjusted pretax rate of return.

The key in all of these valuations is to use the appropriate discount rate, given the cash flows.

When you use the CAPM or its variants to estimate discount rates for publicly traded companies, you are estimating a post-corporate tax and prepersonal tax required rate of return to discount cash flows are that after corporate taxes but before personal taxes (on dividends or capital gains). If you draw on the same public firm risk and return models to estimate discount rates for private businesses, you have to recognize that if the cash flows are after personal taxes, you will have to adjust the discount rates accordingly. Thus, if your cost of equity is 15% and is computed using the CAPM, and the owner faces a 40% tax rate, the post-personal tax cost of equity is 9%:

\[
\text{Post-personal tax cost of equity} = 15\% \times (1 - 0.40) = 9\%
\]

If you decide to compute your cash flows prior to personal taxes, you can continue to use the 15% grossed up cost of equity in your discounting.
To provide an illustration, assume that you have a subchapter S corporation with only one stockholder (who faces a marginal tax rate of 40% on taxable income). Assume that the subchapter S corporation makes $100 in earning that are expected to stay constant in perpetuity and that the CAPM cost of equity for the firm is 10%.

\[
\text{Pretax value} = \frac{\text{Pretax Earnings}}{\text{Prepersonal tax cost of equity}} = \frac{100}{.10} = $1000
\]

\[
\text{Post-tax value} = \frac{\text{Post-tax Earnings}}{\text{Post personal tax cost of equity}} = \frac{100(1 - 40)}{.10(1 - 40)} = $1000
\]

### ILLUSTRATION 24.7: Operating and Net Income

To estimate the cashflows for Chez Pierre, we started with the stated income from the owner’s financial statements. Those statements indicated that the restaurant generated $400,000 in operating income and $240,000 in net income on revenues of $1.2 million in the most recent year:

<table>
<thead>
<tr>
<th>Stated</th>
<th>Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenues $1,200.00</td>
<td>$1,200.00</td>
</tr>
<tr>
<td>– Operating lease expense $120.00</td>
<td></td>
</tr>
<tr>
<td>– Imputed depreciation on leased asset $50.38</td>
<td></td>
</tr>
<tr>
<td>– Wages $200.00</td>
<td>$350.00</td>
</tr>
<tr>
<td>– Material $300.00</td>
<td>$300.00</td>
</tr>
<tr>
<td>– Other operating expenses $180.00</td>
<td>$180.00</td>
</tr>
<tr>
<td>Operating income $400.00</td>
<td>$319.62</td>
</tr>
<tr>
<td>– Imputed interest expenses $0.00</td>
<td>$69.62</td>
</tr>
<tr>
<td>Taxable income $400.00</td>
<td>$300.38</td>
</tr>
<tr>
<td>– Taxes $160.00</td>
<td>$120.15</td>
</tr>
<tr>
<td>Net income $240.00</td>
<td>$180.23</td>
</tr>
</tbody>
</table>

We made two key adjustments. First, we noticed that the owner (and chef) was not paying himself a salary. We added $150,000 to wages to reflect the expected expense associated with a new chef. Second, we have converted operating lease expenses into financial expenses, by capitalizing the lease commitments ($120,000/year for the next 12 years), using the pretax cost of debt of 7.5% that we estimated in Illustration 24.5:

\[
\text{PV of lease commitments} = $120,000 \times (\text{PV of annuity, 7.5%, 12 years}) = $928,230
\]

This conversion then results in two new items on the income statement:

\[
\text{Imputed interest expenses} = $828,233 \times .075 = $69,620
\]

\[
\text{Imputed depreciation} = \text{Current year’s lease expense} - \text{Imputed interest expense} = $120,00 - $69,620 = $50,380
\]

These two adjustments reduce the operating income to $319,620 and the net income to $180,230.
InfoSoft, though a private firm, has essentially been run like a public firm, probably as a lead-in to the initial public offering. This table reflects the operating income for InfoSoft.

**Income Statement—InfoSoft (in ‘000s)**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales and other operating revenues</td>
<td>$10,000</td>
</tr>
<tr>
<td>Operating costs and expenses</td>
<td>$8,300</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$200</td>
</tr>
<tr>
<td>Operating income</td>
<td>$1,500</td>
</tr>
<tr>
<td>Interest expenses</td>
<td>$0</td>
</tr>
<tr>
<td>Taxable income</td>
<td>$1,500</td>
</tr>
<tr>
<td>Taxes</td>
<td>$600</td>
</tr>
<tr>
<td>Net income</td>
<td>$900</td>
</tr>
</tbody>
</table>

**Growth**

The growth rate for a private firm can be estimated by looking at the past (historical growth) or from fundamentals (the reinvestment rate and return on capital). This section will consider some of the issues in estimating private firm growth.

**Estimating Growth**

In estimating growth for publicly traded firms, we noted that we could draw on three sources—historical growth, analyst estimates, and fundamentals. With private firms, we will not find analyst estimates of growth, and historical growth numbers have to be used with caution. The shifting accounting standards that characterize many private firms will mean that reported earnings changes over time may not reflect actual earnings changes. Furthermore, the fact that earnings are measured annually, rather than quarterly, and the reality that private firms tend to be younger than publicly traded firms will mean far less data in the historical growth estimate.

As a consequence of these gaps in past growth and analyst estimates, there is an even greater reliance on fundamentals in private firms. The expected growth rate in operating income is the product of the reinvestment rate and the return on capital, though changes in return on capital in existing assets can create an additional impact.

\[
\text{Expected growth rate} = \text{Reinvestment rate} \times \text{Return on capital}
\]

In making the estimates of reinvestment rates and returns on capital for private firms, we can draw on the experience of publicly traded firms in the business.

**Illustration 24.8: Estimating Growth**

The process of estimating growth is different for the two firms under consideration in this chapter. With Chez Pierre, we are looking at a well run restaurant operating at close to capacity and that is unlikely to grow at rate higher than the inflation rate. Consequently, we will assume a growth rate of 2% in nominal terms, for the next 12 years, which is the remaining lease term. At the end of year 12, we will assume that the business will be liquidated. Since we are assuming a finite life and no real growth, we will also assume a reinvestment rate of zero for the firm.
To estimate the growth rate at InfoSoft, we follow a more conventional route. We first estimate the return that they earn on their capital invested currently, by dividing the after-tax operating income from the most recent year by the book value capital invested\(^3\) at the beginning of the year. We use the operating income from Table 24.8 and used the corporate marginal tax rate of 40%.

\[
\text{Return on Capital} = \frac{\text{After-tax Operating Income}_{2010}}{\text{BV of Debt}_{2009} + \text{BV of equity}_{2009} - \text{Cash}_{2009}}
\]

\[
= \frac{\$1500(1 - .40)}{\$0 + \$5,000 - \$500} = 20.00\%
\]

We then estimate InfoSoft's reinvestment rate by dividing their reinvestment in capital expenditures and working capital in the most recent year by the after-tax operating income. Given that the firm reported capital expenditures of $960,000, depreciation of $200,000 and an increase in non-cash working capital from $100,000 to $150,000, we estimated a reinvestment rate of

\[
\text{Reinvestment Rate} = \frac{960 - 200 + 50}{1500(1 - .40)} = 90.00\%
\]

The expected growth rate in operating income for InfoSoft for the immediate future is based upon the assumption that the return on capital and reinvestment rate will remain unchanged over the next five years.

\[
\text{Expected Growth Rate} = 20\% \times .90 = 18\%
\]

If we had expected the return on capital or the reinvestment rate to change over time, we would have reflected those changes in this growth rate.

**Persistence of Growth**  In valuing publicly traded firms, we generally assumed infinite lives, even though we did allow for the risk that the firm would not survive. With private firms, the perpetual life assumption has to be made with far more caution. Unlike publicly traded firms, where the transition from one CEO to another is common, the transition is much more complicated in a private firm since the owner-manager generally does not want to pass the reins of power to an outsider. Instead, the owner looks to the next generation in his or her family for the successor, a process that is not always successful.

What are the implications for valuation? One is that the terminal value for a private firm will be lower than the terminal value for a publicly traded firm. If we assume, in fact, that the firm will cease operations at some point in time in the future—say when the current owner retires—we would use a liquidation value for the assets as the terminal value. In general, liquidation values are lower than the value of continuing operations. The other is that private firms where owners plan for the transition to the next generation will be worth more than private firms that do not make these arrangements.

Some private firms, especially as they get larger, resemble publicly traded firms in terms of having professional managers. With these firms, the assumption of infinite growth that we used with publicly traded firms can be sustained.

---

\(^3\)The capital invested reflects the value of the research asset.
ILLUSTRATION 24.9: Closure in Valuation and Terminal Values

As we noted earlier, we will assume a 12-year life for Chez Pierre. When the lease ends, we will assume that the restaurant will be closed and that the assets will be liquidated at book value of $500,000.

With Infosof, we are assuming a growing and healthy publicly traded firm, based upon our projections over the next 10 years. The firm should be worth more based upon continuing operations than from liquidation. Consequently, we assume an expected growth rate of 3% beyond year 10 for the firm. As the firm becomes larger, it will become more and more difficult for it to sustain its current return on capital of 20%. We will assume that the return on capital will drop to 12%. These two assumptions yield a reinvestment rate of 25% after year 10.

\[
\text{Reinvestment rate} = \frac{\text{Expected growth rate}}{\text{Return on capital}} = \frac{3\%}{12\%} = 25\%
\]

We will also assume that the beta for Infosof will drop to 1.20 after year 10 and that the firm will use some of its debt capacity (its debt ratio will rise from 0% to 10% and the cost of debt will be 5%). The resulting cost of equity and capital are estimated here:

\[
\text{Cost of equity} = 3.5\% + 1.2(5\%) = 9.5\%
\]

\[
\text{After-tax cost of debt} = 5\% (1 - .40) = 3\%
\]

\[
\text{Cost of capital} = 9.5\% (.9) + 3\% (.1) = 8.85\%
\]

“Key Person” Effect on Value

Young companies, especially in service businesses, are often dependent upon the owner or a few key people for their success. Consequently, the value we estimate for these businesses can change significantly if one or more of these key people will no longer be associated with the firm. To assess a key person discount in valuations, we would suggest that the firm be first valued, with the status quo (with key people involved in the business) and be valued again, with the loss of these individuals built into revenues, earnings, and expected cash flows. To the extent that earnings and cash flows suffer when key people leave, the value of the business will be lower with the loss of these individuals, and the key person discount can then be estimated as follows:

\[
\text{Key person discount} = \frac{\text{Value of firm}_{\text{Status Quo}} - \text{Value of firm}_{\text{Key person lost}}}{\text{Value of firm}_{\text{Status Quo}}}
\]

There is no simple formula that will help in determining how much in cash flows will be lost as a result of the loss of key personnel, since it will vary not only across businesses but across the personnel involved. One way to assess it is to survey existing customers to see how they will respond if the key personnel leave and to then build in this impact into operating forecasts.

To illustrate, assume that you are valuing a restaurant that is being offered for sale by its well-known chef/owner. Assume that the restaurant generated $1 million in
after-tax cash flow last year, has an expected growth rate of 2% and a cost of capital of 12%. The value of the restaurant should be $10 million, based on these inputs:

\[
\text{Value of restaurant}_{\text{Status Quo}} = \frac{\text{Expected cash flow next year}}{\text{Cost of capital} - \text{Expected growth rate}}
\]

\[
= \frac{\$1,000,000 \times (1.02)}{0.12 - 0.02} = \$10,200,000
\]

It is likely, though, that some of the revenues/cash flows from this restaurant can be attributed to the chef and that his departure will cause a drop-off in cash flows. Assume that as the potential buyer, you survey customers in the restaurant and realize that there will be a 20% drop off in cash flow, if the current chef leaves. The value of the restaurant without the key person will be lower:

\[
\text{Value of restaurant}_{\text{Status Quo}} = \frac{\$800,000 \times (1.02)}{0.12 - 0.02} = \$8,160,000
\]

This loss in value would be much greater if there is a likelihood that the chef could open a new competing restaurant.

As the potential seller, the chef can reduce his loss:

- By signing a no-compete legal agreement
- By offering to stay on as chef for a transition period, with the new buyer as the owner

Even with larger companies, the loss of key personnel can have a significant impact on value.

**Iliquidity Discounts**

When you take an equity position in an entity, you generally would like to have the option to liquidate that position if you need to. The need for liquidity arises not only because of cash flow considerations but also because you might want to change your portfolio holdings. With publicly traded firms, liquidation is simple and generally has a low cost—the transaction costs for liquid stocks are a small percent of the value. With equity in a private business, liquidation costs as a percent of firm value can be substantial. Consequently, the value of equity in a private business may need to be discounted for this potential illiquidity. This section will consider the determinants of this discount and how best to estimate it.

**Determinants of Illiquidity Discount**

The illiquidity discount is likely to vary across both firms and buyers, which renders rules of thumb useless. Let us consider first four factors that may cause the discount to vary across firms:

1. **Liquidity of assets owned by the firm.** The fact that a private firm is difficult to sell may be rendered moot if its assets are liquid and can be sold with no
significant loss in value. A private firm with significant holdings of cash and marketable securities should have a lower illiquidity discount than one with factories or other assets for which there are relatively few buyers.

2. **Financial health and cash flows of the firm.** A private firm that is financially healthy should be easier to sell than one that is not healthy. In particular, a firm with strong income and positive cash flows should be subject to a smaller illiquidity discount than one with negative income and cash flows.

3. **Possibility of going public in the future.** The greater the likelihood that a private firm can go public in the future, the lower should be the illiquidity discount attached to its value. In effect, the probability of going public is built into the valuation of the private firm. To illustrate, the owner of a private e-commerce firm in 1998 or 1999 would not have had to apply much of an illiquidity discount to his or her firm’s value, if any, because of the ease with which these firms could be taken public in those years.

4. **Size of the firm.** If we state the illiquidity discount as a percent of the value of the firm, it should become smaller as the size of the firm increases. In other words, the illiquidity discount should be smaller as a percent of firm value for private firms like Cargill and Koch Industries, which are worth billions of dollars, than it should be for a small firm worth $15 million.

The illiquidity discount is also likely to vary across potential buyers because the desire for liquidity varies with individuals. It is likely that those buyers who have deep pockets and see little or no need to cash out their equity positions will attach much lower illiquidity discounts to value for similar firms than buyers that have less of a safety margin.

**Empirical Evidence and Typical Practice** How large is the illiquidity discount attached to private firm valuations? This is a very difficult question to answer empirically because the discount itself cannot be observed. Even if we were able to obtain the terms of all private firm transactions, note that what is reported is the price at which private firms are bought and sold. The value of these firms is not reported, and the illiquidity discount is the difference between the value and the price.

In fact, much of the evidence on illiquidity discounts comes from examining restricted stock at publicly traded firms. Restricted securities are securities issued by a publicly traded company, but not registered with the SEC, that can be sold through private placements to investors but cannot be resold in the open market for a two-year holding period, and only limited amounts can be sold after that. When this stock is issued, the issue price is set much lower than the prevailing market price, which is observable, and the difference is viewed as a discount for illiquidity. The results of three studies that have looked at the magnitude of this discount are summarized as follows:

1. Maher examined restricted stock purchases made by four mutual funds in the period 1969–1973 and concluded that they traded at an average discount of 35.43 percent on publicly traded stock in the same companies.
2. Moroney reported a mean discount of 35 percent for acquisitions of 146 restricted stock issues by 10 investment companies, using data from 1970.
3. Silber examined restricted stock issues from 1984 to 1989 and found that the median discount for restricted stock was 33.75 percent.
In summary, then, there seems to be a substantial discount attached, at least on average, when an investment is not liquid. Much of the practice of estimating illiquidity discounts seems to build on these averages. For instance, rules of thumb often set the illiquidity discount at 20 to 30 percent of estimated value, and there seems to be little or no variation across firms.

Silber (1991) also examined factors that explained differences in discounts across different restricted stocks by relating the size of the discount to observable firm characteristics including revenues and the size of the restricted stock offering. He reported the following regression:

\[
\ln(\text{RPRS}) = 4.33 + 0.036 \ln(\text{REV}) - 0.142 \ln(\text{RBRT}) + 0.174 \text{DERN} + 0.332 \text{DCUST}
\]

where  
- \( \text{RPRS} \) = Restricted stock price/Unrestricted stock price = 1 – Illiquidity discount  
- \( \text{REV} \) = Revenues of the private firm (in millions of dollars)  
- \( \text{RBRT} \) = Restricted block relative to total common stock in %  
- \( \text{DERN} \) = 1 if earnings are positive; 0 if earnings are negative  
- \( \text{DCUST} \) = 1 if there is a customer relationship with the investor; 0 otherwise

The illiquidity discount tends to be smaller for firms with higher revenues, decreases as the block offering decreases, and is lower when earnings are positive and when the investor has a customer relationship with the firm.

These findings are consistent with some of the determinants that we identified in the previous section for the illiquidity premium. In particular, the discounts tend to be smaller for large firms (at least as measured by revenues) and for healthy firms (with positive earnings being the measure of financial health). This would suggest that the conventional practice of using constant discounts across private firms is wrong and that we should be adjusting for differences across firms.

Estimating the Illiquidity Discount  If we do decide to adjust the illiquidity discount to reflect the differences across private firms, we are faced with an estimation question. How are we going to measure these differences and build them into an estimate? There are two ways of doing this. The first is to extend the analysis done for restricted securities into the illiquidity discount; in other words, we could adjust the discount factor for the magnitude of a firm’s revenues and whether it has positive earnings. The second is to apply some of the empirical work that has been done examining the magnitude of the bid-ask spread for publicly traded firms to estimating illiquidity discounts.

Adjusted Discount Factors  Consider again the regression that Silber presents on restricted stock. Not only does it yield a result specific to restricted stock, but it also provides a measure of how much lower the discount should be as a function of revenues. A firm with revenue of $20 million should have a illiquidity discount that is 1.19 percent lower than a firm with revenues of $10 million. Thus we could establish a benchmark discount for a profitable firm with specified revenues (say $10 million) and adjust this benchmark discount for individual firms that have revenues much higher or lower than this number. The regression can also be used to differentiate between profitable and unprofitable firms. Figure 24.1 presents the difference
in illiquidity discounts across both profitable and unprofitable firms with different revenues, using a benchmark discount of 25 percent for a firm with positive earnings and $10 million in revenues.

There are clearly dangers associated with extending a regression run on a small number of restricted stock to estimating discounts for private firms, but it does provide at least a road map for adjusting discount factors.

**Bid-Ask Spread Approach**  
The biggest limitation of using studies based on restricted stock is that the samples are small. We would be able to make far more precise estimates if we could obtain a large sample of firms with illiquidity discounts. We would argue that such a sample exists, if we consider the fact that an asset that is publicly traded is not completely liquid. In fact, liquidity varies widely across publicly traded stock. A small company listed over-the-counter is much less liquid than a company listed on the New York Stock Exchange, which in turn is much less liquid than a large-capitalization company that is widely held. In fact, the difference between the bid price and the ask price that we observe on publicly traded assets can be viewed as a measure of the cost of instant liquidity. An investor who buys an asset, changes his or her mind, and decides to sell the asset immediately will pay the bid-ask spread.

While the bid-ask spread might only be a quarter or half a dollar, it looms as a much larger cost when it is stated as a percent of the price per unit. For a stock that is trading at $2, with a bid-ask spread of ¼, this cost is 12.5 percent. For higher-price and very liquid stocks, the illiquidity discount may be less than 0.5 percent of the price, but it is not zero.

What relevance does this have for illiquidity discounts on private companies? Think of equity in a private company as a stock that never trades. On the continuum

![Figure 24.1 Illiquidity Discounts: Base Discount of 25 Percent for Profitable Firm with $10 Million in Revenues](ch24_p667_701.qxd 12/7/11 2:28 PM Page 687)
just described, you would expect the bid-ask spread to be high for such a stock, and this would essentially measure the illiquidity discount.

To make estimates of the illiquidity discounts using the bid-ask spread as the measure, you would need to relate the bid-ask spread of publicly traded stocks to variables that can be measured for a private business. For instance, you could regress the bid-ask spread against the revenues of the firm and a dummy variable reflecting whether the firm is profitable, and extend the regression done on restricted stocks to a much larger sample. You could even consider the trading volume for publicly traded stocks as an independent variable and set it to zero for a private firm. Using data from the end of 2000, for instance, we regressed the bid-ask spread for Nasdaq stocks against revenues, a dummy variable for positive earnings, cash as a percent of firm value, and trading volume.

\[
\text{Spread} = 0.145 - 0.0022 \ln(\text{Annual revenues}) - 0.015(DERN) - 0.016(\text{Cash/Firm value}) - 0.11(\$ \text{ Monthly trading volume/Firm value})
\]

Plugging in the corresponding values—with a trading volume of zero—for a private firm should yield an estimate of the bid-ask spread for the firm.

**ILLUSTRATION 24.10: Estimating the Illiquidity Discount for Chez Pierre**

Since Chez Pierre is being valued for a private transaction, it is appropriate to consider an illiquidity discount. We can use both approaches described earlier to estimate the magnitude of that discount.

1. **Restricted Stock Approach:** To estimate the illiquidity discount for Chez Pierre, we assume that the base discount for a firm with $10 million in revenues would be 25%. Chez Pierre’s revenues of $1.2 million, being smaller than the typical firm, should result in a higher discount on their value. We estimate the difference in the illiquidity discount between a firm with $10 million in revenue and $1.2 million in revenue to be 3.75%. To do this, we first estimated the illiquidity discount in the Silber equation for a firm with $10 million in revenues.

\[
\text{Illiquidity discount}_{\text{base}} = \frac{100 - \exp[4.33 + 0.036\ln(10) - 0.142\ln(100) + 0.174(1)]}{100} = 48.94\%
\]

We then re-estimated the illiquidity discount with revenues of $1.2 million:

\[
\text{Illiquidity discount}_{\text{Chez Pierre}} = \frac{100 - \exp[4.33 + 0.036\ln(1.2) - 0.142\ln(100) + 0.174(1)]}{100} = 52.69\%
\]

Difference in discount = 52.69% – 48.94% = 3.75%

The estimated illiquidity discount for Chez Pierre would therefore be 28.75%, which is the base discount of 25% adjusted for the additional liquidity discount to reflect smaller revenues that the firm possesses.

2. **Bid-Ask Spread Approach:** We could substitute in the revenues of Chez Pierre, the fact that it has positive earnings and the cash as a percent of revenues held by the firm (1%):

\[
\text{Spread} = 0.145 - 0.0022\ln(\text{Annual revenues}) - 0.015(DERN) - 0.016(\text{Cash/Firm Value}) - 0.11(\$ \text{ Monthly trading volume/Firm Value})
\]

Based on this approach, we would estimate an illiquidity discount of 12.94%.

**iligdisc.xls**: This spreadsheet allows you to estimate the illiquidity discount for private firms using both the restricted stock approach and the bid-ask spread approach.
In the preceding section, we considered how best to estimate the inputs to use in valuing a private firm. As we considered each input, though, we noted that the process of estimation might be different depending on the potential buyer of the firm. With betas, for instance, we argued that the market beta should be used if the potential buyer is a publicly traded firm or a stock market investor (in an initial public offering) and that a total beta should be used if the potential buyer is a private party. We made similar arguments about the cost of debt and cash flows. Table 24.2 summarizes the differences in the way we estimate the inputs to valuation for different valuation motives.

The results of using different approaches to estimating discount rates and cash flows, depending on the potential buyer, can have significant effects on value. In general, a private business that is up for sale will be valued much more highly by a publicly traded firm than by a private entity. This can be traced to the fact that the discount rates are higher when we assume that the buyer is not diversified. Thus the owners of private businesses who are interested in selling their businesses will be well served looking for potential buyers who are publicly traded firms. While they might not be able to extract the entire value, they can try to obtain at least a share of the additional value created because the marginal investors are diversified.

The same implications arise when looking at the alternative of going public. The value that a firm can obtain from a public offering will exceed the value that it will receive from a private entity. The values obtained from an initial public offering and sale to a publicly traded firm will be based on similar discount rates, but may vary because of cost and revenue synergies. If the potential for these synergies is large, selling to a publicly traded firm may result in a higher value than going public.

<table>
<thead>
<tr>
<th>TABLE 24.2</th>
<th>Estimation of Inputs for Valuation: Valuation Motives</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Valuation for Sale to a Private Entity</strong></td>
<td><strong>Valuation for Sale to a Publicly Traded Firm or for an Initial Public Offering</strong></td>
</tr>
<tr>
<td>Cost of equity</td>
<td>Based on total beta, with correlation reflecting diversification of potential buyer</td>
</tr>
<tr>
<td>Cost of debt</td>
<td>May reflect additional spread associated with being a private business</td>
</tr>
<tr>
<td>Operating cash flows</td>
<td>Private business tax rate used in valuation</td>
</tr>
<tr>
<td>Firm life</td>
<td>Finite life terminal value or liquidation value</td>
</tr>
<tr>
<td>Illiquidity discount</td>
<td>Value discounted for illiquidity</td>
</tr>
</tbody>
</table>
ILLUSTRATION 24.11:

ILLUSTRATION 24.12:

AUTHOR: Inserts not received for replacement Illustrations 24.11 and 24.12
Illustration 24.11: Valuing Chez Pierre for a private sale

To value Chez Pierre for a private sale, we draw on the inputs that we have estimated in prior illustrations:

- After-tax operating income in most recent year = $319,620 (1-.40) = $191,770 (from illustration 24.7)
- Cost of capital = 15.27% (from illustration 24.6)
- Expected Growth rate = 2% a year for the next 12 years (from illustration 24.8)
- Reinvestment rate = 0% (from illustration 24.8)

Pre-tax operating income in most recent year = $319,620 (from illustration 24.7)
FCFF in most recent year = $319,620 (1-.40) (1-0) = $191,770

At the end of year 12, we assume that the restaurant will be closed and that the liquidation proceeds will be $500,000. (from illustration 24.9)

The present value of the operating cash flows over the next 12 years is estimated first:

\[
PV_{\text{of FCFF for next 12 years}} = \frac{191,770(1 - \frac{1.02^{12}}{1.1527^{12}})}{.1527 -.02} = \$1,134,121
\]

Adding the present value of the liquidation proceeds (discounted back 12 years at 15.27%) and subtracting out the present value of lease commitments yields the value of equity:

PV of operating cash flows for next 12 years = $1,134,121
PV of liquidation value = $500,000/1.1527^{10} = $90,821
- PV of operating lease commitments = $928,333
Value of equity = $296,709

Based on our estimates for growth and cost of capital, the value of the equity in Chez Pierre is $296,709.

Illustration 24.12: Valuing Infosoft for an initial public offering

To value Infosoft for an initial public offering, we gather the inputs that we have already estimated for the company:

a. Cost of capital: In illustration 24.6, we use a bottom-up market beta for Infosoft and estimated a cost of equity and capital of 11.50%:
   - Cost of equity = 3.5% + 1.60 (5%) = 11.50%
   - Cost of capital = Cost of equity = 11.50%

   In the terminal value computation (in illustration 24.9), we argued for a decrease in beta of 1.20 and an increase in the debt ratio to 10%, leading to a cost of capital of 8.85% in stable growth.

b. Cash flows and growth rates: In illustration 24.7, we presented the current income statement for Infosoft, with pre-tax operating income of $1.5 million and a tax rate of 40%. In illustration 24.8, we estimated a return on capital of 20% and a reinvestment rate of 90% for Infosoft, which we assumed would be maintained for the next 5 years, allowing for expected growth of 19% in earnings.
c. *Terminal value*: In illustration 24.9, we assumed that the firm would be in stable growth after year 10, growing 3% a year, while maintaining a return on capital of 12%. The five years between the high growth period (lasting 5 years) and the stable growth (after year 10) represent a transition phase, where growth rates, reinvestment rates and costs of capital all change from high growth levels to stable growth levels. The table below summarizes the cash flows and the present value (in '000s).

<table>
<thead>
<tr>
<th>Year</th>
<th>EBIT (1-t)</th>
<th>Expected growth</th>
<th>Reinvestment Rate</th>
<th>FCFF</th>
<th>Cost of capital</th>
<th>Cumulated cost of capital</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>$900</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$1,062</td>
<td>18.00%</td>
<td>90.00%</td>
<td>$106</td>
<td>11.50%</td>
<td>1.1150</td>
<td>$95</td>
</tr>
<tr>
<td>2</td>
<td>$1,253</td>
<td>18.00%</td>
<td>90.00%</td>
<td>$125</td>
<td>11.50%</td>
<td>1.2432</td>
<td>$101</td>
</tr>
<tr>
<td>3</td>
<td>$1,479</td>
<td>18.00%</td>
<td>90.00%</td>
<td>$148</td>
<td>11.50%</td>
<td>1.3862</td>
<td>$107</td>
</tr>
<tr>
<td>4</td>
<td>$1,745</td>
<td>18.00%</td>
<td>90.00%</td>
<td>$174</td>
<td>11.50%</td>
<td>1.5456</td>
<td>$113</td>
</tr>
<tr>
<td>5</td>
<td>$2,059</td>
<td>18.00%</td>
<td>90.00%</td>
<td>$206</td>
<td>11.50%</td>
<td>1.7234</td>
<td>$119</td>
</tr>
<tr>
<td>6</td>
<td>$2,368</td>
<td>15.00%</td>
<td>77.00%</td>
<td>$545</td>
<td>10.97%</td>
<td>1.9124</td>
<td>$285</td>
</tr>
<tr>
<td>7</td>
<td>$2,652</td>
<td>12.00%</td>
<td>64.00%</td>
<td>$955</td>
<td>10.44%</td>
<td>2.1121</td>
<td>$452</td>
</tr>
<tr>
<td>8</td>
<td>$2,891</td>
<td>9.00%</td>
<td>51.00%</td>
<td>$1,416</td>
<td>9.91%</td>
<td>2.3214</td>
<td>$610</td>
</tr>
<tr>
<td>9</td>
<td>$3,064</td>
<td>6.00%</td>
<td>38.00%</td>
<td>$1,900</td>
<td>9.38%</td>
<td>2.5391</td>
<td>$748</td>
</tr>
<tr>
<td>10</td>
<td>$3,156</td>
<td>3.00%</td>
<td>25.00%</td>
<td>$2,367</td>
<td>8.85%</td>
<td>2.7638</td>
<td>$856</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3,487</td>
</tr>
</tbody>
</table>

At the end of year 10, the firm is in stable growth and the terminal value is estimated as follows:

$$\text{Terminal value } (\text{000s}) = \frac{\text{EBIT(1- tax rate)_{10}} \times (1+g)(1-\text{Reinvestment Rate})}{(\text{Cost of capital } - g)}$$

$$= \frac{3,156 (1.03)(1-.25)}{(.0885 - .03)} = 41,675$$

Discounting the terminal value back, adding it to the present value of cash flows during the high growth period and subtracting out debt (zero) yields a value of $19,065 million for the equity.

Value of equity = $3,487 + $41,675/2.7638 + 500 = $19,066

d. *Getting to value of equity per share*: To get to value of equity per share, we value the 100,000 equity options that are held by managers and venture capitalists in the company, subtract the value from the value of equity, before dividing by the 1 million shares outstanding.

Value of management options (in '000s) = $1,161

Value of equity = ($19,066 - $1,161)/1000 = $17.90/share
Control Issues

When valuing a firm, you always need to consider the competence and strengths of the management of the firm. With private firms, where the owner is also the manager, this consideration carries special weight, since the owner has absolute control. In a publicly traded firm, in contrast, incompetent management can often be replaced, if enough stockholders can be convinced that it is in their best interests to do so.

There are implications for valuation if a portion of a private firm is offered for sale. If that portion provides a controlling interest (i.e., the right to pick the firm’s management), it should have a substantially higher value than if it does not provide this power. Normally, this would mean that 51 percent of a private firm’s equity should trade at a substantial premium over 49 percent. This applies whether a firm is being sold to a private entity or a publicly traded firm, and may arise in an initial public offering. If, for instance, only nonvoting shares or shares with diluted voting rights are offered to investors in the public offering, they should trade at a discount on shares with full voting rights.

While the intuition about the value of control is simple, estimating how much it is worth is a little more difficult. We will defer a full discussion of the topic until the next chapter, on acquisitions, but we will value it as the difference between two values—the value of the firm run optimally and the value of the firm with the incumbent management. For instance, if the value of a private firm run by incumbent management is $100 million and the value of the firm run optimally is $150 million, the difference in values between the 51 percent and 49 percent shares can be computed as follows:

\[
\text{Value of controlling interest} = 51\% \text{ of optimal value} = .51 \times 150 = \$76.5 \text{ million}
\]

\[
\text{Value of noncontrolling interest} = 49\% \text{ of status quo value} = .49 \times 100 = \$49 \text{ million}
\]

The additional 2 percent interest (from 49 to 51 percent) has a disproportionate effect on value because of control. This value of control will be greatest for private firms that are poorly run and will be close to zero for well-run firms.

In fact, the same approach can be used to compute the discount that nonvoting shares will trade at relative to voting shares in initial public offerings. For instance, assume that this private firm creates 10 million voting shares and offers 70 percent to the public. Since the potential for changing management is created by this offering, the value per share will fall between $10 and $15, depending on the probability that is attached to the management change. Thus, if the probability of the management change is 60 percent, the value per share will be $13.

\[
\text{Value per share} = \left( \frac{\text{Optimal value} - \text{Status-quo value}}{\text{Number of shares} - \text{Number of shares}} \right) \times \text{Probability of change} + \frac{\text{Status-quo value}}{\text{Number of shares}}
\]

\[
= \$100/10 + [(150 - 100) \times .6]/10 = \$13
\]

Now assume that this firm had issued 9 million nonvoting shares, with management retaining 1 million voting shares with complete control. In this case, the nonvoting shares will get little or none of the estimated value change from
optimal management. In fact, the values of the two classes can be estimated as follows:

\[
\text{Value per nonvoting share} = \frac{\text{Status quo value}}{(\text{Number of voting shares} + \text{Nonvoting shares})} = \frac{\$100}{9 + 1} = \$10 \text{ per share}
\]

\[
\text{Value per voting share} = \frac{\text{Status quo value}}{(\text{Number of voting shares} + \text{Nonvoting shares})} \times \frac{(\text{Optimal value} - \text{Status quo value})}{\text{Number of voting shares}} = \frac{\$100}{9 + 1} \times \frac{(\$20 - \$100) \times 0.6}{1} = \$40
\]

The voting shares in this case would trade at an enormous premium over the non-voting shares, but that is because we have assumed that the probability of change is still 60 percent. If the incumbent managers are much more likely to fight a change in management, this probability will drop and reduce the premium with it.

**ILLUSTRATION 24.13: Valuing Voting and Non-voting Shares: Infosoft**

In the last illustration, we valued the equity in InfoSoft at $17.904 million. Based upon the 10 million shares outstanding, we estimated a value per share of $17.90. Assume that the firm decides to create 9 million nonvoting shares and 1 million voting shares. In the initial offering, only the nonvoting shares will be sold to the public, and the current owners will retain all of the voting shares.

To value the voting and nonvoting shares, we need to value InfoSoft under optimal management. Assume that the firm would be worth $20 million under optimal management. The value of the voting and nonvoting shares can then be computed.

\[
\text{Value: nonvoting share} = \frac{\text{Status quo value}}{\frac{\# \text{Voting shares} + \# \text{Nonvoting shares}}{\# \text{Voting shares} + \# \text{Nonvoting shares}}} = \frac{\$17.904}{1 + 9} = \$17.90
\]

Assume that the fact that incumbent managers will retain the voting shares reduces the probability of management change to 25%.

\[
\text{Value per voting share} = \frac{\text{Status quo value}}{\frac{\# \text{Voting shares} + \# \text{Nonvoting shares}}{\# \text{Voting shares} + \# \text{Nonvoting shares}}} \times \frac{(\text{Optimal value} - \text{Status quo value})}{\# \text{Voting Shares}} = \frac{\$17.904}{1 + 9} \times \frac{($20 - \$17.904)(0.25)}{1} = \$23.14
\]

*4InfoSoft was revalued at its optimal debt ratio. We assumed that the existing investment policy was optimal.*
In the previous illustrations we looked at two extremes in the private company valuation spectrum: a private-to-private transaction, where neither the buyer nor the seller was completely undiversified (and thus exposed to total risk) and a private-to-public transaction, an IPO, or sale of a private to a public company, where more conventional valuation approaches work.

There is an intermediate case, where venture capitalists and private equity investors take stakes in private businesses, with the intent of cashing out when the company goes public or is sold to a public entity. Venture capitalists and private equity investors are more diversified than private owners, but they are not as diversified as investors in public markets are for two reasons. The first is that they specialize in a few businesses: Many venture capitalists invest only in biotechnology companies or software businesses. The second is that the size of their positions (which tend to be large) and the need for oversight restrict them from having more than a handful of open positions at any time.

In a sense, if you think of total beta as the appropriate measure for the completely undiversified end of the spectrum and market beta as the right proxy for the completely diversified, venture capitalists and private equity investors fall in the middle, with variations across even these investors. In fact, we can modify the total beta equation to reflect these differences:

\[ \text{VC or Private equity beta} = \frac{\text{Market beta}}{\text{Correlation of VC or PE investor's portfolio with market}} \]

Thus, we are replacing the correlation of the private firm with the market with the correlation of the investor's portfolio. At the limit, a private equity investor like Blackstone or KKR may have so many holdings spread over so many different businesses that the correlation of their portfolios with the market may approach one and the beta that they should use is a market beta.

To see how this will play out, assume that you are valuing a private business operating in a sector where publicly traded companies have an average beta of 1 and where the average correlation of firms with the market is 0.25. Assume that this company will be fully owned by its current owner for two years, will access a technology venture capitalist at the start of year 3, and is expected to either go public or be sold to a publicly traded firm at the end of year five. We estimate the cost of equity at three stages (Risk-free rate = 4%; Equity risk premium = 5%):

**Stage 1:** The nascent business, with a private owner, who is fully invested in that business.
- Perceived beta = \( \frac{1}{0.25} = 4 \)
- Cost of equity = 4% + 4 (5%) = 24%

**Stage 2:** Angel financing provided by specialized venture capitalist, who holds multiple investments, in high technology companies. (Correlation of portfolio with market is 0.5)
- Perceived beta = \( \frac{1}{0.5} = 2 \)
- Cost of equity = 4% + 2 (5%) = 14%

**Stage 3:** Public offering, where investors are retail and institutional investors, with diversified portfolios:
- Perceived beta = 1
- Cost of equity = 4% + 1 (5%) = 9%
Now assume that you have projected cash flows for this company for the next five years and expect it to be a stable growth firm after it goes public in year five, growing 2% a year in perpetuity. The value of the business can be estimated as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected CF</th>
<th>Market beta</th>
<th>Correlation</th>
<th>Beta used</th>
<th>Cost of equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$100</td>
<td>1</td>
<td>0.25</td>
<td>4</td>
<td>24.00%</td>
</tr>
<tr>
<td>2</td>
<td>$125</td>
<td>1</td>
<td>0.25</td>
<td>4</td>
<td>24.00%</td>
</tr>
<tr>
<td>3</td>
<td>$150</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>14.00%</td>
</tr>
<tr>
<td>4</td>
<td>$165</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>14.00%</td>
</tr>
<tr>
<td>5</td>
<td>$170</td>
<td>1</td>
<td>0.5</td>
<td>2</td>
<td>14.00%</td>
</tr>
<tr>
<td>Terminal</td>
<td>$175</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>9.00%</td>
</tr>
</tbody>
</table>

| Cumulated COE | $2,500 |
| PV | $80.65 | $81.30 | $85.57 | $82.57 | $1,172.07 |
| Value of firm | $1,502 |

Note that using the private owner’s cost of equity (24%) forever would have yielded too low a value ($1,221) and using the market beta cost of equity (9%) forever would resulted in too high a value ($2,165).

## Precash and Postcash Valuations

When valuing private companies, many analysts draw a distinction between precash and postcash valuations. In general, this is done especially when an infusion of cash is anticipated either from venture capitalists or from an initial public offering. The precash valuation values the firm before the cash influx and the postcash valuation values it after.

There are two reasons why the two valuations may be different. The first is that the firm may face capital rationing constraints without the infusion of the cash, resulting in a scaling down of how much the firm can reinvest. If the firm’s return on capital is greater than the cost of capital, this will cause the value to be lower before the cash influx. The second is that the value of cash and marketable securities will be added to the value of the operating assets to arrive at firm value. After a large cash influx, firms may have excess cash to invest in marketable securities, which when added to the value of operating assets will increase value. If the cash is taken out of the firm, though, by the existing owners, you should not add the cash to the value.

Which of these two values should be used to estimate the value per share in a public offering? Since stockholders in the firm will hold stock in the postcash firm, the postcash value should be used. In the case of a venture capitalist, though, the answer may be different. If the venture capitalist has bargaining power—she is the only person who is interested in providing venture capital—she can ask for a share of the firm value based on the precash valuation, arguing that the increase in value is feasible only with the additional venture capital. If two or more venture capitalists are interested in the firm, odds are that the postcash valuations will be the basis for deciding how much of the firm will be yielded to the venture capitalist.
ILLUSTRATION 24.14: Valuing a Private Equity Stake

Assume that you work for a publicly traded firm and have been asked to value a potential stake in a small, privately held firm that wants you to invest $10 million in its equity, which it plans to use to expand operations. First, you would value the private firm assuming that you do not invest the $10 million. Based on the projected cash flows, assume that you value the equity in the firm at $30 million:

\[
\text{Precash valuation} = 30 \text{ million}
\]

Now assume that your investment of $10 million will allow the firm to grow faster and that the present value of the expected cash flows is $50 million for the equity. (This present value does not include the cash inflow of $10 million from the private equity investment.)

\[
\text{Postcash valuation} = 50 \text{ million} + 10 \text{ million} = 60 \text{ million}
\]

The key question, assuming that you decide to make this investment, is the percentage of the private firm you should demand in return for the $10 million investment. At the minimum, you would demand a share of the postcash valuation:

\[
\text{Share of ownership}_{\text{minimum}} = \frac{\text{Cash invested}}{\text{Postcash valuation}} = \frac{10}{60} = 16.66\%
\]

However, you would bargain for a larger share. At the limit, you could argue for a share of the precash valuation:

\[
\text{Share of ownership}_{\text{maximum}} = \frac{\text{Cash investment}}{(\text{Precash valuation} + \text{Cash investment})} = \frac{10}{(30 + 10)} = 25\%
\]

Relative Valuation of Private Businesses

The essence of relative valuation is that you value a firm, based upon how much the market is paying for similar firms. This premise is clearly more challenging for private businesses. Notwithstanding these problems, analysts have tried to extend the relative valuation practices that have been developed for public companies into the private business space. In general, the biggest area of difference across analysts who value private businesses lies in where they go to get the comparable firms. Some analysts focus on transaction prices paid for other private businesses, arguing that these businesses are likely to have more in common with the young business being valued. Other analysts, distrustful of private transaction prices, draw on the market prices of publicly traded companies in the same business, and try to adjust for differences in fundamentals.

Private Transaction Multiples Since we are valuing a young, private business, it seems logical that we should look at what others have paid for similar businesses in the recent past. That is effectively the foundation on which private transaction multiples are based. In theory, at least, we pull together a dataset of other young, private businesses, similar to the one that we are valuing (same business, similar size, and at the same stage in the life cycle), that have been bought/sold and the transaction values. We then scale these values to a common variable (revenues, earnings or something even sector-specific) and compute a typical multiple that acquirers have been willing to pay. Applying this multiple to the same variable for the company being valued should yield an estimated value for the company.

Potential Problems While the biggest problem used to be the absence of organized databases of private business transactions, that is no longer the case. Many private services offer databases (for a price) that contain this data, but other problems remain:

\[5\] BIZCOMPS, IBA Market Data and Pratt Stats all provide transaction data for private businesses.
Arms-length transactions: One of the perils of using prices from private transactions is that some of them are not arms-length transactions, where the price reflects just the business being sold. In effect, the price includes other services and side factors that may be specific to the transaction. Thus, a doctor selling a medical practice may get a higher price because he agrees to stay on for a period of time after the transaction to ease the transition.

Timing differences: Private business transactions are infrequent and reflect the fact that the same private business will not be bought and sold dozens of times during a particular period. Unlike public firms, where the current price can be used to compute the multiples for all firms at the same point in time, private transactions are often staggered across time. A database of private transactions can therefore include transactions from June 2008 and December 2008, a period when the public markets lost almost 45 percent of their value.

Scaling variable: To compare firms of different scale, we generally divide the market price by a standardizing variable. With publicly traded firms, this can take the form of revenues (Price/Sales, EV/Sales), earnings (PE, EV/EBITDA), or book value. While we could technically do the same with private transactions, there are two potential roadblocks. The first is that young firms have little to show in terms of current revenues and earnings, and what they do show may not be a good indication of their ultimate potential. The second is that there are broad differences in accounting standards across private businesses and these differences can result in bottom lines that are not quite equivalent.

Nonstandardized equity: As we noted in the last section, equity claims in young private businesses can vary widely in terms of cash flow, control claims, and illiquidity. The transaction price for equity in a private business will reflect the claims that are embedded in the equity in that business and may not easily generalize to equity in another firm with different characteristics.

Non-U.S. firms: Most of the transaction databases that are available and accessible today are databases of transactions of private businesses in the United States. As we are called upon increasingly to value young businesses in other markets, some of which are riskier, emerging markets, it is not clear how or even whether this data can be used in that context.

Usefulness and Best Practices
So, when is it appropriate to use private transaction data to value a young private business? As a general rule, this approach works best for small businesses that plan to stay small and private, rather than expand their reach and perhaps go public. It also helps if the firm being valued is in a business where there are not only a large number of other private businesses but also where transactions are common. For instance, this approach should work well for valuing a medical/dental practice or a small, retail business. It will get more difficult to apply for firms that are in unique or unusual businesses.

If we decide to employ private company transactions to value a young business, there are five general practices that can help to deliver more dependable valuations:

1. Scale to variables that are less affected by discretionary choices: As a counter to the problem of wide differences in accounting and operating standards across private companies, we can focus on variables where discretionary choice matters less. For instance, multiples of revenues (which are more difficult to fudge or manipulate) should be preferred to multiples of earnings. We could even scale...
value to units specific to the business being valued—number of patients for a
general medical practice or the number of customers for a plumbing business.

2. **Value businesses, not equity:** In Chapter 4, we classified multiples into equity
multiples (where equity value is scaled to equity earnings or book value) and
enterprise value multiples (where the value of the business is scaled to operat-
ing earnings, cash flows, or the book value of capital). Given the wide differ-
ences in equity claims and the use of debt across private businesses, it is better
to focus on enterprise value multiples rather than equity multiples. In other
words, it is better to value the entire business and then work out the value of
equity than it is to value equity directly.

3. **Start with a large dataset:** Since transactions with private businesses are infre-
fquent, it is best to start with a large dataset of companies and collect all trans-
action data. This will then allow us to screen the data for transactions that look
suspicious (and are thus likely to fail the arm’s-length test).

4. **Adjust for timing differences:** Even with large datasets of private transactions,
there will be timing differences across transactions. While this is not an issue in a
period where markets are stable, we should make adjustments to the value
(even if they are crude) to account for the timing differences. For instance, us-
ing June 2008 and December 2008 as the transaction dates, we would reduce
the transaction prices from June 2008 by the drop in the public market (a small
cap index like the Russell 5000 dropped by about 40 percent over that period)
to make the prices comparable.

5. **Focus on differences in fundamentals:** The notion that the value of a business
depends on its fundamentals—growth, cash flows, and risk—cannot be aban-
donned just because we are doing relative valuation. The estimated value is
likely to be more reliable if we can collect other measures of the transacted pri-
vate businesses that reflect these fundamentals. For instance, it would be useful
to obtain not only the transaction prices of private businesses but also the
growth in revenues recorded in these businesses in the period prior to the trans-
actions and the age of the business (to reflect maturity and risk). We can ex-
plain the data to see if there is a relationship between transaction value and
these variables, and if there is one, to build it into the valuation.

**Public Multiples** It is far easier to obtain timely data on pricing and multiples for
publicly traded firms. In fact, for those analysts who do not have access to private
transaction data, this is the only option when it comes to relative valuation. The
peril, though, is that we are extending the pricing lessons that we learn from look-
ing at more mature, publicly traded firms to a young private business.

**Problems** The issues we face in applying public market multiples to private busi-
nesses, especially early in the life cycle, are fairly obvious:

- **Life cycle affects fundamentals:** If we accept the premise that only those young
  firms that make it through the early phase of the life cycle and succeed are
  likely to go public, we also have to accept the reality that public firms will have
different fundamentals from private firms. Generally, public firms will be
  larger, have less potential for growth, and have more established markets than
private businesses, and these differences will manifest themselves in the multi-
plies investors pay for public companies.
Survival: A related point is that there is a high probability of failure in young firms. However, this probability of failure should decrease as firms establish their product offerings, and those that go public should have a greater chance of surviving than younger private firms. The former should therefore trade at higher market values, for any given variable such as revenues, earnings, or book value, holding all else (growth and risk) constant.

Diversified versus undiversified investors: When we discussed estimating risk and discount rates for young, private businesses, we noted the different perspectives on risk that diversified investors in public companies have, relative to equity investors in private businesses, and how that difference can manifest itself as higher costs of equity for the latter. When we use multiples of earnings or revenues, obtained from a sample of publicly traded firms with diversified investors, to value a private business with undiversified investors, we will overvalue the latter.

Scaling variable: Assuming that we are able to obtain a reasonable multiple of revenues or earnings from our public company dataset, we face one final problem. Young firms often have very little revenues to show in the current year, and many will be losing money; the book value is usually meaningless. Applying a multiple to any one of these measures will result in strange valuations.

Liquidity: Since equity in publicly traded companies is more liquid than equity in private businesses, the value obtained by using public multiples will be too high if used for a private business. Just as we had to adjust for illiquidity in intrinsic valuation, we have to adjust for illiquidity with relative valuation.

Usefulness and Best practices What types of private businesses are best valued using public company multiples? Generally, young companies that aspire to not only reach a larger market and either go public or be acquired by a public company are much better candidates for this practice. In effect, we are valuing the company for what it wants to be, rather than what it is today.

There are five simple practices that can not only prevent egregious valuation errors but also lead to better valuations:

1. Use forward revenues/earnings: One of the problems we noted with using multiples on young companies is that the current operations of the company do not provide much in terms of tangible results: Revenues are very small, and earnings are negative. One solution is to forecast the operating results of the firm further down the life cycle and use these forward revenues and earnings as the basis for valuation. In effect, we will estimate the value of the business in five years, using revenues or earnings from that point in time.

2. Adjust the multiple for your firm’s characteristics at time of valuation: If we are valuing the firm five years down the road, we have to estimate a multiple that is appropriate for the firm at that point in time, rather than today. Consider a simple illustration. Assume that you have a company that is expected to generate a compounded revenue growth of 50 percent a year for the next five years, as it scales from being a very small firm to a more established enterprise. Assume that revenue growth after year 5 will drop to a more moderate compounded annual rate of 10 percent. The multiple that we apply to revenues or earnings in year 5 should reflect an expected growth rate of 10 percent (and not 50 percent).
3. Adjust for survival: When we estimated the intrinsic value for young firms, we allowed for the possibility of failure by adjusting the value for the probability that the firm would not make it. We should stick with that principle, since the value based upon future revenues/earnings is implicitly based upon the assumption that the firm survives and succeeds.

4. Adjust for nondiversification: The value estimated for the firm or equity, based upon future earnings and revenues, has to be discounted back to the present to arrive at the value today. By using the techniques that we developed for adjusting the beta and cost of equity for private businesses in the intrinsic value section, we can discount for the forecasted future value of the business by a high enough rate, to reflect the non-diversification of equity investors today. In effect, we are assuming that he firm will go public in the future year (where the multiple is applied) and that the nondiversification issue will dissipate.

5. Adjust for illiquidity: In the last section on intrinsic valuation, we presented different ways of estimating illiquidity discounts for equity in private businesses. We could adopt the same techniques to adjust the public multiple value for illiquidity.

CONCLUSION

The value of a private firm is the present value of the cash flows it is expected to generate, discounted back at a rate that reflects both the risk in the private firm and the mix of debt and equity it uses. While this statement is identical to the one used to describe the value of a publicly traded firm, there are differences in the way we estimate these inputs for private firms, and even among private firms, depending on the motive for the valuation.

When valuing a private firm for sale to an individual or private entity, we have to consider three specific issues. The first is that the cost of equity, which we have hitherto assumed to be determined purely by the risk that cannot be diversified, might have to be adjusted for the fact that the potential buyer is not well diversified. The second is that equity holdings in private businesses are illiquid, leading to a discount on the estimated value. The discounts on restricted stock issues made by publicly traded firms or the bid-ask spreads of these firms may provide us with useful information on how large this discount should be. The third is that a controlling interest in equity of a private firm can trade at a significant premium over a minority interest.

The valuation of a private firm for sale to a publicly traded firm or initial public offering follows a much more conventional route. We can continue to assume that the cost of equity should be based only on nondiversifiable risk and there is no need for an illiquidity discount. There can still be a control value if less than a controlling interest is sold to the publicly traded firm or if nonvoting shares are issued in the initial public offering.

QUESTIONS AND SHORT PROBLEMS

In the problems following, use an equity risk premium of 5.5 percent if none is specified.

1. You have been asked to value Barrista Espresso, a chain of espresso coffee shops that have opened on the East Coast of the United States.

   • The company had earnings before interest and taxes of $10.50 million in the most recent year on revenues of $50 million. However, the founders of
the company had never charged themselves a salary, which would have amounted to $1 million if based on comparable companies.

- The tax rate is 36% for all firms, and working capital is 10% of revenues.
- The capital expenditures in the most recent year amounted to $4.5 million, while depreciation was only $1 million.
- Earnings, revenues, and net capital expenditures are expected to grow 30% a year for five years, and 6% after that forever.
- The comparable firms have an average beta of 1.3567 and an average D/E ratio of 13.65%. The average correlation with the market is 0.50. Barrista Espresso is expected to maintain a debt ratio of 12% and face a cost of debt of 8.75%. The risk-free rate is 6%, and the market risk premium is 5.5%.

a. Estimate the value of Barrista Espresso as a firm.
b. Estimate the value of equity in Barrista Espresso.
c. Would your valuation be different if you were valuing the firm for an IPO?

2. You have valued a business, using discounted cash flow models, at $250 million for a private sale. The business, which does make money, had revenues of $200 million in the most recent year. (The average firm has revenues of $10 million.) How much of a liquidity discount would you apply to this firm:

a. Based on the Silber regression?
b. Based on correcting the average discount (25%) for the size of the firm?

3. You are valuing a bed-and-breakfast in Vermont with the following information:

- The business had pretax operating income of $100,000 in the most recent year. This income has grown 5% a year for the past three years, and is expected to continue growing at that rate for the foreseeable future.
- About 40% of this operating income can be attributed to the fact that the owner is a master chef. He does not plan to stay on if the business is sold.
- The business is financed equally with debt and equity. The pretax cost of borrowing is 8%. The beta for publicly traded firms in the hospitality business is 1.10. The Treasury bond rate is 7%, the market risk premium is 5.5%, and the tax rate is 40%.
- The capital maintenance expenditure, net of depreciation, was $10,000 in the most recent year, and it is expected to grow at the same rate as operating income.
- The business is expected to have an operating life of 10 years, after which the building will be sold for $500,000, net of capital gains taxes.

a. Value the business for sale.
b. How much would the value change if the owner offered to stay on for the next three years?

4. You have been asked by the owner of Tectonics Software, a small firm that produces and sells computer software, to come up with an estimate of value for the firm for an initial public offering. The firm had revenues of $20 million in the most recent year, on which it made earnings before interest and taxes of $2 million. The firm had debt outstanding of $10 million, on which pretax interest expenses amounted to $1 million. The book value of equity is $10 million. The average unlevered beta of publicly traded software firms is 1.20, and the average market value of equity of these firms is, on average, three times the book value of equity. All firms face a 40% tax rate. Capital expenditures amounted to $1 million in the most recent year and were twice the depreciation charge in that year. Both items are expected to grow at the same rate as revenues for the next five years. The
return on capital after year 5 is expected to be 15%. The revenues of this firm are expected to grow 20% a year for the next five years and 5% after that, and the operating margins will remain at existing levels. The Treasury bond rate is 6%.

a. Estimate the cost of capital for the firm.
b. Estimate the value of the equity in the firm.
c. If the firm plans to issue 1 million shares, estimate the value per share.

5. How would your answer to (4) change if you were valuing Tectonics Software for sale to a private individual? The individual in question has a portfolio that is not diversified and has a correlation of 0.60 with the market index. In addition, use the following bid-ask spread equation to estimate the illiquidity discount:

\[
\text{Bid-ask spread} = 0.14 - 0.015 \ln(\text{Revenues})
\]

Estimate the value of equity in the private transaction.