Present Value: Solutions

Problem 1

a. Current Savings Needed = $ 500,000/1.110 = $ 192,772

b. Annuity Needed = $ 500,000 (APV,10%,10 years) = $ 31,373

Problem 2

Present Value of $ 1,500 growing at 5% a year for next 15 years = $ 18,093

Future Value = $ 18093 (1.08^15) = $ 57,394

Problem 3

Annual Percentage Rate = 8%

Monthly Rate = 8%/12 = 0.67%

Monthly Payment needed for 30 years = $ 200,000(APV,0.67%,360) = $ 1,473

Problem 4

a. Discounted Price Deal

Monthly Cost of borrowing $ 18,000 at 9% APR = $ 373.65

[A monthly rate of 0.75% is used]

b. Special Financing Deal 17.98245614

Monthly Cost of borrowing $ 20,000 at 3% APR = $ 359.37

The second deal is the better one.

Problem 5

a. Year-end Annuity Needed to have $ 100 million available in 10 years= $ 6.58

[FV = $ 100, r = 9%, n = 10 years]

b. Year-beginning Annuity Needed to have $ 100 million in 10 years = $ 6.04

Problem 6

Value of 15-year corporate bond; 9% coupon rate; 8 % market interest rate

Assuming coupons are paid semi-annually,

Value of Bond = 45*(1-1.04^(-30))/.04+1000/1.04^30 = $ 1,086.46

If market interest rates increase to 10%,

Value of Bond = 45*(1-1.05^(-30))/.05+1000/1.05^30 = $ 923.14

The bonds will trade at par only if the market interest rate = coupon rate.

Problem 7

Value of Stock = 1.50 (1.06)/ (.13 - .06) = $ 22.71

Problem 8

Value of Dividends during high growth period = $ 1.00 (1.15)(1-1.15^5/1.125^5)/(.125-.15)

$ 5.34

Expected Dividends in year 6 = $ 1.00 (1.15)^5*1.06*2 = $ 4.26

Expected Terminal Price = $ 4.26/(.125-.06) = $ 65.54

Value of Stock = $ 5.34 + $ 65.54/1.125^5 = $ 41.70

Problem 9

Expected Rate of Return = (1000/300)^(1/10) - 1 = 12.79%

Problem 10

Effective Annualized Interst Rate = (1+.09/52)^52 - 1 = 9.41%

Problem 11

Annuity given current savings of $ 250,000 and n=25 = $ 17,738.11

Problem 12

PV of first annuity - $ 20,000 a year for next 10 years = $ 128,353.15
PV of second annuity discounted back 10 years = $ 81,326.64
Sum of the present values of the annuities = $ 209,679.79

If annuities are paid at the start of each period,

PV of first annuity - $ 20,000 at beginning of each year= $ 148,353.15
PV of second annuity discounted back 10 years = $ 88,646.04
Sum of the present values of the annuities = $ 236,999.19

Problem 13

PV of deficit reduction can be computed as follows –

Year	Deficit Reductio	PV
1	$ 25.00	$ 23.15
2	$ 30.00	$ 25.72
3	$ 35.00	$ 27.78
4	$ 40.00	$ 29.40
5	$ 45.00	$ 30.63
6	$ 55.00	$ 34.66
7	$ 60.00	$ 35.01
8	$ 65.00	$ 35.12
9	$ 70.00	$ 35.02
10	$ 75.00	$ 34.74
Sum	$ 500.00	$ 311.22

The true deficit reduction is $ 311.22 million.

Problem 14

a. Annuity needed at 6% = 1.89669896 (in billions)
b. Annuity needed at 8% = 1.72573722 (in billions)
Savings = 0.17096174 (in billions)

This cannot be viewed as real savings, since there will be greater risk associated with the higher-return investments.

Problem 15

a.

Year	Nominal	PV
0	$5.50	$5.50
1	$4.00	$3.74
2	$4.00	$3.49
3	$4.00	$3.27
4	$4.00	$3.05
5	$7.00	$4.99
	$28.50	$24.04

b. Let the sign up bonus be reduced by X.
Then the cash flow in year 5 will have to be raised by X + 1.5 million,
to get the nominal value of the contract to be equal to $30 million.
Since the present value cannot change,

$$X - (X+1.5)/1.075 = 0$$
$$X (1.075 - 1) = 1.5$$

X = 1.5/ (1.075 -1) = $3.73 million

The sign up bonus has to be reduced by $3.73 million and the final year's cash flow has to be increased by $5.23 million, to arrive at a contract with a nominal value of $30 million and a present value of $24.04 million.

Problem 16

	Chatham	South Orange
Mortgage	$300,000	$200,000
Monthly Paym	$2,201	$1,468
Annual Payme	$26,416	$17,610
Property Tax	$6,000	$12,000
Total Payment	$32,416	$29,610

b. Mortgage payments will end after 30 years. Property taxes are not only a perpetuity; they are a growing perpetuity. Therefore, they are likely to be more onerous.

c. If property taxes are expected to grow at 3% annually forever,

PV of property taxes = Property tax * (1 +g) / (r -g)

For Chatham, PV of property tax = $6000 *1.03/(.08-.03) = $123,600

For South Orange, PV of property tax = $12,000 *1.03/(.08-.03) = $247,200

To make the comparison, add these to the house prices,

Cost of the Chatham house = $400,000 + $123,600 = $523,600

Cost of the South Orange house = $300,000 + $247,200 = $547,200

The Chatham house is cheaper.

Problem 17

a. Monthly Payments at 10% on current loan =	$ 1,755.14
b. Monthly Payments at 9% on refinanced mortgage =	$ 1,609.25
Monthly Savings from refinancing =	$ 145.90
c. Present Value of Savings at 8% for 60 months =	$ 7,195.56
Refinancing Cost = 3% of $ 200,000 =	$6,000
d. Annual Savings needed to cover $ 6000 in refinancing cost=	$ 121.66
Monthly Payment with Savings = $ 1755.14 - $ 121.66 =	$ 1,633.48
Interest Rate at which Monthly Payment is $ 1633.48 =	9.17%

Problem 18

a. Present Value of Cash Outflows after age 65 = $ 300,000 + PV of $ 35,000 each year for 35 years = $ 707,909.89

b. FV of Current Savings of $ 50,000 =	$ 503,132.84
Shortfall at the end of the 30th year =	$ 204,777.16
Annuity needed each year for next 30 years for FV of $ 204777 =	$ 1,807.66
c. Without the current savings,	
Annuity needed each year for 25 years for FV of $ 707910 =	$ 9,683.34

Problem 19

a. Estimated Funds at end of 10 years:

FV of $ 5 million at end of 10th year =	$ 10.79	(in millions)
FV of inflows of $ 2 million each year for next 5 years =	$ 17.24	
- FV of outflows of $ 3 million each year for years 6-10 =	$ 17.60	
= Funds at end of the 10th year =	$ 10.43	
b. Perpetuity that can be paid out of these funds = $ 10.43 (.08) =	$ 0.83	

Problem 20

a. Amount needed in the bank to withdraw $ 80,000 each year for 25 years =	$ 1,127,516
b. Future Value of Existing Savings in the Bank =	$ 407,224
Shortfall in Savings = $ 1127516 - $ 407224 =	$ 720,292
Annual Savings needed to get FV of $ 720,292 =	$ 57,267
c. If interest rates drop to 4% after the 10th year,	
Annuity based upon interest rate of 4% and PV of $ 1,127,516 =	$ 72,174.48

Problem 21

Year	Coupon	Face Value	PV
1	$ 50.00		$ 46.30
2	$ 50.00		$ 42.87
3	$ 50.00		$ 39.69
4	$ 50.00		$ 36.75
5	$ 50.00		$ 34.03
6	$ 60.00		$ 37.81
7	$ 70.00		$ 40.84
8	$ 80.00		$ 43.22
9	$ 90.00		$ 45.02
10	$ 100.00	$ 1,000.00	$ 509.51
		Sum =	$ 876.05

Problem 22

a. Value of Store = $ 100,000 (1.05)/(.10-.05) = $ 2,100,000

b. Growth rate needed to justify a value of $ 2.5 million,

100000(1+g)/(.10-g) = 2500000

Solving for g,

g = 5.77%