The Promise and Peril of Real Options

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Abstract

In recent years, practitioners and academics have made the argument that traditional discounted cash flow models do a poor job of capturing the value of the options embedded in many corporate actions. They have noted that these options need to be not only considered explicitly and valued, but also that the value of these options can be substantial. In fact, many investments and acquisitions that would not be justifiable otherwise will be value enhancing, if the options embedded in them are considered. In this paper, we examine the merits of this argument. While it is certainly true that there are options embedded in many actions, we consider the conditions that have to be met for these options to have value. We also develop a series of applied examples, where we attempt to value these options and consider the effect on investment, financing and valuation decisions.

In finance, the discounted cash flow model operates as the basic framework for most analysis. In investment analysis, for instance, the conventional view is that the net present value of a project is the measure of the value that it will add to the firm taking it. Thus, investing in a positive (negative) net present value project will increase (decrease) value. In capital structure decisions, a financing mix that minimizes the cost of capital, without impairing operating cash flows, increases firm value and is therefore viewed as the optimal mix. In valuation, the value of a firm is the present value of the expected cash flows from the assets of the firm.

In recent years, this framework has come under some fire for failing to consider the options that are embedded in each of these actions. For instance, the net present value of a project does not capture the values of the options to delay, expand or abandon a project. When comparing across investments, the traditional approach of picking the model with the highest return or net present value may shortchange investments that offer a firm more flexibility in operations and investing. A financing model that focuses on minimizing the current cost of capital does not consider the value of financial flexibility that comes from having excess debt capacity. In a similar vein, firms that hold back on returning cash to their stockholders and accumulate large cash balances might also be guided by the desire for financing flexibility. The value of equity, obtained from a discounted cash flow valuation model, does not measure the option to control, and if necessary, liquidate the firm that equity investors possess, and it ignores other options that might be owned by the firm, including patents, licenses and rights to natural reserves. In acquisition valuation, the strategic options that might be opened up for the acquiring firm, as a result of the transaction, are often not considered in valuation.

In light of these options that seem to be everywhere, there are some theorists and many practitioners who believe that we should consider these options when analyzing corporate decisions. There is no clear unanimity among this group as to what they would like to see done. Some top managers and consultants would like to use real options as a rhetorical tool that can be used to justify investment, financing and acquisition decisions; they feel that while there are embedded options in most decisions, they cannot be valued with any precision. There are others who argue that we should try to quantitatively estimate the value of these options, and build them into the decision process.

In this paper, we explore the existence of options in business decisions and find that they are ubiquitous. We also, however, lay out the conditions that need to be fulfilled for a real option to not only exist but to have significant value. Finally, we examine how best to obtain the inputs needed to value these options and incorporate them into decision making. We come down firmly on the side of those who believe that this can and should be done.

This paper begins with a short introduction to options, the determinants of option value and the basics of option pricing. We do not spend much time on the technicalities of option pricing, though we present some of the special issues that come up when valuing real options. We then looks at option applications in three parts. The first part includes options embedded in investments or projects, including the options to expand, delay and abandon a project. Included in this section is the discussion of strategic options, the value of research and development and natural resource reserves. The second part of the analysis looks at options in firm valuation. In particular, we look at the liquidation option that equity investors possess, and how much value it creates, especially in the context of highly levered, risky firms. The third part considers options in financing decisions. We consider the value of flexibility as an option and the use of options in the design of securities to reduce the cost of financing and default risk.

Basics of Option Pricing

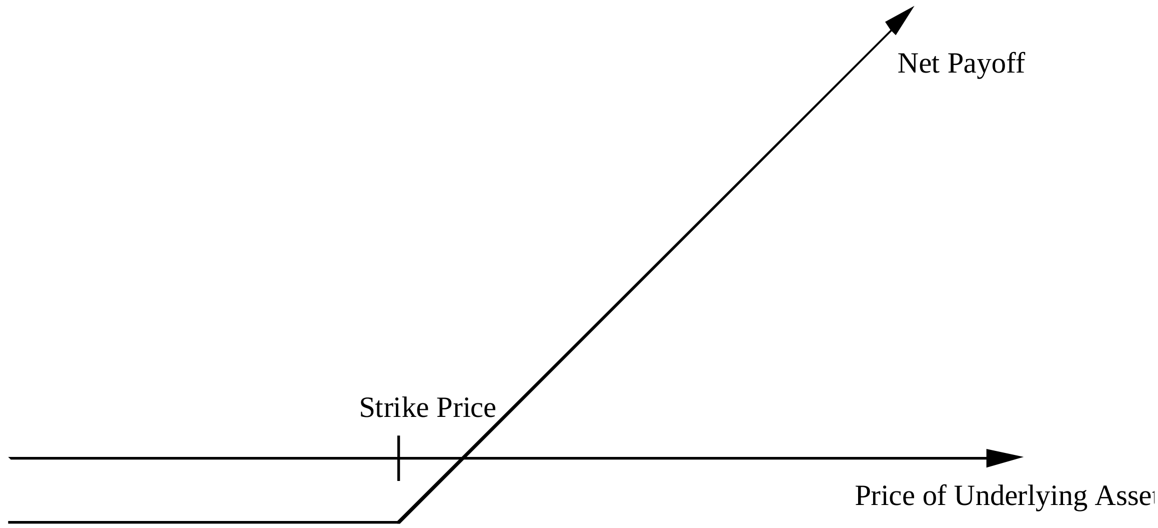
An option provides the holder with the right to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option. Since it is a right and not an obligation, the holder can choose not to exercise the right and allow the option to expire. There are two types of options - call options and put options.

Call and Put Options: Description and Payoff Diagrams

A call option gives the buyer of the option the right to buy the underlying asset at a fixed price, called the strike or the exercise price, at any time prior to the expiration date of the option: the buyer pays a price for this right. If at expiration, the value of the asset is less than the strike price, the option is not exercised and expires worthless. If, on the other hand, the value of the asset is greater than the strike price, the option is exercised - the buyer of the option buys the stock at the exercise price and the difference between the asset value and the exercise price comprises the gross profit on the investment. The net profit on the investment is the difference between the gross profit and the price paid for the call initially.

A payoff diagram illustrates the cash payoff on an option at expiration. For a call, the net payoff is negative (and equal to the price paid for the call) if the value of the underlying asset is less than the strike price. If the price of the underlying asset exceeds the strike price, the gross payoff is the difference between the value of the underlying asset and the strike price, and the net payoff is the difference between the gross payoff and the price of the call. This is illustrated in the figure below:

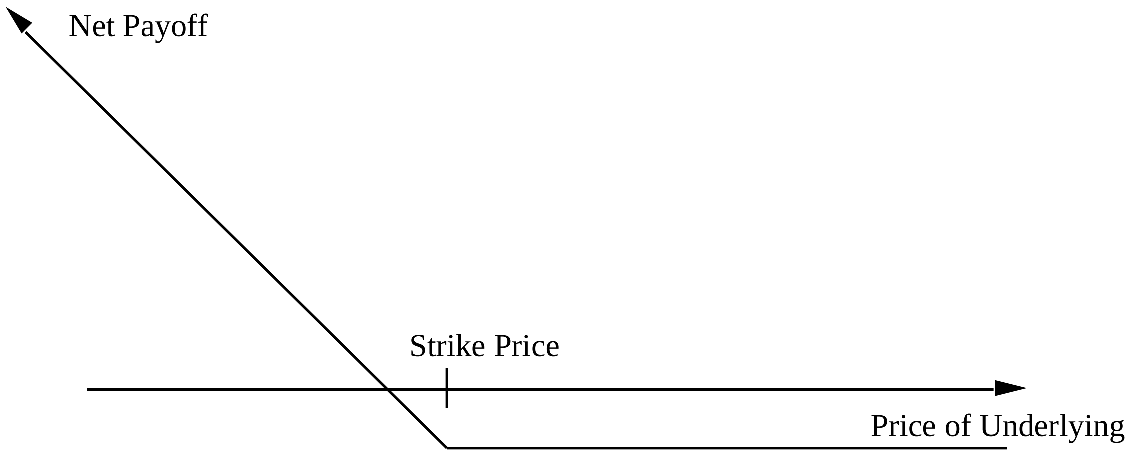
Figure 1: Payoff on Call Option



A put option gives the buyer of the option the right to sell the underlying asset at a fixed price, again called the strike or exercise price, at any time prior to the expiration date of the option. The buyer pays a price for this right. If the price of the underlying asset is greater than the strike price, the option will not be exercised and will expire worthless. If on the other hand, the price of the underlying asset is less than the strike price, the owner of the put option will exercise the option and sell the stock a the strike price, claiming the difference between the strike price and the market value of the asset as the gross profit. Again, netting out the initial cost paid for the put yields the net profit from the transaction.

A put has a negative net payoff if the value of the underlying asset exceeds the strike price, and has a gross payoff equal to the difference between the strike price and the value of the underlying asset if the asset value is less than the strike price. This is summarized in the figure below.

Figure 2: Payoff on Put Option



Determinants of Option Value

The value of an option is determined by a number of variables relating to the underlying asset and financial markets.

1. Current Value of the Underlying Asset : Options are assets that derive value from an underlying asset. Consequently, changes in the value of the underlying asset affect the value of the options on that asset. Since calls provide the right to buy the underlying asset at a fixed price, an increase in the value of the asset will increase the value of the calls. Puts, on the other hand, become less valuable as the value of the asset increase.

2. Variance in Value of the Underlying Asset: The buyer of an option acquires the right to buy or sell the underlying asset at a fixed price. The higher the variance in the value of the underlying asset, the greater the value of the option. This is true for both calls and puts. While it may seem counter-intuitive that an increase in a risk measure (variance) should increase value, options are different from other securities since buyers of options can never lose more than the price they pay for them; in fact, they have the potential to earn significant returns from large price movements.

3. Dividends Paid on the Underlying Asset: The value of the underlying asset can be expected to decrease if dividend payments are made on the asset during the life of the option. Consequently, the value of a call on the asset is a decreasing function of the size of

expected dividend payments, and the value of a put is an increasing function of expected dividend payments. A more intuitive way of thinking about dividend payments, for call options, is as a cost of delaying exercise on in-the-money options. To see why, consider a option on a traded stock. Once a call option is in the money, i.e, the holder of the option will make a gross payoff by exercising the option, exercising the call option will provide the holder with the stock, and entitle him or her to the dividends on the stock in subsequent periods. Failing to exercise the option will mean that these dividends are foregone.

4. Strike Price of Option: A key characteristic used to describe an option is the strike price. In the case of calls, where the holder acquires the right to buy at a fixed price, the value of the call will decline as the strike price increases. In the case of puts, where the holder has the right to sell at a fixed price, the value will increase as the strike price increases.

5. Time To Expiration On Option: Both calls and puts become more valuable as the time to expiration increases. This is because the longer time to expiration provides more time for the value of the underlying asset to move, increasing the value of both types of options. Additionally, in the case of a call, where the buyer has to pay a fixed price at expiration, the present value of this fixed price decreases as the life of the option increases, increasing the value of the call.

6. Riskless Interest Rate Corresponding To Life Of Option: Since the buyer of an option pays the price of the option up front, an opportunity cost is involved. This cost will depend upon the level of interest rates and the time to expiration on the option. The riskless interest rate also enters into the valuation of options when the present value of the exercise price is calculated, since the exercise price does not have to be paid (received) until expiration on calls (puts). Increases in the interest rate will increase the value of calls and reduce the value of puts.

The table below summarizes the variables and their predicted effects on call and put prices.

Table 1: Summary of Variables Affecting Call and Put Prices

	Effect on

Factor	*Call Value*	*Put Value*
Increase in underlying asset's value	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

American Versus European Options: Variables Relating To Early Exercise

A primary distinction between American and European options is that American options can be exercised at any time prior to its expiration, while European options can be exercised only at expiration. The possibility of early exercise makes American options more valuable than otherwise similar European options; it also makes them more difficult to value. There is one compensating factor that enables the former to be valued using models designed for the latter. In most cases, the time premium associated with the remaining life of an option and transactions costs makes early exercise sub-optimal. In other words, the holders of in-the-money options will generally get much more by selling the option to someone else than by exercising the options.

While early exercise is not optimal generally, there are at least two exceptions to this rule. One is a case where the underlying asset pays large dividends, thus reducing the value of the asset, and any call options on that asset. In this case, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value as a consequence of the dividend payment. The other exception arises when an investor holds both the underlying asset and deep in-the-money puts on that asset at a time when interest rates are high. In this case, the time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

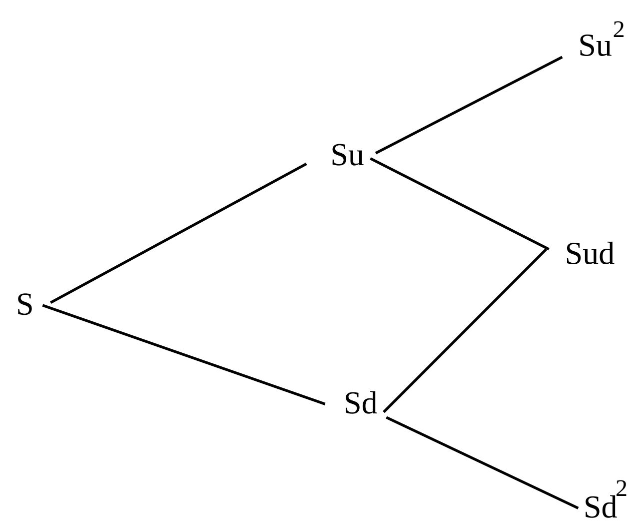
Option Pricing Models

Option pricing theory has made vast strides since 1972, when Black and Scholes published their path-breaking paper providing a model for valuing dividend-protected European options. Black and Scholes used a "replicating portfolio" — a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued— to come up with their final formulation. While their derivation is mathematically complicated, there is a simpler binomial model for valuing options that draws on the same logic.

The Binomial Model

The binomial option pricing model is based upon a simple formulation for the asset price process, in which the asset, in any time period, can move to one of two possible prices. The general formulation of a stock price process that follows the binomial is shown in the figure below.

Figure 3: General Formulation for Binomial Price Path



In this figure, S is the current stock price; the price moves up to Su with probability p and down to Sd with probability 1-p in any time period.

Creating A Replicating Portfolio

The objective in creating a replicating portfolio is to use a combination of risk-free borrowing/lending and the underlying asset to create the same cash flows as the option being valued. The principles of arbitrage apply here, and the value of the option must be equal to the value of the replicating portfolio. In the case of the general formulation above, where stock prices can either move up to Su or down to Sd in any time period, the replicating portfolio for a call with strike price K will involve borrowing \$B and acquiring Δ of the underlying asset, where:

Δ = Number of units of the underlying asset bought = $(C_u - C_d)/(Su - Sd)$

where,

C_u = Value of the call if the stock price is Su

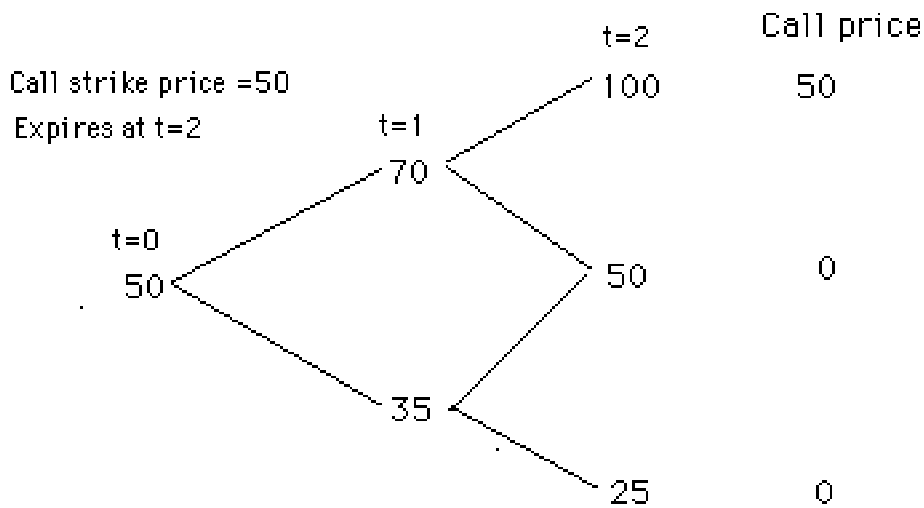
C_d = Value of the call if the stock price is Sd

In a multi-period binomial process, the valuation has to proceed iteratively; i.e., starting with the last time period and moving backwards in time until the current point in time. The portfolios replicating the option are created at each step and valued, providing the values for the option in that time period. The final output from the binomial option pricing model is a statement of the value of the option in terms of the replicating portfolio, composed of Δ shares (option delta) of the underlying asset and risk-free borrowing/lending.

Value of the call = Current value of underlying asset * Option Delta - Borrowing needed to replicate the option

An Example of Binomial valuation

Assume that the objective is to value a call with a strike price of 50, which is expected to expire in two time periods, on an underlying asset whose price currently is 50 and is expected to follow a binomial process:



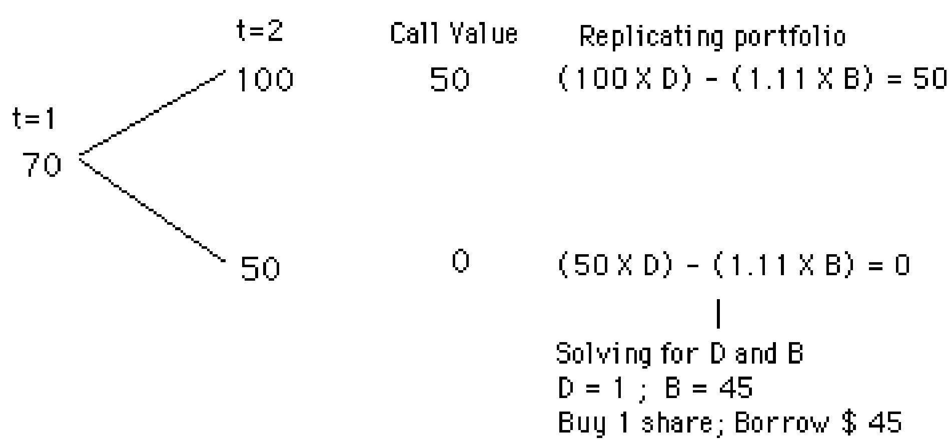
Now assume that the interest rate is 11%. In addition, define

Δ = Number of shares in the replicating portfolio

B = Dollars of borrowing in replicating portfolio

The objective is to combined Δ shares of stock and B dollars of borrowing to replicate the cash flows from the call with a strike price of $ 50. This can be done iteratively, starting with the last period and working back through the binomial tree.

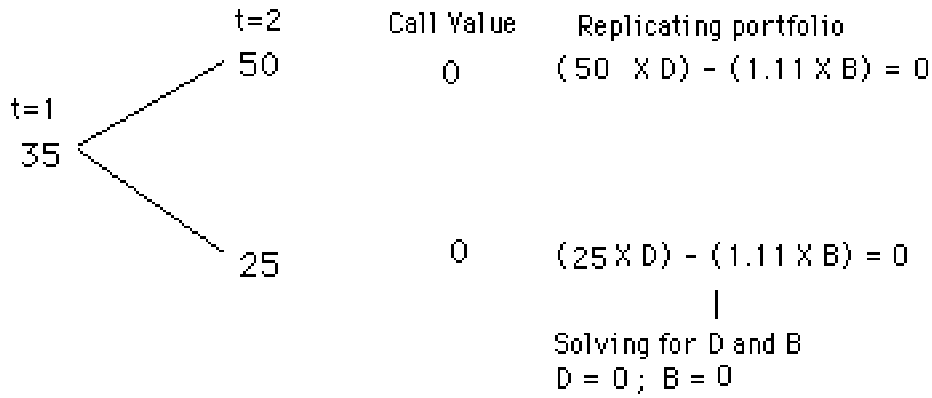
Step 1: Start with the end nodes and work backwards:



Thus, if the stock price is $70 at t=1, borrowing $45 and buying one share of the stock will give the same cash flows as buying the call. The value of the call at t=1, if the stock price is $70, is therefore:

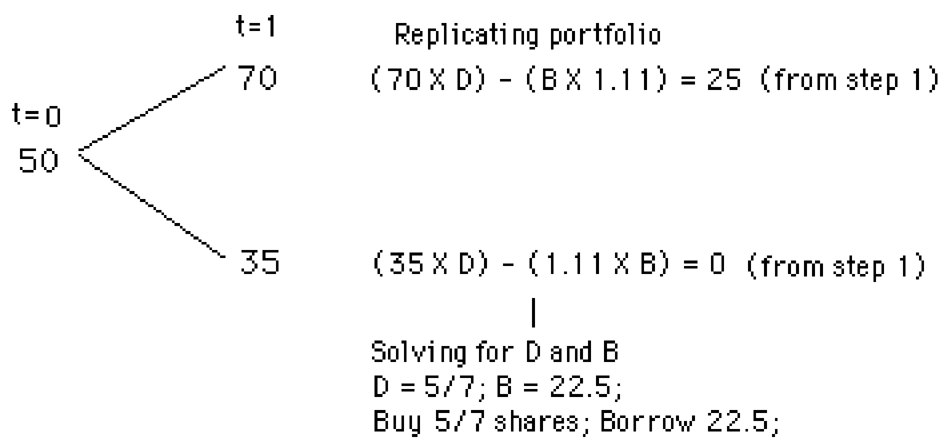
Value of Call = Value of Replicating Position = 70 Δ - B = 70-45 = 25

Considering the other leg of the binomial tree at t=1,



If the stock price is 35 at t=1, then the call is worth nothing.

Step 2: Move backwards to the earlier time period and create a replicating portfolio that will provide the cash flows the option will provide.



In other words, borrowing $22.5 and buying 5/7 of a share will provide the same cash flows as a call with a strike price of $50. The value of the call therefore has to be the same as the value of this position.

Value of Call = Value of replicating position = 5/7 X Current stock price - $ 22.5 = $ 13.20

The Determinants of Value

The binomial model provides insight into the determinants of option value. The value of an option is not determined by the expected price of the asset but by its current price, which, of course, reflects expectations about the future. This is a direct consequence of arbitrage. If the option value deviates from the value of the replicating portfolio, investors can create an arbitrage position, i.e., one that requires no investment, involves no risk, and delivers positive returns. To illustrate, if the portfolio that replicates the call costs more than the call does in the market, an investor could buy the call, sell the replicating portfiolio and be guaranteed the difference as a profit. The cash flows on the two positions will offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the price movements (u and d) increase, and with increases in the interest rate.

The Black-Scholes Model

While the binomial model provides an intuitive feel for the determinants of option value, it requires a large number of inputs, in terms of expected future prices at each node. The Black-Scholes model is not an alternative to the binomial model; rather, it is one limiting case of the binomial.

The binomial model is a discrete-time model for asset price movements, including a time interval (t) between price movements. As the time interval is shortened, the limiting distribution, as t approaches 0, can take one of two forms. If as t approaches 0, price changes become smaller, the limiting distribution is the normal distribution and the price process is a continuous one. If as t approaches 0, price changes remain large, the limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps. The

Black-Scholes model applies when the limiting distribution is the normal distribution,[1] and it explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Model

The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected. Thus, neither the possibility of early exercise nor the payment of dividends affects the value of options in this model.

The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the ln(value) of the underlying asset

The model itself can be written as:

Value of call = $S\,N(d_1) - K\,e^{-rt}\,N(d_2)$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

[1] Stock prices cannot drop below zero, because of the limited liability of stockholders in publicly listed firms. Hence, stock prices, by themselves, cannot be normally distributed, since a normal distribution requires some probability of infinitely negative values. The distribution of the natural logs of stock prices is assumed to be log-normal in the Black-Scholes model. This is why the variance used in this model is the variance in the log of stock prices.

The process of valuation of options using the Black-Scholes model involves the following steps:

Step 1: The inputs to the Black-Scholes are used to estimate d_1 and d_2.

Step 2: The cumulative normal distribution functions, $N(d_1)$ and $N(d_2)$, corresponding to these standardized normal variables are estimated.

Step 3: The present value of the exercise price is estimated, using the continuous time version of the present value formulation:

Present value of exercise price = $K e^{-rt}$

Step 4: The value of the call is estimated from the Black-Scholes model.

The Replicating Portfolio in the Black-Scholes

The determinants of value in the Black-Scholes are the same as those in the binomial - the current value of the stock price, the variability in stock prices, the time to expiration on the option, the strike price, and the riskless interest rate. The principle of replicating portfolios that is used in binomial valuation also underlies the Black-Scholes model. In fact, embedded in the Black-Scholes model is the replicating portfolio.

$$\text{Value of call} = S\,\underbrace{N(d_1)}_{\text{Buy } N(d_1) \text{ shares}} \quad \underbrace{- K e^{-rt} N(d_2)}_{\text{Borrow this amount}}$$

N(d1), which is the number of shares that are needed to create the replicating portfolio is called the <u>option delta</u>. This replicating portfolio is self-financing and has the same value as the call at every stage of the option's life.

The probabilities, $N(d_1)$ and $N(d_2)$, embedded in the option pricing model also have some use in analysis. They represent, in approximate terms, the range of probability that the option will be in the money at expiration, i.e, the probability that S>K. Since $N(d_1)$ will always be greater than $N(d_2)$, it represents the upper end of the range.

Model Limitations and Fixes

The version of the Black-Scholes model presented above does not take into account the possibility of early exercise or the payment of dividends, both of which impact the value of options. Adjustments exist, which while not perfect, provide partial corrections to value.

1. Dividends

The payment of dividends reduces the stock price. Consequently, call options will become less valuable and put options more valuable as dividend payments increase. One approach to dealing with dividends to estimate the present value of expected dividends paid by the underlying asset during the option life and subtract it from the current value of the asset to use as "S" in the model. Since this becomes impractical as the option life becomes longer, we would suggest an alternate approach. If the dividend yield (y = dividends/ current value of the asset) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S\, e^{-yt}\, N(d_1) - K\, e^{-rt}\, N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

From an intuitive standpoint, the adjustments have two effects. First, the value of the asset is discounted back to the present at the dividend yield to take into account the expected drop in value from dividend payments. Second, the interest rate is offset by the dividend yield to reflect the lower carrying cost from holding the stock (in the replicating portfolio). The net effect will be a reduction in the value of calls, with the adjustment, and an increase in the value of puts.

2. Early Exercise

The Black-Scholes model is designed to value European options, i.e. options that cannot be exercised until the expiration day. Most options that we consider are American options, which can be exercised anytime before expiration. Without working through the mechanics of valuation models, an American option should always be worth at least as much and generally more than a European option because of the early exercise option. There are three basic approaches for dealing with the possibility of early exercise. The first is to continue to use the unadjusted Black-Scholes, and regard the resulting value as a floor or conservative estimate of the true value. The second approach is to value the option to each potential exercise date. With options on stocks, this basically requires that we value options to each ex-dividend day and chooses the maximum of the estimated call values. The third approach is to use a modified version of the binomial model to consider the possibility of early exercise.

While it is difficult to estimate the prices for each node of a binomial, there is a way in which variances estimated from historical data can be used to compute the expected up and down movements in the binomial. To illustrate, if σ^2 is the variance in ln(stock prices), the up and down movements in the binomial can be estimated as follows:

$$u = \text{Exp}\,[(r - \sigma^2/2)(T/m) + \sqrt{(\sigma^2 T/m)}]$$

$$d = \text{Exp}\,[(r - \sigma^2/2)(T/m) - \sqrt{(\sigma^2 T/m)}]$$

where u and d are the up and down movements per unit time for the binomial, T is the life of the option and m is the number of periods within that lifetime. Multiplying the stock price at each stage by u and d will yield the up and the down prices. These can then be used to value the asset.

3. The Impact Of Exercise On The Value Of The Underlying Asset

The derivation of the Black-Scholes model is based upon the assumption that exercising an option does not affect the value of the underlying asset. This may be true for listed options on stocks, but it is not true for some types of options. For instance, the exercise of warrants increases the number of shares outstanding and brings fresh cash into the firm, both of which will affect the stock price.[2] The expected negative impact (dilution) of exercise will decrease the value of warrants compared to otherwise similar call options. The adjustment for dilution in the Black-Scholes to the stock price is fairly simple. The stock price is adjusted for the expected dilution from the exercise of the options. In the case of warrants, for instance:

$$\text{Dilution-adjusted } S = (S\ n_S + W\ n_W) / (n_S + n_w)$$

where

S = Current value of the stock n_W = Number of warrants outstanding

W = Market value of warrants outstanding n_S = Number of shares outstanding

When the warrants are exercised, the number of shares outstanding will increase, reducing the stock price. The numerator reflects the market value of equity, including both stocks and warrants outstanding. The reduction in S will reduce the value of the call option.

There is an element of circularity in this analysis, since the value of the warrant is needed to estimate the dilution-adjusted S and the dilution-adjusted S is needed to estimate the value of the warrant. This problem can be resolved by starting the process off with an estimated value of the warrant (say, the exercise value), and then iterating with the new estimated value for the warrant until there is convergence.

Valuing Puts

The value of a put is can be derived from the value of a call with the same strike price and the same expiration date through an arbitrage relationship that specifies that:

$$C - P = S - K\ e^{-rt}$$

where C is the value of the call and P is the value of the put (with the same life and exercise price).

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[2] Warrants are call options issued by firms, either as part of management compensation contracts or to raise equity.

This arbitrage relationship can be derived fairly easily and is called put-call parity. To see why put-call parity holds, consider creating the following portfolio:

(a) Sell a call and buy a put with exercise price K and the same expiration date "t"

(b) Buy the stock at current stock price S

The payoff from this position is riskless and always yields K at expiration (t). To see this, assume that the stock price at expiration is S*:

Position	*Payoffs at t if S*>K*	*Payoffs at t if S*<K*
Sell call	-(S*-K)	0
Buy put	0	K-S*
Buy stock	S*	S*
Total	*K*	*K*

Since this position yields K with certainty, its value must be equal to the present value of K at the riskless rate ($K e^{-rt}$).

$$S+P-C = K e^{-rt}$$

$$C - P = S - K e^{-rt}$$

This relationship can be used to value puts. Substituting the Black-Scholes formulation for the value of an equivalent call,

$$\text{Value of put} = S e^{-yt} (N(d_1) - 1) - K e^{-rt} (N(d_2) - 1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

A Few Caveats On Applying Option Pricing Models

The option pricing models described in the preceding chapter can be used to value any asset that has the characteristics of an option, with some caveats. In subsequent sections, we will apply option pricing theory in a variety of contexts. In many of the cases described, the options being valued are not on financially traded assets (such as stocks or commodities) but are real options (such as those on projects or natural resources reserves). We begin by offering a few caveats on the application of option pricing models to these cases and suggesting some adjustments that might need to be made to these models.

1. The Underlying Asset Is Not Traded: Option pricing theory, as presented in both the binomial and the Black-Scholes models, is built on the premise that a replicating portfolio can be created using the underlying asset and riskless lending or borrowing. While this is a perfectly justifiable assumption in the context of listed options on traded stocks, it becomes less defensible when the underlying asset is not traded, and arbitrage is therefore not feasible. When the options valued are on assets that are not traded, the values from option pricing models have to be interpreted with caution.

2. The Price Of The Asset Follows A Continuous Process : As noted earlier, the Black-Scholes option pricing model is derived under the assumption that the underlying asset's price process is continuous (i.e., there are no price jumps). If this assumption is violated, as it is with many real options, the model will underestimate the value of deep out-of-the-money options. One solution is to use higher variance estimates to value deep out-of-the-money options and lower variance estimates for at-the-money or in-the-money options; another is to use an option pricing model that explicitly allows for price jumps, though the inputs to these models are often difficult to estimate.[3]

3. The Variance Is Known And Does Not Change Over The Life Of The Option: The assumption option pricing models make that the variance is known and does not change over the option lifetime is not unreasonable when applied to listed short-term options on traded stocks. When option pricing theory is applied to long-term real options, however, there are problems with this assumption, since the variance is unlikely to remain constant

[3] Jump process models that incorporate the Poisson process require inputs on the probability of price jumps, the average magnitude, and the variance, all of which can be estimated, but with a significant amount of noise.

over extended periods of time and may in fact be difficult to estimate in the first place. Again, modified versions of the option pricing model exist that allow for changing variances, but they require that the process by which variance changes be modeled explicitly.

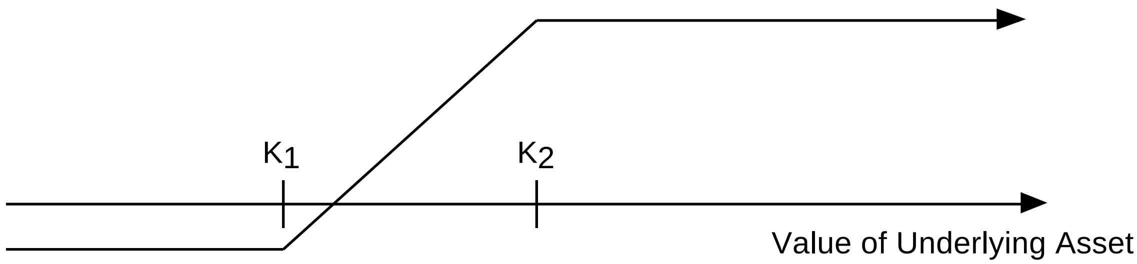
4. Exercise Is Instantaneous: The option pricing models are based upon the premise that the exercise of an option is instantaneous. This assumption may be difficult to justify with real options, however; exercise may require building a plant or constructing an oil rig, for example, actions that do not happen in an instant. The fact that exercise takes time also implies that the true life of a real option is often less than the stated life. Thus, while a firm may own the rights to an oil reserve for the next ten years, the fact that it takes several years to extract the oil reduces the life of the natural resource option the firm owns.

Barrier, Compound and Rainbow Options

So far in our discussion of option pricing, we have not considered more complicated options that often arise in analysis. In this section, we consider three variations on the simple option. The first is a barrier option, where the option value is capped if the price of the underlying asset exceeds a pre-specified level. The second is the compound option, which is an option on an option. The third is a rainbow option, where there is more than one source of uncertainty affecting the value of the option.

Capped and Barrier Options

Assume that you have a call option with a strike price of $ 25 on an asset. In an unrestricted call option, the payoff on this option will increase as the underlying asset's price increases above $ 25. Assume, however, that if the price reaches $ 50, the payoff is capped at $ 25. The payoff diagram on this option is as follows:



This option is called a capped call. Notice, also, that once the price reaches $ 50, there is no time premium associated with the option anymore and that the option will therefore be exercised. Capped calls are part of a family of options called barrier options, where the payoff on and the life of the option is a function of whether the underlying asset reaches a certain level during a specified period.

The value of a capped call will always be lower than the value of the same call without the payoff limit. A simple approximation of this value can be obtained by valuing the call twice, once with the given exercise price, and once with the cap, and taking the difference in the two values. In the above example, then, the value of the call with an exercise price of K_1 and a cap at K_2 can be written as:

Value of Capped Call = Value of call ($K=K_1$) - Value of call ($K=K_2$)

Barrier options can take many forms. In a knockout option, an option ceases to exist if the option reaches a certain price. In the case of a call option, this knock-out price is usually set below the strike price, and this option is called a down-and-out option. In the case of a put option, the knock-out price will be set above the exercise price and this option is called an up-and-out option. Like the capped call, these options will be worth less than their unrestricted counterparts. Many real options have limits on potential upside and/or knock-out provisions, and ignoring these limits can result in the overstatement of the value of these options.

Compound Options

Some options derive their value, not from an underlying asset, but from other options. These options are called compound options. Compound options can take any of four forms - a call on a call, a put on a put, a call on a put and a put on a call. Geske (1979) developed the analytical formulation for valuing compound options by replacing the standard normal distribution used in a simple option model with a bi-variate normal distribution in the calculation.

In the context of real options, the compound option process can get complicated. Consider, for instance, the option to expand a project that we will consider in the next section. While we will value this option using a simple option pricing model, in reality, there could be multiple stages in expansion, with each stage representing an option for the following stage. In this case, we will under value the option by considering it as a simple rather than a compound option.

Notwithstanding this discussion, the valuation of compound options become progressively more difficult as we add more options to the chain. In this case, rather than wreck the valuation on the shoals of estimation error, it may be better to accept the conservative estimate that is provided with a simple valuation model as a floor on the value.

Rainbow Options

In a simple option, the only source of uncertainty is the price of the underlying asset. There are some options that derive their value from two or more sources of uncertainty, and these options are called rainbow options. Using the simple option pricing model to value such options can lead to biased estimates of value. As an example, consider an undeveloped oil reserve as an option, where the firm that owns the reserve has the right to develop the reserve. Here, there are two sources of uncertainty. The first is obviously the price of oil, and the second is the quantity of oil that is in the reserve.

To value this undeveloped reserve, we can make the simplifying assumption that we know the quantity of the reserves with certainty. In reality, however, uncertainty about the quantity will affect the value of this option and make the decision to exercise more difficult[4].

Options in Investment Analysis / Capital Budgeting

In traditional investment analysis, a project or new investment should be accepted only if the returns on the project exceed the hurdle rate; in the context of cash flows and discount rates, this translates into projects with positive net present values. The limitation with this view of the world, which analyzes projects on the basis of expected cash flows and discount rates, is that it fails to consider fully the myriad options that are usually associated with many investments.

In this section, we will analyze three options that are embedded in capital budgeting projects. The first is the option to delay a project, especially when the firm has exclusive rights to the project. The second is the option to expand a project to cover new products or markets some time in the future. The third is the option to abandon a project if the cash flows do not measure up to expectations.

The Option to Delay a Project

Projects are typically analyzed based upon their expected cash flows and discount rates at the time of the analysis; the net present value computed on that basis is a measure of its value and acceptability at that time. Expected cash flows and discount rates change over time, however, and so does the net present value. Thus, a project that has a negative net present value now may have a positive net present value in the future. In a competitive environment, in which individual firms have no special advantages over their competitors

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[4] The analogy to a listed option on a stock would be the case where you do not know what the stock price is with certainty when you exercise the option. The more uncertain you are about the stock price, the more margin for error you have to give yourself when you exercise the option to ensure that you are in fact earning a profit.

in taking projects, this may not seem significant. In an environment in which a project can be taken by only one firm (because of legal restrictions or other barriers to entry to competitors), however, the changes in the project's value over time give it the characteristics of a call option.

In the abstract, assume that a project requires an initial up-front investment of X, and that the present value of expected cash inflows computed right now is V. The net present value of this project is the difference between the two:

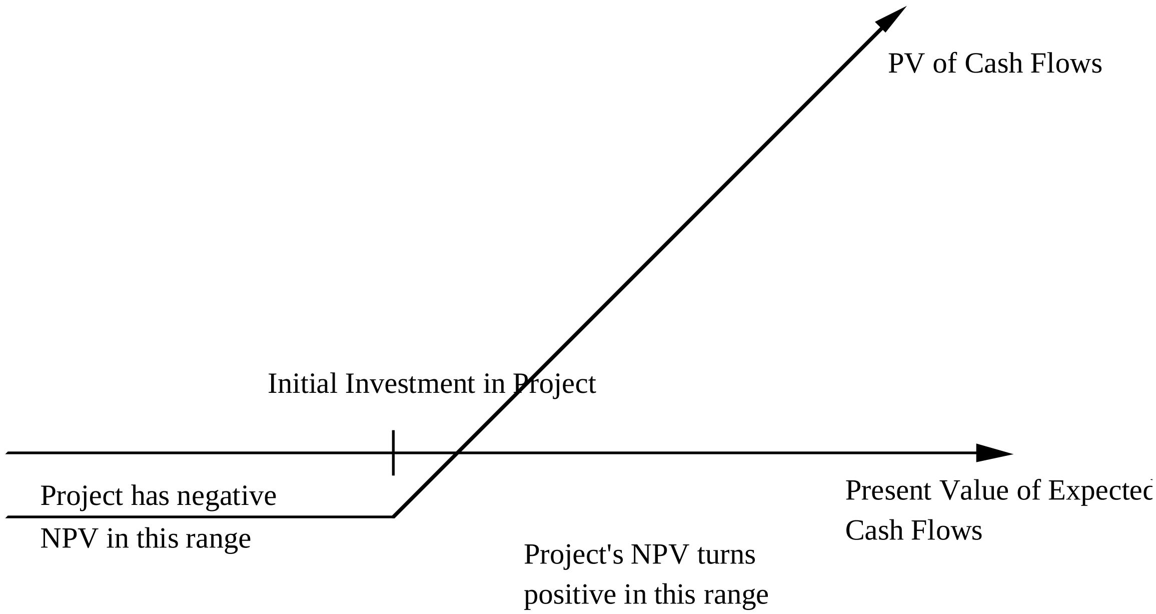
$$NPV = V - X$$

Now assume that the firm has exclusive rights to this project for the next n years, and that the present value of the cash inflows may change over that time, because of changes in either the cash flows or the discount rate. Thus, the project may have a negative net present value right now, but it may still be a good project if the firm waits. Defining V again as the present value of the cash flows, the firm's decision rule on this project can be summarized as follows:

If	$V > X$	Take the project: Project has positive net present value
	$V < X$	Do not take the project: Project has negative net present value

If the firm does not take the project, it incurs no additional cash flows, though it will lose what it originally invested in the project. This relationship can be presented in a payoff diagram of cash flows on this project, as shown in Figure 4, assuming that the firm holds out until the end of the period for which it has exclusive rights to the project:

Figure 4: The Option to Delay a Project



Note that this payoff diagram is that of a call option — the underlying asset is the project, the strike price of the option is the investment needed to take the project; and the life of the option is the period for which the firm has rights to the project. The present value of the cash flows on this project and the expected variance in this present value represent the value and variance of the underlying asset.

Obtaining the Inputs for Valuing the Option to Delay

On the surface, the inputs needed to apply option pricing theory to valuing the option to delay are the same as those needed for any option. We need the value of the underlying asset, the variance in that value, the time to expiration on the option, the strike price, the riskless rate and the equivalent of the dividend yield (cost of delay). Actually estimating these inputs for product patent valuation can be difficult, however.

Value Of The Underlying Asset

In the case of product options, the underlying asset is the project itself. The current value of this asset is the present value of expected cash flows from initiating the project now, not including the up-front investment, which can be obtained by doing a standard capital budgeting analysis. There is likely to be a substantial amount of noise in the cash flow estimates and the present value, however. Rather than being viewed as a problem, this uncertainty should be viewed as the reason for why the project delay option has value. If the expected cash flows on the project were known with certainty and were not expected to change, there would be no need to adopt an option pricing framework, since there would be no value to the option.

Variance in the value of the asset

As noted in the prior section, there is likely to be considerable uncertainty associated with the cash flow estimates and the present value that measures the value of the asset now. This is partly because the potential market size for the product may be unknown, and partly because technological shifts can change the cost structure and profitability of the product. The variance in the present value of cash flows from the project can be estimated in one of three ways.

1. If similar projects have been introduced in the past, the variance in the cash flows from those projects can be used as an estimate. This may be the way that a consumer product company like Gilette might estimate the variance associated with introducing a new blade for its razors.
2. Probabilities can be assigned to various market scenarios, cash flows estimated under each scenario and the variance estimated across present values. Alternatively, the probability distributions can be estimated for each of the inputs into the project analysis - the size of the market, the market share and the profit margin, for instance - and simulations used to estimate the variance in the present values that emerge. This approach tends to work best when there are only one or two sources[5] of significant uncertainty about future cash flows.

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[5] In practical terms, the probability distributions for inputs like market size and market share can often be obtained from market testing.

3. The variance in firm value of firms involved in the same business (as the project being considered) can be used as an estimate of the variance. Thus, the average variance in firm value of firms involved in the software business can be used as the variance in present value of a software project.

The value of the option is largely derived from the variance in cash flows - the higher the variance, the higher the value of the project delay option. Thus, the value of a option to do a project in a stable business will be less than the value of one in an environment where technology, competition and markets are all changing rapidly.

Exercise Price On Option

A project delay option is exercised when the firm owning the rights to the project decides to invest in it. The cost of making this investment is the exercise price of the option. The underlying assumption is that this cost remains constant (in present value dollars) and that any uncertainty associated with the product is reflected in the present value of cash flows on the product.

Expiration Of The Option And The Riskless Rate

The project delay option expires when the rights to the project lapse; investments made after the project rights expire are assumed to deliver a net present value of zero as competition drives returns down to the required rate. The riskless rate to use in pricing the option should be the rate that corresponds to the expiration of the option. While this input can be estimated easily when firms have the explicit right to a project (through a license or a patent, for instance), it becomes far more difficult to obtain when firms only have a competitive advantage to take a project. Since competitive advantages fade over time, the number of years for which the firm can be expected to have these advantages is the life of the option.

Cost of Delay (Dividend Yield)

There is a cost to delaying taking a project, once the net present value turns positive. Since the project rights expire after a fixed period, and excess profits (which are the source of positive present value) are assumed to disappear after that time as new competitors emerge, each year of delay translates into one less year of value-creating cash flows.[6] If the cash flows are evenly distributed over time, and the life of the patent is n years, the cost of delay can be written as:

$$\text{Annual cost of delay} = \frac{1}{n}$$

Thus, if the project rights are for 20 years, the annual cost of delay works out to 5% a year. Note, though, that this cost of delay rises each year , to 1/19 in year 2, 1/18 in year 3 and so on, making the cost of delaying exercise larger over time.

Concept Check: In a normal option, it almost never pays to exercise early. Why, in the case of a project option, might this not hold true?

Valuing the Option to Delay a Project: An Illustration

Assume that you have are interested in acquiring the exclusive rights to market a new product that will make it easier for people to access their email on the road. If you do acquire the rights to the product, you estimate that it will cost you $ 500 million up-front to set up the infrastructure needed to provide the service. Based upon your current projections, you believe that the service will generate only $ 100 million in after-tax cash flows each year. In addition, you expect to operate without serious competition for the next 5 years.

[6] A value-creating cashflow is one that adds to the net present value because it is in excess of the required return for investments of equivalent risk.

From a purely static standpoint, the net present value of this project can be computed by taking the present value of the expected cash flows over the next 5 years. Assuming a discount rate of 15% (based on the riskiness of this project), we obtain the following net present value for the project:

NPV of project = - 500 million + $ 100 million (PV of annuity, 15%, 5 years)

= - 500 million + $ 335 million = - $ 165 million

This project has a negative net present value.

The biggest source of uncertainty on this project is the number of people who will be interested in this product. While the current market tests indicate that you will capture a relatively small number of business travelers as your customers, the test also indicates a possibility that the potential market could get much larger over time. In fact, a simulation of the project's cash flows yields a standard deviation of the 42% in the present value of the cash flows, with an expected value of $ 335 million.

To value the exclusive rights to this project, we first define the inputs to the option pricing model:

Value of the Underlying Asset (S) = PV of Cash Flows from Project if introduced now

= $ 335 million

Strike Price (K) = Initial Investment needed to introduce the product = $ 500 million

Variance in Underlying Asset's Value = 0.42^2 = 0.1764

Time to expiration = Period of exclusive rights to product = 5 years

Dividend Yield = 1/Life of the patent = 1/5 = 0.20

Assume that the 5-year riskless rate is 5%. The value of the option can be estimated as follows:

Call Value= 335 $\exp^{(-0.2)(5)}$ (0.2250) -500 ($\exp^{(-0.05)(5)}$ (0.0451)= $ 10.18 million

The rights to this product, which has a negative net present value if introduced today, is $ 10.18 million. Note though that the likelihood that this project will become viable before expiration is low (4.5%-22.5%) as measured by N(d1) and N(d2).

Concept Check: If you were negotiating with the owner of this product, would you offer to pay $ 10.18 million for the rights to this drug? Why or why not?

Practical Considerations

While it is quite clear that the option to delay is embedded in many projects, there are several problems associated with the use of option pricing models to value these options. First, the *underlying asset in this option, which is the project, is not traded,* making it difficult to estimate its value and variance. We would argue that the value can be estimated from the expected cash flows and the discount rate for the project, albeit with error. The variance is more difficult to estimate, however, since we are attempting the estimate a variance in project value over time.

Second, the *behavior of prices over time may not conform to the price path assumed by the option pricing models.* In particular, the assumption that value follows a diffusion process, and that the variance in value remains unchanged over time, may be difficult to justify in the context of a project. For instance, a sudden technological change may dramatically change the value of a project, either positively or negatively.

Third, *there may be no specific period for which the firm has rights to the project.* Unlike the example above, in which the firm had exclusive rights to the project for 20 years, often the firm's rights may be less clearly defined, both in terms of exclusivity and time. For instance, a firm may have significant advantages over its competitors, which may, in turn, provide it with the virtually exclusive rights to a project for a period of time. The rights are not legal restrictions, however, and could erode faster than expected. In such cases, the expected life of the project itself is uncertain and only an estimate. In the valuation of the rights to the product, in the previous section, we used a life for the option of 5 years, but competitors could in fact enter sooner than we anticipated. Alternatively, the barriers to entry may turn out to be greater than expected, and allow the firm to earn excess returns for longer than 5 years. Ironically, uncertainty about the expected life of the option can increase the variance in present value, and through it, the expected value of the rights to the project.

Implications Of Viewing The Right To Delay A Project As An Option

Several interesting implications emerge from the analysis of the option to delay a project as an option. First, a project may have a negative net present value based upon expected cash flows currently, but it may still be a “valuable” project because of the option characteristics. Thus, while a negative net present value should encourage a firm to reject a project, it should not lead it to conclude that the rights to this project are worthless. Second, a project may have a positive net present value but still not be accepted right away. This is because the firm may gain by waiting and accepting the project in a future period, for the same reasons that investors do not always exercise an option just because it is in the money. This is more likely to happen if the firm has the rights to the project for a long time, and the variance in project inflows is high. To illustrate, assume that a firm has the patent rights to produce a new type of disk drive for computer systems and that building a new plant will yield a positive net present value right now. If the technology for manufacturing the disk drive is in flux, however, the firm may delay taking the project in the hopes that the improved technology will increase the expected cash flows and consequently the value of the project. It has to weigh this off against the cost of delaying taking the project, which will be the cash flows that will be forsaken by not taking the project. Third, factors that can make a project less attractive in a static analysis can actually make the rights to the project more valuable. As an example, consider the effect of uncertainty about how long the firm will be able to operate without competition and earn excess returns. In a static analysis, increasing this uncertainty increases the riskiness of the project and may make it less attractive. When the project is viewed as an option, an increase in the uncertainty may actually make the option more valuable, not less.

Case 1: Valuing a Patent

A product patent provides a firm with the right to develop and market a product. The firm will do so only if the present value of the expected cash flows from the product sales exceed the cost of development, however, as shown in Figure 5. If this does not occur, the firm can shelve the patent and not incur any further costs. If I is the present value of the costs of developing the product, and V is the present value of the expected cash flows from development, the payoffs from owning a product patent can be written as:

Payoff from owning a product patent = V - I if V> I

= 0 if V ≤ I

Thus, a product patent can be viewed as a call option, where the product itself is the underlying asset.

Figure 5: Payoff to Introducing Product

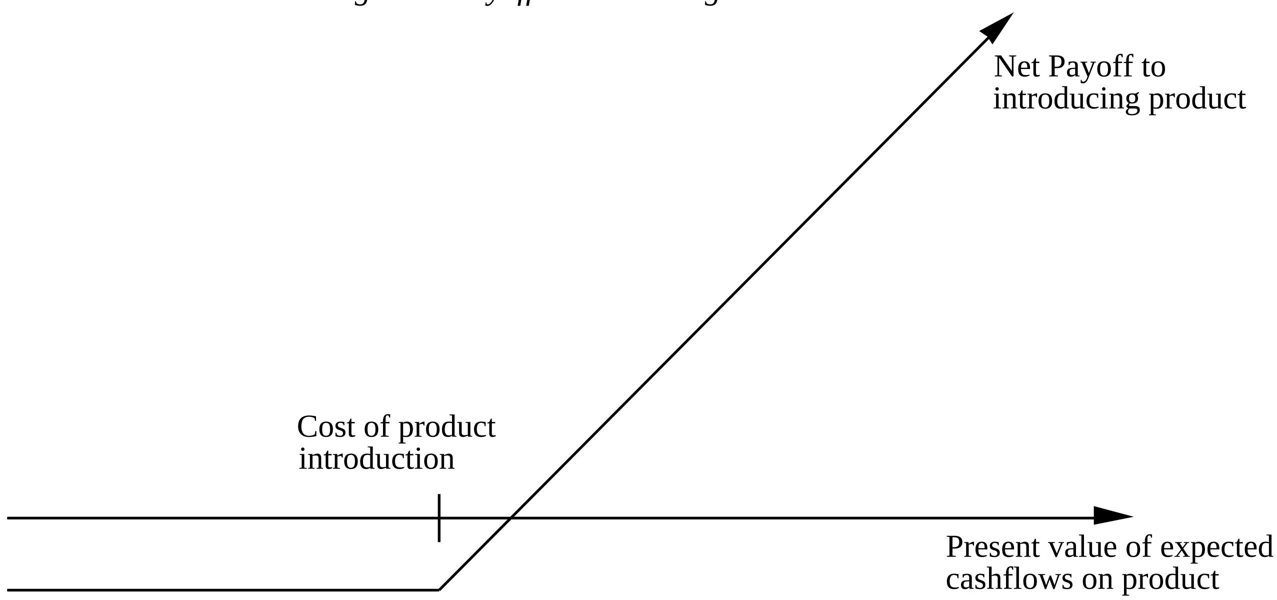

Illustration: Valuing a Patent: Avonex in 1997

Biogen is a bio-technology firm with a patent on a drug called Avonex, which has passed FDA approval to treat multiple sclerosis. Assume that you are trying to value the patent to Biogen and that you arrive at the following estimates for use in the option pricing model:

1. An internal analysis of the drug today, based upon the potential market and the price that the firm can expect to charge, yields a present value of cash flows of $ 3.422 billion, prior to considering the initial development cost.
2. The initial cost of developing the drug for commercial use is estimated to be $2.875 billion, if the drug is introduced today.
3. The firm has the patent on the drug for the next 17 years, and the current long-term treasury bond rate is 6.7%.
4. While it is difficult to do reasonable simulations of the cash flows and present values, the average variance in firm value for publicly traded bio-technology firms is 0.224.

It is assumed that the potential for excess returns exists only during the patent life, and that competition will wipe out excess returns beyond that period. Thus, any delay in introducing the drug, once it becomes viable, will cost the firm one year of patent-protected excess returns. (For the initial analysis, the cost of delay will be 1/17, next year it will be 1/16, the year after 1/15 and so on.)

Based on these assumptions, we obtain the following inputs to the option pricing model.

Present Value of Cash Flows from Introducing the Drug Now = S = $ 3.422 billion

Initial Cost of Developing Drug for Commercial Use (today) = K = $ 2.875 billion

Patent Life = t = 17 years

Riskless Rate = r = 6.7% (17-year Treasury Bond rate)

Variance in Expected Present Values = σ^2 = 0.224 (Industry average firm variance for bio-tech firms)

Expected Cost of Delay = y = 1/17 = 5.89%

These yield the following estimates for d and N(d):

d1 = 1.1362 N(d1) = 0.8720

d2 = -0.8512 N(d2) = 0.2076

Plugging back into the option pricing model, we get:

Value of the patent = 3,422 exp(-0.0589)(17) (0.8720) - 2,875 (exp(-0.067)(17) (0.2076)= $ 907 million

To provide a contrast, the net present value of this project is only $ 547 million:

NPV = $3,422 million - $ 2,875 million = $ 547 million

The time premium on this option suggests that the firm will be better off waiting rather than developing the drug immediately, the cost of delay notwithstanding. However, the cost of delay will increase over time, and make exercise (development) more likely.

Potential Refinements

In the course of this discussion, we have made a number of simplifying assumptions, to make it easier to estimate a value. For instance,

1. We assumed that all of the uncertainty in the patent value came from the present value of the cash flows, and that the initial investment is known with certainty. In reality, the initial investment is also estimated with noise, and the option value might have to reflect it in one of two ways. The first is to move the uncertainty into S, which is the estimate of the expected value of the cash flows. The second is to do the entire analysis in scaled terms. To illustrate what I mean by this, consider the patent just described with an initial investment of $3,422 million and an initial investment of $2,875 million. Assume that both estimates can change over time. The analysis can be recast in the following units:

S = (Present Value of Cash Flows/ Initial Investment) = 3422/2875 = 1.1903

K = Initial Investment in scaled terms = 1.00

σ^2 = Variance in the present value index over time (rather than the present value of the cash flows) = 0.224 (I have assumed it to be the same in this case)

All of the other inputs remain unchanged. The option value will be estimated as a percent of the initial investment, and works out as follows:

Value of the option = 0.3154 or 31.54% of the initial investment

Since we used the same variance estimate in both cases, the value of the option is still $ 907 million. To the extent that the variance in the ratio of present value of cash flows to the initial investment can be different[7] from the variance of the present value of the cash flows, the value of the option could have changed with the rescaling.

2. We assumed that the excess returns are restricted to the patent life, and that they disappear the instant the patent expires. In the pharmaceutical sector, the expiration of a a patent does not necessarily mean the loss of excess returns. In fact, many firms continue to be able to charge a premium price for their products and earn excess returns, even after the patent expires, largely as a consequence of the brand name image that they built up over the project life. A simple way of adjusting for this reality is to increase the present value of the cash flows on the project (S) and decrease the cost of delay (y) to reflect this reality. The net effect is a greater likelihood that firms will delay commercial development, while they wait to collect more information and assess market demand.

In the course of making these refinements, it is worth keeping in mind that an approximate estimate of the value of an option will suffice in most cases.

Case 2: Valuing Natural Resource Options

In a natural resource investment, the underlying asset is the natural resource and the value of the asset is based upon two variables - (1) the estimated quantity, and (2) the price of the resource. Thus, in a gold mine, for example, the underlying asset is the value of the estimated gold reserves in the mine, based upon the current price of gold. In most such investments, there is an initial cost associated with developing the resource; the difference between the value of the asset extracted and the cost of the development is the profit to the owner of the resource (see Figure 5). Defining the cost of development as X, and the

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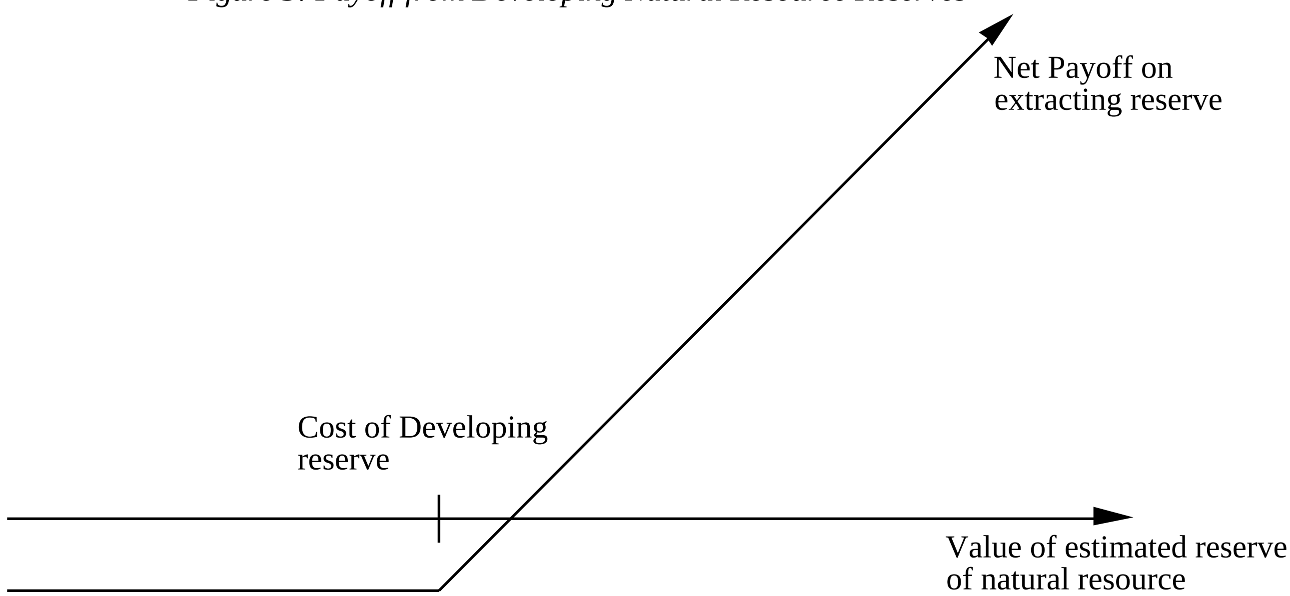
[7] In general, if the variance in S is σ_S^2 and the variance in the initial investment is σ_K^2, the variance in S/K can be written as follows:

estimated value of the resource as V, the potential payoffs on a natural resource option can be written as follows:

Payoff on natural resource investment	$= V - X$	if $V > X$
	$= 0$	if $V \leq X$

Thus, the investment in a natural resource option has a payoff function similar to a call option.

Figure 5: Payoff from Developing Natural Resource Reserves



To value a natural resource investment as an option , we need to make assumptions about a number of variables:

1. Available reserves of the resource: Since this is not known with certainty at the outset, it has to be estimated. In an oil tract, for instance, geologists can provide reasonably accurate estimates of the quantity of oil available in the tract.

2. Estimated cost of developing the resource: The estimated development cost is the exercise price of the option. Again, a combination of knowledge about past costs and the specifics of the investment have to be used to come up with a reasonable measure of development cost.

3. Time to expiration of the option: The life of a natural resource option can be defined in one of two ways. First, if the ownership of the investment has to be relinquished at the end of a fixed period of time, that will be the life of the option. In many off-shore oil leases, for instance, the oil tracts are leased to the oil company for several years. The second approach is based upon the inventory of the resource and the capacity output rate, as well as estimates of the number of years it would take to exhaust the inventory. Thus, a gold mine with a mine inventory of 3 million ounces and a capacity output rate of 150,000 ounces a year will be exhausted in 20 years, which is defined as the life of the natural resource option.

4. Variance in value of the underlying asset: The variance in the value of the underlying asset is determined by two factors – (1) variability in the price of the resource, and (2) variability in the estimate of available reserves. In the special case where the quantity of the reserve is known with certainty, the variance in the underlying asset's value will depend entirely upon the variance in the price of the natural resource; in this case, the option can be valued like any other simple option. In the more realistic case where the quantity of the reserve and the oil price can change over time, the option becomes more difficult to value; here, the firm may have to invest in stages to exploit the reserves.

5. Cost of Delay: The net production revenue as a percentage of the market value of the reserve is the equivalent of the dividend yield and is treated the same way in calculating option values. An alternative way of thinking about this cost is in terms of a cost of delay. Once a natural resource option is in-the-money (Value of the reserves > Cost of developing these reserves), the firm by not exercising the option is costing itself the production revenue it could have generated by developing the reserve.

An important issue in using option pricing models to value natural resource options is the effect of development lags on the value of these options. Since the resources cannot be extracted instantaneously, a time lag has to be allowed between the decision to extract the resources and the actual extraction. A simple adjustment for this lag is to take the adjust the value of the developed reserve for the loss of cash flows during the development period. Thus, if there is a one-year lag in development, the current value of the developed reserve will be discounted back one year at the net production revenue/asset value ratio[8] (which we also called the dividend yield above).

Illustration: Valuing a Gold Mine[9]

Consider a gold mine with an estimated inventory of 1 million ounces and a capacity output rate of 50,000 ounces per year. The price of gold is expected to grow 3% a year. The firm owns the rights to this mine for the next 20 years. The cost of opening the mine is $100 million, and the average production cost is $250 per ounce; once initiated, the production cost is expected to grow 5% a year. The standard deviation in gold prices is 20%, and the current price of gold is $375 per ounce. The riskless rate is 6%. The inputs to the model are as follows:

Value of the underlying asset = Present Value of expected gold sales (@ 50,000 ounces a year) = $(50{,}000 * 375) * [1- (1.03^{20}/1.09^{20})]/(.09-.03) - (50{,}000*250)*[1-(1.05^{20}/1.09^{20})]/(.09-.05)$ = \$ 211.79 million - \$ 164.55 million = \$ 47.24 million

Exercise price = Cost of opening mine = $100 million

Variance in ln(gold price) = 0.04

Time to expiration on the option = 20 years

Riskless interest rate = 6%

Dividend Yield = Loss in production for each year of delay = 1 / 20 = 5%

[8] Intuitively, it may seem like the discounting should occur at the riskfree rate. The simplest way of explaining why we discount at the dividend yield is to consider the analogy with a listed option on a stock. Assume that on exercising a listed option on a stock, you had to wait six months for the stock to be delivered to you. What you lose is the dividends you would have received over the six month period by holding the stock. Hence, the discounting is at the dividend yield.

[9] The following is a simpler version of the example presented in Brennan and Schwartz, applying option pricing theory to value a gold mine.

(Note: It will take 20 years to empty the mine, and the firm owns the rights for 20 years. Every year of delay implies a loss of one year of production.)

Based upon these inputs, the Black-Scholes model provides the following value for the call:

$d_1 = -0.1676$ $N(d_1) = 0.4334$

$d_2 = -1.0621$ $N(d_2) = 0.1441$

Call Value = 47.24 $\exp^{(-0.05)(20)}$ (0.4334) - 100 ($\exp^{(-0.09)(20)}$ (0.1441)= \$ 3.19 million

The value of the mine as an option is \$ 3.19 million, in contrast the static capital budgeting analysis would have yielded a net present value of -\$52.76 million (\$47.24 million - \$100 million). The additional value accrues directly from the mine's option characteristics.

Concept Check: Assume that the price of gold increases by 5%. Will the value of the option increase by an equivalent amount? Why or why not?

Illustration: Valuing an Oil Reserve[10]

Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil; the cost of developing the reserve is expected to be \$ 600 million, and the development lag is two years. The firm has the rights to exploit this reserve for the next 20 years, and the marginal value per barrel of oil is \$12 currently[11] (price per barrel - marginal cost per barrel). Once developed, the net production revenue each year will be 5% of the value of the reserves. The riskless rate is 8%, and the variance in ln(oil prices) is 0.03.

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[10] The following is a simplified version of the illustration provided by Siegel, Smith and Paddock to value an offshore oil property.

[11] For simplicity, we will assume that while this marginal value per barrel of oil will grow over time, the present value of the marginal value will remain unchanged at \$ 12 per barrel. If we do not make this

Given this information, the inputs to the Black-Scholes can be estimated as follows:

Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $12 * 50 /(1.05)^2 = $ 544.22

If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period; hence, the discounting of the reserve back at the dividend yield.

Exercise Price = Cost of developing reserve = $ 600 million (assumed to be fixed over time)

Time to expiration on the option = 20 years

In this example, we assume that the only uncertainty is in the price of oil, and the variance therefore becomes the variance in ln(oil prices).

Variance in the value of the underlying asset = 0.03

Riskless rate =8%

Dividend Yield = Net production revenue / Value of reserve = 5%

Based upon these inputs, the Black-Scholes model provides the following value for the call:

d_1 = 1.0359 N(d_1) = 0.8498

d_2 = 0.2613 N(d_2) = 0.6030

Call Value= 544 .22 exp$^{(-0.05)(20)}$ (0.8498) -600 (exp$^{(-0.08)(20)}$ (0.6030)= $ 97.08 million

This oil reserve, though not viable at current prices, is still a valuable property because of its potential to create value if oil prices go up.

Uncertainty about Quantity of Reserves

In the example above, we assumed that there was no uncertainty about the quantity of the reserve. Realistically, the oil company has an estimate of the reserve of 50 million

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assumption, we will have to estimate the present value of the oil that will be extracted over the extraction period.

barrels but does not know it with certainty. If we introduce uncertainty about the quantity of the reserve into the analysis, the effect on option value is uncertain. The expected value of the oil may be unchanged, but the variance in the value of the reserve will change. In fact, if the variance in the ln(estimated reserve quantity) is $\sigma_{\text{oil price}}^2$ and the variance in ln(oil prices) is σ_{qty}^2 , the variance[12] in the value of the reserve can be written as:

$$\sigma_{\text{reserve value}}^2 = \sigma_{\text{oil price}}^2 + \sigma_{\text{qty}}^2$$

We are assuming that there is no correlation between the estimated quantity of oil in a reserve and the price per barrel of oil. Thus, in the above example, if we introduced a variance of 0.05 in the natural log of the estimated reserve of 50 million barrels, we obtain the following estimate for the variance in the value of the reserve:

$$\sigma_{\text{reserve value}}^2 = 0.05 + 0.03 = 0.08$$

This would be the variance we would use in the option pricing model to estimate a new value for the reserve. By itself, the increased variance should increase the value of the option, but it introduces more uncertainty into the exercise decision. At the point of exercise, the oil company will not have a clear estimate of the value of the reserves, and thus might be exercising when it should not (value of the reserves is less than the cost of developing the reserve).

Concept Check: Assume that oil prices increase by $ 5 per barrel today and drop back by $ 5 tomorrow. Will these changes affect the value of the oil reserve? If so, why? If not, why not?

[12] This is the variance of a product of two variables. If c = ln(ab), then variance in c can be written as aq function of the variances of ln(a) and ln(b).

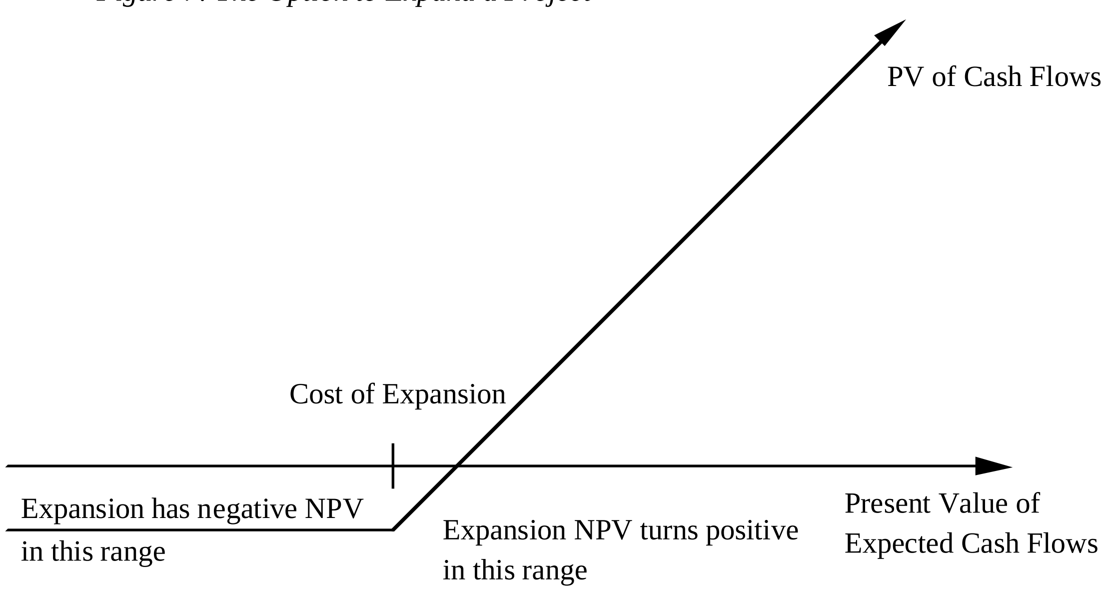
Variance (ln (c)) = Variance (ln(ab)) = Variance (ln(a)) + Variance (ln(b)) + Covariance(ln(a), ln(b))

The Option to Expand a Project

In some cases, firms take projects because doing so allows them either to take on other projects or to enter other markets in the future. In such cases, it can be argued that the initial projects are options allowing the firm to take other projects, and the firm should therefore be willing to pay a price for such options. A firm may accept a negative net present value on the initial project because of the possibility of high positive net present values on future projects.

To examine this option using the framework developed earlier, assume that the present value of the expected cash flows from entering the new market or taking the new project is V, and the total investment needed to enter this market or take this project is X. Further, assume that the firm has a fixed time horizon, at the end of which it has to make the final decision on whether or not to take advantage of this opportunity. Finally, assume that the firm cannot move forward on this opportunity if it does not take the initial project. This scenario implies the option payoffs shown in Figure 7.

Figure 7: The Option to Expand a Project



As you can see, at the expiration of the fixed time horizon, the firm will enter the new market or take the new project if the present value of the expected cash flows at that point in time exceeds the cost of entering the market.

IN PRACTICE: Valuing an Option to Expand: The Home Depot

Assume that The Home Depot is considering opening a small store in France. The store will cost 100 million FF to build, and the present value of the expected cash flows from the store is 120 million FF. Thus, by itself, the store has a negative NPV of 20 million FF.

Assume, however, that by opening this store, the Home Depot acquires the option to expand into a much larger store any time over the next 5 years. The cost of expansion will be 200 million FF, and it will be undertaken only if the present value of the expected cash flows exceeds 200 million FF. At the moment, the present value of the expected cash flows from the expansion is believed to be only 150 million FF. If it were not, the Home Depot would have opened to larger store right away. The Home Depot still does not know much about the market for home improvement products in France, and there is considerable uncertainty about this estimate. The variance is 0.08.

The value of the option to expand can now be estimated, by defining the inputs to the option pricing model as follows:

Value of the Underlying Asset (S) = PV of Cash Flows from Expansion, if done now

=150 million FF

Strike Price (K) =Cost of Expansion = 200 million FF

Variance in Underlying Asset's Value = 0.08

Time to expiration = Period for which expansion option applies = 5 years

Assume that the five-year riskless rate is 6%. The value of the option can be estimated as follows:

Call Value= 150 $\exp^{(-0.06)(5)}$ (0.6314) -200 ($\exp^{(-0.06)(20)}$ (0.3833)= 37.91 million FF

This value can be added on to the net present value of the original project under consideration.

NPV of Store = 80 million FF - 100 million FF = -20 million

Value of Option to Expand = 37.91 million FF

NPV of store with option to expand = -20 million + 37.91 million = 17.91 mil FF

The Home Depot should open the smaller store, even though it has a negative net present value, because it acquires an option of much greater value, as a consequence.

Practical Considerations

The practical considerations associated with estimating the value of the option to expand are similar to those associated with valuing the option to delay. In most cases, firms with options to expand have no specific time horizon by which they have to make an expansion decision, making these open-ended options, or, at best, options with arbitrary lives. Even in those cases where a life can be estimated for the option, neither the size nor the potential market for the product may be known, and estimating either can be problematic. To illustrate, consider the Home Depot example discussed above. While we adopted a period of five years, at the end of which the Home Depot has to decide one way or another on its future expansion in France, it is entirely possible that this time frame is not specified at the time the store is opened. Futhermore, we have assumed that both the cost and the present value of expansion are known initially. In reality, the firm may not have good estimates for either before opening the first store, since it does not have much information on the underlying market.

Concept Check: Firms that require their initial ventures in new markets to carry their own weight (i.e., have positive net present values) are much less likely to enter these markets. Comment.

Implications

The option to expand is implicitly used by firms to rationalize taking projects that may have negative net present value, but provide significant opportunities to tap into new markets or sell new products. While the option pricing approach adds rigor to this argument by estimating the value of this option, it also provides insight into those occasions when it is most valuable. In general, the option to expand is clearly more valuable for more volatile businesses with higher returns on projects (such as biotechnology or computer software), than in stable businesses with lower returns (such as housing, utilities or automobile production).

Strategic Considerations/Options

In many acquisitions or investments, the acquiring firm believes that the transaction will give it competitive advantages in the future. These competitive advantages can range the gamut, and include:

1. Entrée into a Growing or Large Market: An investment or acquisition may allow the firm to enter a large or potentially large market much sooner than it otherwise would have been able to do so. A good example of this would be the acquisition of a Mexican retail firm by a US firm, with the intent of expanding into the Mexican market.
2. Technological Expertise: In some cases, the acquisition is motivated by the desire to acquire a proprietary technology, that will allow the acquirer to expand either its existing market or into a new market.
3. Brand Name: Firms sometime pay large premiums over market price to acquire firms with valuable brand names, because they believe that these brand names can be used for expansion into new markets in the future.

While all of these potential advantages may be used to justify initial investments that do not meet financial benchmarks (negative net present value projects, acquisition premiums …), not all of them create valuable options. As we will see later in the paper, the value of the option is derived from the degree to which these competitive advantages, assuming that they do exist, translate into sustainable excess returns.

Research, Development and Test Market Expenses

Firms that spend considerable amounts of money on research and development and test marketing are often stymied when they try to evaluate these expenses, since the payoffs are often in terms of future projects. At the same time, there is the very real possibility that after the money has been spent, the products or projects may turn out not to be viable; consequently, the expenditure is treated as a sunk cost. In fact, it can be argued that R & D has the characteristics of a call option —the amount spent on the R&D is the cost of the call option, and the projects or products that might emerge from the research provide the payoffs on the options. If these products are viable (i.e., the present value of the cash inflows exceeds the needed investment), the payoff is the difference between the two; if not, the project will not be accepted, and the payoff is zero.

Several logical implications emerge from this view of R & D. First, research expenditures should provide much higher value for firms that are in volatile technologies or businesses, since the variance in product or project cash flows is positively correlated with the value of the call option. Thus, Minnesota Mining and Manufacturing (3M), which expends a substantial amount on R&D on basic office products, such as the Post-it pad, should receive less value[13] for its research than does Amgen, whose research primarily concerns bio-technology products. Second, the value of research and the optimal amount to be spent on research will change over time as businesses mature. The best example is the pharmaceutical industry - pharmaceutical companies spent most of the 1980s investing substantial amounts in research and earning high returns on new products, as the health

[13] This statement is based on the assumption that the quality of research is the same at both firm, though the research is in different businesses, and that the only difference is in the volatility of the underlying businesses.

care business expanded. In the 1990s, however, as health care costs started leveling off and the business matured, many of these companies found that they were not getting the same payoffs on research and started cutting back. Some companies moved research dollars from conventional drugs to bio-technology products, where the uncertainty about future cash flows remains high.

Concept Check: This approach pre-supposes that the research is applied and directed towards finding commercial products. Would the same arguments apply for basic research (such as the research done on basic theory at universities and some research institutions like Bell Labs)? Why or why not?

Multi-Stage Projects/ Investments

When entering new businesses or taking new investments, firms sometimes have the option to enter the business in stages. While doing so may reduce potential upside, it also protects the firm against downside risk, by allowing it, at each stage, to gauge demand and decide whether to go on to the next stage. In other words, a standard project can be recast as a series of options to expand, with each option being dependent on the previous one. There are two propositions that follow:

4. Some projects that do not look good on a full investment basis may be value creating if the firm can invest in stages.
5. Some projects that look attractive on a full investment basis may become even more attractive if taken in stages.

The gain in value from the options created by multi-stage investments has to be weighed off against the cost. Taking investments in stages may allow competitors who decide to enter the market on a full scale to capture the market. It may also lead to higher costs at each stage, since the firm is not taking full advantage of economies of scale.

There are several implications that emerge from viewing this choice between multi-stage and one-time investments in an option framework. The projects where the gains will be largest from making the investment in multiple stages include:

5. Projects where there are significant barriers to entry from competitors entering the market, and taking advantage of delays in full-scale production. Thus, a firm with a patent on a product or other legal protection against competition pays a much smaller price for starting small and expanding as it learns more about the product
6. Projects where there is significant uncertainty about the size of the market and the eventual success of the project. Here, starting small and expanding allows the firm to reduce its losses if the product does not sell as well as anticipated, and to learn more about the market at each stage. This information can then be useful in subsequent stages in both product design and marketing.
7. Projects where there is a substantial investment needed in infrastructure (large fixed costs) and high operating leverage. Since the savings from doing a project in multiple stages can be traced to investments needed at each stage, they are likely to be greater in firms where those costs are large. Capital intensive projects as well as projects that require large initial marketing expenses (a new brand name product for a consumer product company) will gain more from the options created by taking the project in multiple stages.

When are real options valuable? Some Key Tests

While the argument that some or many investment have valuable strategic or expansion options embedded in them has great allure, there is a danger that this argument will be used to justify poor investments. In fact, acquirers have long justified huge premiums on acquisitions on synergistic and strategic grounds. To prevent real options from falling into the same black hole, we need to be more rigorous in our measurement of the value of real options.

Quantitative Estimation

When real options are used to justify a decision, the justification has to be in more than qualitative terms. In other words, managers who argue for taking a project with poor returns or paying a premium on an acquisition on the basis of real options, should be required to value these real options and show, in fact, that the economic benefits exceed the costs. There will be two arguments made against this requirement. The first is that real options cannot be easily valued, since the inputs are difficult to obtain and often noisy. The second is that the inputs to option pricing models can be easily manipulated to back up whatever the conclusion might be. While both arguments have some basis, a noisy estimate is better than no estimate at all, and the process of quantitatively trying to estimate the value of a real option is, in fact, the first step to understanding what drives it value.

Key Tests

Not all investments have options embedded in them, and not all options, even if they do exist, have value. To assess whether an investment creates valuable options that need to be analyzed and valued, three key questions need to be answered affirmatively.

1. <u>Is the first investment a pre-requisit for the later investment/expansion? If not, how necessary is the first investment for the later investment/expansion?</u> Consider our earlier analysis of the value of a patent or the value of an undeveloped oil reserve as options. A firm cannot generate patents without investing in research or paying another firm for the patents, and it cannot get rights to an undeveloped oil reserve without bidding on it at a government auction or buying it from another oil company. Clearly, the initial investment here (spending on R&D, bidding at the auction) is required for the firm to have the second option. Now consider the Home Depot investment in a French store and the option to expand into the French market later. The initial store investment provides the Home Depot with information about market potential, without which presumably it is unwilling to expand into the larger market. Unlike the patent and undeveloped reserves illustrations, the initial investment is not a pre-requisit for the

second, though management might view it as such. The connection gets even weaker when we look at one firm acquiring another to have the option to be able to enter a large market. Acquiring an internet service provider to have a foothold in the internet retailing market or buying a Brazilian brewery to preserve the option to enter the Brazilian beer market would be examples of such transactions.

2. Does the firm have an exclusive right to the the later investment/expansion? If not, does the initial investment provide the firm with significant competitive advantages on subsequent investments? The value of the option ultimately derives not from the cash flows generated by then second and subsequent investments, but from the excess returns generated by these cash flows. The greater the potential for excess returns on the second investment, the greater the value of the option in the first investment. The potential for excess returns is closely tied to how much of a competitive advantage the first investment provides the firm when it takes subsequent investments. At one extreme, again, consider investing in research and development to acquire a patent. The patent gives the firm that owns it the exclusive rights to produce that product, and if the market potential is large, the right to the excess returns from the project. At the other extreme, the firm might get no competitive advantages on subsequent investments, in which case, it is questionable as to whether there can be any excess returns on these investments. In reality, most investments will fall in the continuum between these two extremes, with greater competitive advantages being associated with higher excess returns and larger option values.

3. How sustainable are the competitive advantages? In a competitive market place, excess returns attract competitors, and competition drives out excess returns. The more sustainable the competitive advantages possessed by a firm, the greater will be the value of the options embedded in the initial investment. The sustainability of competitive advantages is a function of two forces. The first is the nature of the competition; other things remaining equal, competitive advantages fade much more quickly in sectors

where there are aggressive competitors. The second is the nature of the competitive advantage. If the resource controlled by the firm is finite and scarce (as is the case with natural resource reserves and vacant land), the competitive advantage is likely to be sustainable for longer periods. Alternatively, if the competitive advantage comes from being the first mover in a market or technological expertise, it will come under assault far sooner. The most direct way of reflecting this in the value of the option is in its life; the life of the option can be set to the period of competitive advantage and only the excess returns earned over this period counts towards the value of the option.

The Option to Abandon a Project

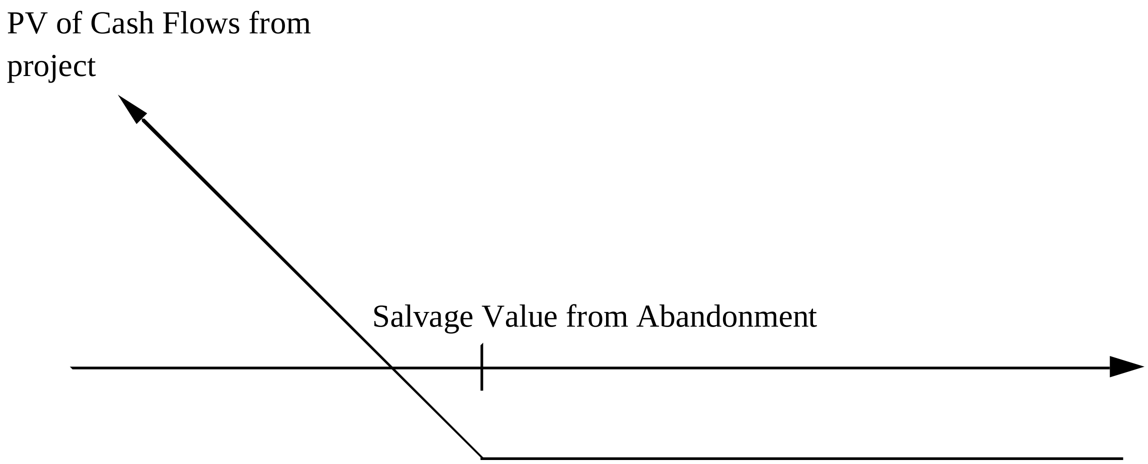
The final option to consider here is the option to abandon a project when its cash flows do not measure up to expectations. One way to reflect this value is through decision trees. This approach has limited applicability in most real world investment analyses; it typically works only for multi-stage projects, and it requires inputs on probabilities at each stage of the project. The option pricing approach provides a more general way of estimating and building in the value of abandonment into the value of an option. To illustrate, assume that V is the remaining value on a project if it continues to the end of its life, and L is the liquidation or abandonment value for the same project at the same point in time. If the project has a life of n years, the value of continuing the project can be compared to the liquidation (abandonment) value — if the value from continuing is higher, the project should be continued; if the value of abandonment is higher, the holder of the abandonment option could consider abandoning the project –

Payoff from owning an abandonment option $= 0$ if $V > L$

$= L - V$ if $V \leq L$

These payoffs are graphed in Figure 9, as a function of the expected stock price.

Figure 9: The Option to Abandon a Projectt



Unlike the prior two cases, the option to abandon takes on the characteristics of a put option.

Valuing an Option to Abandon; An Illustration

Assume that a firm is considering taking a 10-year project that requires an initial investment of $ 100 million in a real estate partnership, where the present value of expected cash flows is $ 110 million. While the net present value of $ 10 million is small, assume that the firm has the option to abandon this project anytime in the next 10 years, by selling its share of the ownership to the other partners for $ 50 million. The variance in the present value of the cash flows from being in the partnership is 0.09.

The value of the abandonment option can be estimated by determining the characteristics of the put option:

Value of the Underlying Asset (S) = PV of Cash Flows from Project

= $ 110 million

Strike Price (K) = Salvage Value from Abandonment = $ 50 million

Variance in Underlying Asset’s Value = 0.06

Time to expiration = Period for which the firm has abandonment option = 10 years

Assume that the ten-year riskless rate is 6%, and that the property is not expected to lose value over the next 10 years. The value of the put option can be estimated as follows:

Call Value = 110 (0.9737) -50 (exp$^{(-0.06)(10)}$ (0.8387) = \$ 84.09 million

Put Value= \$ 84.09 - 110 + 50 exp$^{(-0.06)(10)}$ = \$ 1.53 million

The value of this abandonment option has to be added on to the net present value of the project of \$ 10 million, yielding a total net present value with the abandonment option of \$ 11.53 million. Note though that abandonment becomes a more and more attractive option as the remaining project life decreases, since the present value of the remaining cash flows will decrease.

Practical Considerations

In the above analysis, we assumed, rather unrealistically, that the abandonment value was clearly specified up front and that it did not change during the life of the project. This may be true in some very specific cases, in which an abandonment option is built into the contract. More often, however, the firm has the option to abandon, and the salvage value from doing so can be estimated with noise up front. Further, the abandonment value may change over the life of the project, making it difficult to apply traditional option pricing techniques. Finally, it is entirely possible that abandoning a project may not bring in a liquidation value, but may create costs instead; a manufacturing firm may have to pay severance to its workers, for instance. In such cases, it would not make sense to abandon, unless the cash flows on the project are even more negative.

We also assumed that the real estate investment was not expected to lose value over time. On a real project, there may be a loss in value as the project ages. This expected loss in value, on an annual basis, can be built in as a dividend yield and used to value the abandonment option. It will make the option more valuable.

Implications

The fact that the option to abandon has value provides a rationale for firms to build the operating flexibility to scale back or terminate projects if they do not measure up to expectations. It also indicates that firms that focus on generating more revenues by offering their customers the option to walk away from commitments may be giving up more than they gain, in the process.

Escape Clauses in Contracts

The first, and most direct way, is to build operating flexibility contractually with those parties that are involved in the project. Thus, contracts with suppliers may be written on an annual basis, rather than long term, and employees may be hired on a temporary basis, rather than permanently. The physical plant used for a project may be leased on a short term basis, rather than bought, and the financial investment may be made in stages rather than as an initial lump sum. While there is a cost to building in this flexibility, the gains may be much larger, especially in volatile businesses.

Customer Incentives

On the other side of the transaction, offering abandonment options to customers and partners in joint ventures can have a negative impact on value. As an example, assume that a firm that sells its products on multi-year contracts offers customers the option to cancel the contract at any time. While this may sweeten the deal and increase sales, there is likely to be a substantial cost. In the event of a recession, firms that are unable to meet their obligations are likely to cancel their contracts. When there is sufficient volatility in income, any benefits gained by the initial sale (obtained by offering the inducement of cancellation by the buyer) may be offset by the cost of the option provided to customers.

Valuing Equity as an option

In traditional discounted cash flows models, a firm is valued by estimating cash flows over a long time horizon (often over an infinite period) and discounting the cash flows back at a discount rate that reflects the riskiness of the cash flows. The value of equity is obtained by subtracting the value of debt from firm value. In this section, we will argue that discounted cash flow models understate the value of equity in firms with high financial leverage and negative operating income, since they do not reflect the option that equity investors have to liquidate the firm's assets.

The General Framework

The equity in a firm is a residual claim, that is, equity holders lay claim to all cash flows left over after other financial claimholders (debt, preferred stock etc.) have been satisfied. If a firm is liquidated, the same principle applies; equity investors receive whatever is left over in the firm after all outstanding debt and other financial claims are paid off. The principle of limited liability protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm. The payoff to equity investors, on liquidation can therefore be written as:

Payoff to equity on liquidation	$= V - D$	if $V > D$
	$= 0$	if $V \leq D$

where

V = Liquidation Value of the firm

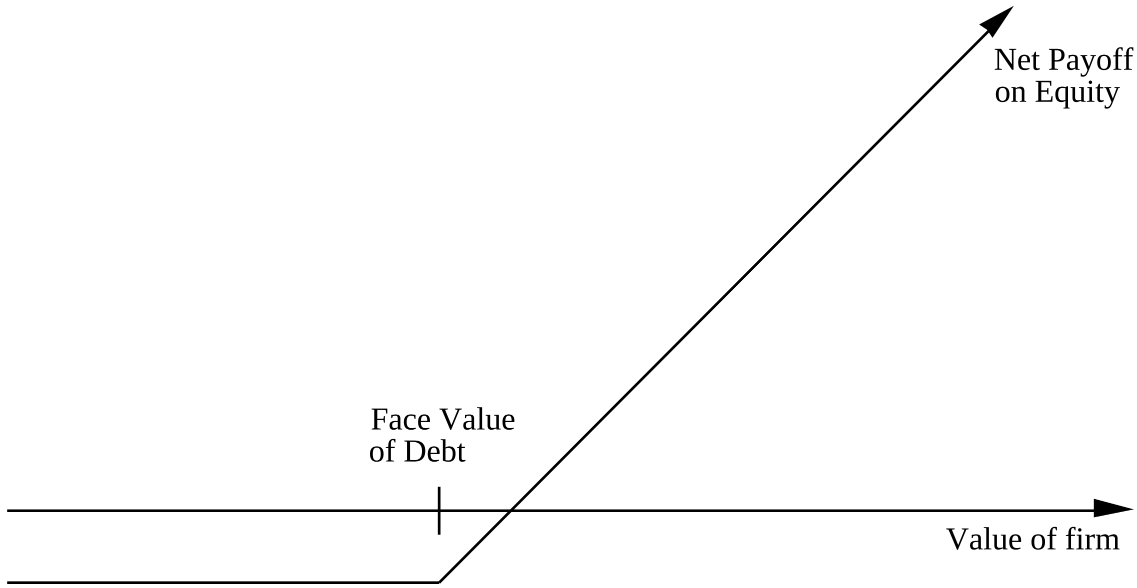
D = Face Value of the outstanding debt and other external claims

A call option, with a strike price of K, on an asset with a current value of S, has the following payoffs:

Payoff on exercise	$= S - K$	if $S > K$
	$= 0$	if $S \leq K$

Equity can thus be viewed as a call option the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercise price) paid off, as shown in Figure 10.

Figure 10: Payoff on Equity as Option on a Firm



If the debt in the firm is a single issue of zero-coupon bonds with a fixed lifetime, and the firm can be liquidated by equity investors at any time prior, the life of equity as a call option corresponds to the life of the bonds.

Application To Valuation: A Simple Example

Assume that you have a troubled airline, whose assets are currently valued at $100 million; the standard deviation in this asset value is 40%. Further, assume that the face value of debt is $80 million (it is zero coupon debt with 10 years left to maturity). If the ten-year treasury bond rate is 10%, how much is the equity worth? What should the interest rate on debt be?

At the outset, the information provided may seem inadequate to answer these questions. The use of the option pricing approach provides a solution, however. The parameters of equity as a call option are as follows:

Value of the underlying asset = S = Value of the firm = $ 100 million

Exercise price = K = Face Value of outstanding debt = $ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call:

$d_1 = 1.5994$ $N(d_1) = 0.9451$

$d_2 = 0.3345$ $N(d_2) = 0.6310$

Value of the call = 100 (0.9451) - 80 $\exp^{(-0.10)(10)}$ (0.6310) = \$75.94 million

Value of the outstanding debt = \$100 - \$75.94 = \$24.06 million

Interest rate on debt = (\$ 80 / \$24.06)$^{1/10}$ -1 = 12.77%

Implications

The first implication of viewing equity as a call option is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt. While the firm will be viewed as troubled by investors, accountants, and analysts, its equity is not worthless. In fact, just as deep out-of-the-money traded options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, equity commands value because of the time premium on the option (the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

Revisiting the preceding example, assume that the value of the firm is only \$50 million, well below the face value of the outstanding debt (\$80 million). Assume that all the other inputs remain unchanged. The parameters of equity as a call option are as follows:

Value of the underlying asset = S = Value of the firm = \$ 50 million

Exercise price = K = Face Value of outstanding debt = \$ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the call:

$d_1 = 1.0515$ $N(d_1) = 0.8534$

$d_2 = -0.2135$ $N(d_2) = 0.4155$

Value of the call = 50 (0.8534) - 80 $\exp^{(-0.10)(10)}$ (0.4155) = \$30.44 million

Value of the bond= \$50 - \$30.44 = \$19.56 million

As you can see, the equity in this firm has substantial value, because of the option characteristics of equity. This might explain why equity in firms that are essentially bankrupt still can have value.

Obtaining Option Pricing Inputs - Some Real World Problems

The examples used thus far to illustrate the use of option pricing theory to value equity have made some simplifying assumptions. Among them are the following:

1. There are only two claimholders in the firm - debt and equity.
2. There is only one issue of debt outstanding, and it can be retired at face value.
3. The debt has a zero coupon and no special features (convertibility, put clauses etc.)
4. The value of the firm and the variance in that value can be estimated.

Each of these assumptions is made for a reason. First, by restricting the claimholders to two, the problem is made more tractable; introducing other claimholders, such as preferred stock makes it more difficult to arrive at a result, albeit not impossible. Second, by assuming only one zero-coupon debt issue that can be retired at face value anytime prior to maturity, the features of the debt are made to correspond closely to the features of the strike price on a standard option. Third, if the debt is coupon debt, or more than one debt issue is outstanding, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations. Finally, knowing the value of the firm and the variance in that value makes the option pricing possible, but it also raises an interesting question about the usefulness of option pricing in the valuation context. If the bonds of the firm are publicly traded, the market value of the debt can be subtracted from the value of the firm to obtain the value of equity much more directly. The option pricing approach does have its advantages, however. Specifically, when the debt of a firm is not publicly traded, option pricing theory can provide an estimate of value for the equity in the firm. Even when the debt is publicly traded, the bonds may not be correctly valued, and the option pricing framework can be useful in evaluating the values of debt and equity. Finally, relating the values of debt and equity to the variance in firm value provides some insight into the redistributive effects of actions taken by the firm.

Application to valuation

Since most firms do not fall into the neat framework developed above (having only one zero-coupon bond outstanding), some compromises have to be made to use this model in valuation.

Value of the Firm

The value of the firm can be obtained in one of two ways. In the first, the market values of outstanding debt and equity are cumulated, assuming that all debt and equity are traded, to obtain firm value. The option pricing model then reallocates the firm value between debt and equity. This approach, while simple, is internally inconsistent. We start with one set of values for debt and equity, and using the option pricing model, end up with entirely different values for each.

In the second, the market values of the assets of the firm are estimated, either by discounting expected cash flows at the weighted average cost of capital or by using prices from a market that exists for these assets. The one consideration that we need to keep in mind is that the value of the firm in an option pricing model should be the value obtained on liquidation. This may be less than the total firm value, which includes expected future investments, and also be reduced to reflect the cost of liquidation. If the firm value is being estimated using a discounted cash flow model then, this would suggest that only assets in place[14] should be considered while estimating firm value.

Variance in Firm value

The variance in firm value can be obtained directly if both stocks and bonds in the firm trade in the market place. Defining σ_e^2 as the variance in the stock price and σ_d^2 as the variance in the bond price, w_e as the market-value weight of equity and w_d as the market-value weight of debt, the variance in firm value can be written as:[15]

$$\sigma^2_{firm} = w_e^2\, \sigma_e^2 + w_d^2\, \sigma_d^2 + 2\, w_e\, w_d\, \rho_{ed}\, \sigma_e\, \sigma_d$$

where ρ_{ed} is the correlation between the stock and the bond prices.

When the bonds of the firm are not traded, the variance of similarly rated bonds is used as the estimate of σ_d^2 and the correlation between similarly rated bonds and the firm's stock is used as the estimate of ρ_{ed}.

When companies get into financial trouble, both of the approaches above can yield strange results as both stock prices and bond prices become more volatile. An alternative that often yields more reliable estimates is to use the average variance in firm value for other firms in the sector.

Maturity of the Debt

Most firms have more than one debt issue on their books, and much of the debt comes with coupons. Since the option pricing model allows for only one input for the time to expiration, these multiple bonds issues and coupon payments have to be compressed into one measure; that is, these multiple issues have to be converted into one equivalent zero-coupon bond. One solution, which takes into account both the coupon payments and the maturity of the bonds, is to estimate the duration of each debt issue and calculate a face-

[14] Technically, this can be done by putting the firm into stable growth, and valuing it as a stable growth firm, where reinvestments are used to either preserve or augment existing assets.

value-weighted average of the durations of the different issues. This value-weighted duration is then used as a measure of the time to expiration of the option.

Face Value of Debt

When firms have more than one debt issue on their books, the face value of debt that is used has to include all of the principal outstanding on the debt. In addition, as coupons come due on existing debt, they need to be added on to the face value of debt to reflect the added obligations. In fact, to stay consistent with the notion of converting all debt into the equivalent of a zero-coupon bond, it may make sense to cumulate expected coupon (or interest) payments[16] on the debt over its lifetime and add it to the face value of debt.

Valuing Equity as an option - Eurotunnel

Eurotunnel was the firm that was created to build and ultimately profit from the tunnel under the English Channel, linking England and France. While the tunnel was readied for operations in the early 1990s, it never was a commercial success and reported significant losses each year after opening. In early 1998, Eurotunnel had a book value of equity of -£117 million, and in 1997, the firm had reported earnings before interest and taxes of -£56 million and net income of -£685 million. By any measure, it was a firm in financial trouble.

Much of the financing for the tunnel had come from debt, and at the end of 1997, Eurotunnel had debt obligations in excess of £ 8,000 million, including expected coupon payments. The following table summarizes the outstanding debt at the firm, with our estimates of the expected duration for each class of debt:

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[15] This is an extension of the variance formula for a two-asset portfolio.

[16] If we do not cumulate the coupons and add them to the face value, we will tend to understate the value of the debt.

Debt Type	*Face Value (including cumulated coupons)*	*Duration*
Short term	£ 935	0.50
10 year	£ 2435	6.7
20 year	£ 3555	12.6
Longer	£ 1940	18.2
Total	£8,865 mil	10.93 years

The firm's only significant asset is its ownership of the tunnel and we estimated the value of this asset, from its expected cash flows and the appropriate cost of capital. The assumptions we made were as follows:

- Revenues will grow 5% a year in perpetuity.
- The cost of goods sold which was 85% of revenues in 1997 will drop to 65% of revenues by 2002 and stay at that level.
- Capital spending and depreciation will grow 5% a year in perpetuity.
- There are no working capital requirements.
- The debt ratio, which was 95.35% at the end of 1997, will drop to 70% by 2002. The cost of debt is 10% in high growth period and 8% after that.
- The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years (as the leverage decreases).

The long term bond rate at the time of the valuation was 6%. Based on these assumptions, we estimated the cash flows to be as follows:

	1	*2*	*3*	*4*	*5*
Revenues	470	494	519	545	572
- COGS	400	395	389	381	372
- Depreciation	141	145	150	154	159
EBIT	(71)	(47)	(20)	9	41
- EBIT*t	(25)	(16)	(7)	3	14

EBIT (1-t)	(46)	(30)	(13)	6	27
+ Depreciation	141	145	150	154	159
- Capital Spending	46	48	49	51	52
- Chg. Working Capital					
Free CF to Firm	49	67	87	110	133
Terminal Value					2710
Present Value	46	59	72	84	2051

The value of the assets of the firm is £2,312 million.

The final input we estimated was the standard deviation in firm value. Since there are no directly comparable firms, we estimated the standard deviations in Eurotunnel stock and debt to be as follows:

Standard deviation in Eurotunnel stock price = 41%

Standard deviation in Eurotunnel bond price = 17%

We also estimated a correlation of 0.50 between Eurotunnel stock and bond prices, and the average market debt to capital ratio during the period was 85%. Combining these inputs, we estimated the standard deviation in firm value to be

$\sigma_{firm}^2 = (0.15)^2\ (0.41)^2 + (0.85)^2\ (0.17)^2 + 2\ (0.15)\ (0.85)(0.5)(0.41)(0.17) = 0.0335$

In summary, the inputs to the option pricing model were as follows:

Value of the underlying asset = S = Value of the firm = £2,312 million

Exercise price = K = Face Value of outstanding debt = £8,865 mil

Life of the option = t = Weighted average duration of debt = 10.93 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335

Riskless rate = r = Treasury bond rate corresponding to option life = 6%

Based upon these inputs, the Black-Scholes model provides the following value for the call:

$d_1 = -0.8337$ $N(d_1) = 0.2023$

$d_2 = -1.4392$ $N(d_2) = 0.0751$

Value of the call = 2,312 (0.2023) – 8,865 $\exp^{(-0.06)(10.93)}$ (0.0751) = £ 122 million

Eurotunnel's equity was trading at £ 150 million in 1997.

The option pricing framework in addition to yielding a value for Eurotunnel equity also yields some valuable insight into the drivers of value for this equity. While it is certainly important that the firm try to bring costs under control and increase operating margins, the two most critical variables determining its value are the life of the options and the variance in firm value. Any action that increases (decreases) the option life will have a postive (negative) effect on equity value. For instance, when the French government put pressure on the bankers who had lent money to Eurotunnel to ease restrictions and allow the firm more time to repay its debt, equity investors benefited, as their options became more long term. Similarly, an action that increases the volatility of expected firm value will increase the value of the option.

Option Pricing in Capital Structure and Dividend Policy Decisions

Option pricing theory can be applied to capital structure decisions in a number of ways. One is to illustrate the conflict between stockholders and bondholders when it comes to investment analysis and conglomerate mergers. A second is in the design and valuation of debt, equity, and hybrid securities. A third is to examine the value of financial flexibility, that is often cited by firms that choose not to use excess debt capacity and pay out what they can in dividends.

The Conflict between Bondholders and Stockholders

Stockholder and bondholders have different objective functions, and this can lead to agency problems, whereby stockholders expropriate wealth from bondholders. The conflict can manifest itself in a number of ways. For instance, stockholders have an incentive to take riskier projects than bondholders, and to pay more out in dividends than bondholders would like them to. The conflict between bondholders and stockholders can be illustrated dramatically using the option pricing methodology developed in the previous section.

Taking on Risky Projects

Since equity is a call option on the value of the firm, other things remaining equal, an increase in the variance in the firm value will lead to an increase in the value of equity. It is therefore conceivable that stockholders can take risky projects with negative net present values, which, while making them better off, may make the bondholders and the firm less valuable. To illustrate, consider the firm with a value of assets of $100 million, a face value of zero-coupon ten-year debt of $80 million and a standard deviation in the value of the firm of 40%, that we valued in the earlier illustration. The equity and debt in this firm were valued as follows:

Value of Equity = $75.94 million

Value of Debt = $24.06 million

Value of Firm == $100 million

Now assume that the stockholders have the opportunity to take a project with a negative net present value of -$2 million; the project is a very risky one that will push up the standard deviation in firm value to 50%. The equity as a call option can then be valued using the following inputs:

Value of the underlying asset = S = Value of the firm = $ 100 million - $2 million = $ 98 million (The value of the firm is lowered because of the negative net present value project)

Exercise price = K = Face Value of outstanding debt = $ 80 million

Life of the option = t = Life of zero-coupon debt = 10 years

Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.25

Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Based upon these inputs, the Black-Scholes model provides the following value for the equity and debt in this firm.

Value of Equity = $77.71

Value of Debt = $20.29

Value of Firm = $98.00

The value of equity rises from $75.94 million to $ 77.71 million, even though the firm value declines by $2 million. The increase in equity value comes at the expense of bondholders, who find their wealth decline from $24.06 million to $20.19 million.

Conglomerate Mergers

Bondholders and stockholders may experience the conflict in the case of conglomerate mergers, where the variance in earnings and cash flows of the combined firm can be expected to decline because the merging firms have earning streams that are not perfectly correlated. In these mergers, the value of the combined equity in the firm will decrease after the merger because of the decline in variance; consequently, bondholders will gain. Stockholders can reclaim some or all of this lost wealth by utilizing their higher debt capacity and issuing new debt. To illustrate, suppose you are provided with the following information on two firms , Lube and Auto (car service) and Gianni Cosmetics (a cosmetics manufacturer) that hope to merge.

	Lube & Auto	*Gianni Cosmetics*	
Value of the firm	$100 million	$ 150 million	
Face Value of Debt	$ 80 million	$ 50 million	(Zero-coupon debt)
Maturity of debt	10 years	10 years	
Std. Dev. in firm value	40 %	50 %	
Correlation between firm cash flows	0.4		

The ten-year bond rate is 10%.

The variance in the value of the firm after the acquisition can be calculated as follows:

Variance in combined firm value $= w_1^2\, \sigma_1^2 + w_2^2\, \sigma_2^2 + 2\, w_1\, w_2\, \rho_{12}\, \sigma_1\, \sigma_2$

$= (0.4)^2\, (0.16) + (0.6)^2\, (0.25) + 2\, (0.4)\, (0.6)\, (0.4)\, (0.4)\, (0.5)$

$= 0.154$

The values of equity and debt in the individual firms and the combined firm can then be estimated using the option pricing model:

	Lube & Auto	Gianni	Combined firm
Value of equity in the firm	$75.94	$134.47	$ 207.43
Value of debt in the firm	$24.06	$ 15.53	$ 42.57
Value of the firm	$100.00	$150.00	$ 250.00

The combined value of the equity prior to the merger is $ 210.41 million; it declines to $207.43 million after that. The wealth of the bondholders increases by an equal amount. There is a transfer of wealth from stockholders to bondholders, as a consequence of the merger. Thus, conglomerate mergers that are not followed by increases in leverage are likely to cause this type of redistribution of wealth across claimholders in the firm.

Security Design and Valuation

An understanding of how options work and how to value them is particularly valuable for firms attempting to design securities that include option features. In designing financing, firms should try to match the cash flows on their financing as closely as possible with the cash flows generated by the assets. By doing so, they reduce the likelihood of default risk and increase debt capacity. Combining options with straight bonds can sometimes allow a firm to accomplish this matching, as in the following cases:

- *Convertible bonds*: A convertible bond is a combination of a conversion option and a straight bond. Convertible bonds allow firms with high growth potential, high volatility in earnings and cash flows, and low cash flows currently to borrow without exposing themselves to significant default risk.
- *Commodity bonds*: A commodity bond, which is a bond where the coupon rate is tied to commodity prices, is a combination of an option on a commodity (such as gold or oil) and a straight bond. Commodity firms whose earnings tend to move with commodity prices can gain by using these bonds

· *Catastrophe bonds*: A catastrophe bond allows for the suspension of coupon payments and/or the reduction of principal day, in the event of a specified catastrophe. For insurance companies, which are often exposed to large liabilities in the event of a catastrophe (such as an earthquake or a hurricane), they provide a relatively default risk-free approach to borrowing.

Value of Financial Flexibility

When making financial decisions, managers consider the effects such decisions will have on their capacity to take new projects or meet unanticipated contingencies in future periods. Practically, this translates into firms maintaining excess debt capacity or larger cash balances than are warranted by current needs, to meet unexpected future requirements. While maintaining this financing flexibility has value to firms, it also has a cost; the large cash balances might earn below market returns, and excess debt capacity implies that the firm is giving up some value and has a higher cost of capital.

One reason that a firm maintains large cash balances and excess debt capacity in order to have the option to take unexpected projects that promise high returns, in the future. To value financial flexibility as an option, consider the following framework. A firm has expectations about how much it will need to reinvest in future periods, based upon its own past history and current conditions in the industry. On the other side of the ledger, a firm also has expectations about how much it can raise from internal funds and its normal access to capital markets in future periods. Assume that there is volatility in the expectation about future reinvestment needs; for simplicity, we will assume that the capacity to generate funds is known to the firm. The advantage (and value) of having excess debt capacity or large cash balances is that the firm can meet any reinvestment needs in excess of funds available using its debt capacity. The payoff from these projects, however, comes from the excess returns that the firm expects to make on them. To value financial flexibility on an annualized basis, therefore, we will use the following measures:

Input to Model	*Measure*	*Estimation Approach*
S	Expected Annual Reinvestment Needs as % Firm Value	Use historical average of (Net Cap Ex + Change in Non-cash Working Capital)/ Market Value of Firm
K	Annual Reinvestment Needs as percent of firm value that can be raised without financing flexibility	If firm does not want to or cannot use external financing: (Net Income - Dividend + Depreciation)/Market Value of Firm If firm uses external capital (bank debt, bonds or equity) regularly: (Net Income + Depreciation + Net External Financing)/Market Value of Firm
σ^s	Variance in reinvestment needs	Variance in the expected reinvestment as percent of firm value (using historical data)
t	1 year	To get an annual estimate of the value of flexibility

We estimated these inputs for the Home Depot, starting with the reinvestments as a percent of firm value. The following table summarizes these numbers from 1989-1998:

Year	*Reinvestment Needs*	*Firm Value*	*Reinvestment Needs as percent of Firm Value*	*ln(Reinvestment Needs)*
1989	$175	$2,758	6.35%	-2.7563329

1990	$374	$3,815	9.80%	-2.3224401
1991	$427	$5,137	8.31%	-2.4874405
1992	$456	$7,148	6.38%	-2.7520951
1993	$927	$9,239	10.03%	-2.2992354
1994	$1,176	$12,477	9.43%	-2.3617681
1995	$1,344	$15,470	8.69%	-2.4432524
1996	$1,086	$19,535	5.56%	-2.8897065
1997	$1,589	$24,156	6.58%	-2.7214279
1998	$1,817	$30,219	6.01%	-2.8112841

Average Reinvestment needs as % of Firm Value = 7.71%

Standard Deviation in ln(Reinvestment Needs) = 22.36%

We followed up by estimating internal funds as a percent of firm value, using the sum of net income and depreciation as a measure of internal funds:

Year	*Net Income*	*Depreciation*	*Firm Value*	*Internal Funds/Value*
1989	$112	$21	$2,758	4.82%
1990	$163	$34	$3,815	5.16%
1991	$249	$52	$5,137	5.86%
1992	$363	$70	$7,148	6.06%
1993	$457	$90	$9,239	5.92%
1994	$605	$130	$12,477	5.89%
1995	$732	$181	$15,470	5.90%
1996	$938	$232	$19,535	5.99%
1997	$1,160	$283	$24,156	5.97%
1998	$1,614	$373	$30,219	6.58%

Internal funds, on average, were 5.82% of firm value between 1989 and 1998. Since the firm uses almost no external debt and has only one small bond issue outstanding, the firm made up the difference between its reinvestment needs (7.71%) and internal fund generation (5.82%) by issuing equity. We will assume, looking forward, that the Home Depot no longer wants to issue new equity.

The Home Depot's current debt ratio is 4.55%, and its current cost of capital is 9.51%. Its optimal debt ratio is 20%, and its cost of capital at that debt level is 9.17%. Finally, the Home Depot in 1998, earned a return on capital of 16.37%, and we will assume that this is the expected return on new projects, as well.

S = Expected Reinvestment Needs as percent of Firm Value = 7.71%

K = Reinvestment needs that can be financed without flexibility = 5.82%

t = 1 year

σ^2 = Variance in ln(Net Capital Expenditures) = $(.2236)^2$ = .05

With a riskfree rate of 6%, the option value that we estimate using these inputs is .02277. We then converted this option value into a measure of value over time, by multiplying the value by the annual excess return, and then assuming that the firm foregoes this excess returns forever[17]:

Value of Flexibility = .02277 (Return on Capital – Cost of Capital)/ Cost of Capital

= .02277 (.1637 - .0951)/.0951 = 1.6425%

On an annual basis, the flexibility generated by the excess debt capacity is worth 1.6425% of firm value at the Home Depot, which is well in excess of the savings (9.51% - 9.17% = 0.34%) in the cost of capital that would be accomplished, if it used up the excess debt capacity.

The one final consideration here is that this estimate does not consider the fact that the Home Depot does not have unlimited financial flexibility. In fact, assume that excess

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[17] We are assuming that the project that a firm is unable to take because it lacks financial flexibility is lost forever, and that the excess returns on this project would also have lasted forever. Both assumptions are strong, and may result in the overstatement of the lost value.

debt capacity of the Home Depot (which is 15.45%, the difference between the optimal debt ratio and the current debt ratio) is the upside limit on financial flexibility. We can value the effect of this limit, by valuing a call with the same parameters as the call described above, but with a strike price of 21.27% (15.45% + 5.82%). In this case, the effect on the value of flexibility is negligible of imposing this constraint.

Implications

Looking at financial flexibility as an option yields valuable insights on when financial flexibility is most valuable. Using the framework developed above, for instance, we would argue that:

4. Other things remaining equal, firms operating in businesses where projects earn substantially higher returns than their hurdle rates should value flexibility more than those that operate in stable businesses where excess returns are small. This would imply that firms such as Microsoft and Dell, that earn large excess returns on their projects, can use the need for financial flexibility as the justification for holding large cash balances and excess debt capacity.
8. Since a firm's ability to fund these reinvestment needs is determined by its capacity to generate internal funds, other things remaining equal, financial flexibility should be worth less to firms with large and stable earnings, as a percent of firm value. Firms that have small or negative earnings, and therefore much lower capacity to generate internal funds, will value flexibility more.
9. Firms with limited internal funds can still get away with little or no financial flexibility if they can tap external markets for capital – bank debt, bonds and new equity issues. Other things remaining equal, the greater the capacity (and willingness) of a firm to raise funds from external capital markets, the less should be the value of flexibility. This may explain why private or small firms, which have far less access to capital, will value financial flexibility more than larger firms. The existence of corporate bond markets can also make a difference in how much flexibility is valued. In markets where

firms cannot issue bonds and have to depend entirely upon banks for financing, there is less access to capital and a greater need to maintain financial flexibility. In the Home Depot example above, a willingness to tap external funds – debt or equity – would reduce the value of flexibility substantially.

10. The need for and the value of flexibility is a function of how uncertain a firm is about future reinvestment needs. Firms with predictable reinvestment needs should value flexibility less than firms in sectors where reinvestment needs are volatile on a period-to-period basis.

Conclusion

Option pricing theory has wide applicability in corporate finance and we have explored a wide range of these applications in this chapter. We began the chapter with a discussion of some of the measurement issues that make the pricing of real options more difficult than the pricing of options on financial assets. We then considered three options embedded in investment projects - the option to expand a project, the option to abandon a project and product patents as options. In all of these cases, the underlying asset was the project and the options added value to the project. We then posed the argument that equity could be viewed as a call option on the firm, and that this would suggest that equity would have value even when the firm value was less than the outstanding claims on it. Furthermore, viewing equity as an option allows us to consider the conflict between stockholders and bondholders much more clearly and provides us with insights on why conglomerates may make stockholders worse off, while making bondholders better off.