Solutions to Present Value Problems

Present Value: Solutions

Problem 1
a. Current Savings Needed = $500,000/1.110 = $ 192,772
b. Annuity Needed = $500,000 (APV,10%,10 years) = $31,373

Problem 2
Present Value of $1,500 growing at 5% a year for next 15 years = $ 18,093
Future Value = $18093 (1.08^15) = $57,394

Problem 3
Annual Percentage Rate = 8%
Monthly Rate = 8%/12 = 0.67%
Monthly Payment needed for 30 years = $200,000(APV,0.67%,360) = $1,473

Problem 4
a. Discounted Price Deal
Monthly Cost of borrowing $18,000 at 9% APR = $373.65
[ A monthly rate of 0.75% is used]
b. Special Financing Deal
Monthly Cost of borrowing $20,000 at 3% APR = $359.37
The second deal is the better one.

Problem 5
a. Year-end Annuity Needed to have $100 million available in 10 years = $6.58
[FV = $100, r = 9%, n = 10 years]
b. Year-beginning Annuity Needed to have $100 million in 10 years = $6.04

Problem 6
Value of 15-year corporate bond; 9% coupon rate; 8% market interest rate
Assuming coupons are paid semi-annually,
Value of Bond = 45*(1-1.04^(-30))/.04+1000/1.04^30 = $1,086.46
If market interest rates increase to 10%,
Value of Bond = 45*(1-1.05^(-30))/.05+1000/1.05^30 = $923.14
The bonds will trade at par only if the market interest rate = coupon rate.

Problem 7
Value of Stock = 1.50 (1.06)/ (.13 -.06) = $22.71

Problem 8
Value of Dividends during high growth period = $1.00 (1.15)(1-1.15^5/1.125^5)/(1.125-.15)
Expected Dividends in year 6 = $1.00 (1.15)^5*1.06*2 = $4.26
Expected Terminal Price = $4.26/(.125-.06) = $65.54
Value of Stock = $5.34 + $65.54/1.125^5 = $41.70

Problem 9
Expected Rate of Return = (1000/300)^ (1/10) - 1 = 12.79%

Problem 10
Effective Annualized Interest Rate = (1+.09/52)^52 - 1 = 9.41%
Problem 11
Annuity given current savings of $250,000 and $25 = $17,738.11

Problem 12
PV of first annuity - $20,000 a year for next 10 years = $128,353.15
PV of second annuity discounted back 10 years = $81,326.64
Sum of the present values of the annuities = $209,679.79

If annuities are paid at the start of each period,
PV of first annuity - $20,000 at beginning of each year = $148,353.15
PV of second annuity discounted back 10 years = $88,646.04
Sum of the present values of the annuities = $236,999.19

Problem 13
PV of deficit reduction can be computed as follows –

<table>
<thead>
<tr>
<th>Year</th>
<th>Deficit Reduction</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$25.00</td>
<td>$23.15</td>
</tr>
<tr>
<td>2</td>
<td>$30.00</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>$75.00</td>
<td>$34.74</td>
</tr>
<tr>
<td>Sum</td>
<td>$500.00</td>
<td>$311.22</td>
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</tbody>
</table>

The true deficit reduction is $311.22 million.

Problem 14
a. Annuity needed at 6% = 1.89669896 (in billions)
b. Annuity needed at 8% = 1.72573722 (in billions)
Savings = 0.17096174 (in billions)
This cannot be viewed as real savings, since there will be greater risk associated with the higher-return investments.

Problem 15
a. Year Nominal PV
0    $5.50    $5.50
1    $4.00    $3.74
2    $4.00    $3.49
3    $4.00    $3.27
4    $4.00    $3.05
5    $7.00    $4.99
   $28.50    $24.04

b. Let the sign up bonus be reduced by X.
Then the cash flow in year 5 will have to be raised by X + 1.5 million, to get the nominal value of the contract to be equal to $30 million.
Since the present value cannot change,
X - (X+1.5)/1.075 = 0
X (1.075 - 1) = 1.5
Solutions to Present Value Problems

\[ X = \frac{1.5}{(1.075 - 1)} = 3.73 \text{ million} \]

The sign up bonus has to be reduced by $3.73 million and the final year's cash flow has to be increased by $5.23 million, to arrive at a contract with a nominal value of $30 million and a present value of $24.04 million.

**Problem 16**

<table>
<thead>
<tr>
<th></th>
<th>Chatham</th>
<th>South Orange</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage</td>
<td>$300,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>Monthly Paym</td>
<td>$2,201</td>
<td>$1,468</td>
</tr>
<tr>
<td>Annual Paym</td>
<td>$26,416</td>
<td>$17,610</td>
</tr>
<tr>
<td>Property Tax</td>
<td>$6,000</td>
<td>$12,000</td>
</tr>
<tr>
<td>Total Payment</td>
<td>$32,416</td>
<td>$29,610</td>
</tr>
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</table>

b. Mortgage payments will end after 30 years. Property taxes are not only a perpetuity; they are a growing perpetuity. Therefore, they are likely to be more onerous.

c. If property taxes are expected to grow at 3% annually forever,

\[ \text{PV of property taxes} = \text{Property tax} \times \frac{1 + g}{r - g} \]

For Chatham, \( \text{PV of property tax} = 6000 \times \frac{1.03}{0.08 - 0.03} = 123,600 \)

For South Orange, \( \text{PV of property tax} = 12,000 \times \frac{1.03}{0.08 - 0.03} = 247,200 \)

To make the comparison, add these to the house prices,

Cost of the Chatham house = $400,000 + $123,600 = $523,600

Cost of the South Orange house = $300,000 + $247,200 = $547,200

The Chatham house is cheaper.

**Problem 17**

a. Monthly Payments at 10% on current loan = $1,755.14

b. Monthly Payments at 9% on refinanced mortgage = $1,609.25

Monthly Savings from refinancing = $145.90

c. Present Value of Savings at 8% for 60 months = $7,195.56

Refinancing Cost = 3% of $200,000 = $6,000

d. Annual Savings needed to cover $6,000 in refinancing cost = $121.66

Monthly Payment with Savings = $1755.14 - $121.66 = $1,633.48

Interest Rate at which Monthly Payment is $1633.48 = 9.17%

**Problem 18**

a. Present Value of Cash Outflows after age 65 = $300,000 + PV of $35,000 each year for 35 years = $707,909.89

b. FV of Current Savings of $50,000 = $503,132.84

Shortfall at the end of the 30th year = $204,777.16

Annuity needed each year for next 30 years for FV of $204777 = $1,807.66

c. Without the current savings,

Annuity needed each year for 25 years for FV of $707910 = $9,683.34

**Problem 19**

a. Estimated Funds at end of 10 years:

- FV of $5 million at end of 10th year = $10.79 (in millions)
- FV of inflows of $2 million each year for next 5 years = $17.24
- FV of outflows of $3 million each year for years 6-10 = $17.60

= Funds at end of the 10th year = $10.43

b. Perpetuity that can be paid out of these funds = $10.43 (.08) = $0.83
Problem 20
a. Amount needed in the bank to withdraw $80,000 each year for 25 years = $1,127,516
b. Future Value of Existing Savings in the Bank = $407,224
   Shortfall in Savings = $1,127,516 - $407,224 = $720,292
   Annual Savings needed to get FV of $720,292 = $57,267
c. If interest rates drop to 4% after the 10th year,
   Annuity based upon interest rate of 4% and PV of $1,127,516 = $72,174.48

Problem 21
<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon</th>
<th>Face Value</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>10</td>
<td>$100.00</td>
<td>$1,000.00</td>
<td>$ 509.51</td>
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Sum = $876.05

Problem 22
a. Value of Store = $100,000 (1.05)/(0.10-0.05) = $2,100,000
b. Growth rate needed to justify a value of $2.5 million,
   100000(1+g)/(0.10-g) = 2500000
   Solving for g, g = 5.77%