Option Pricing Theory and Applications

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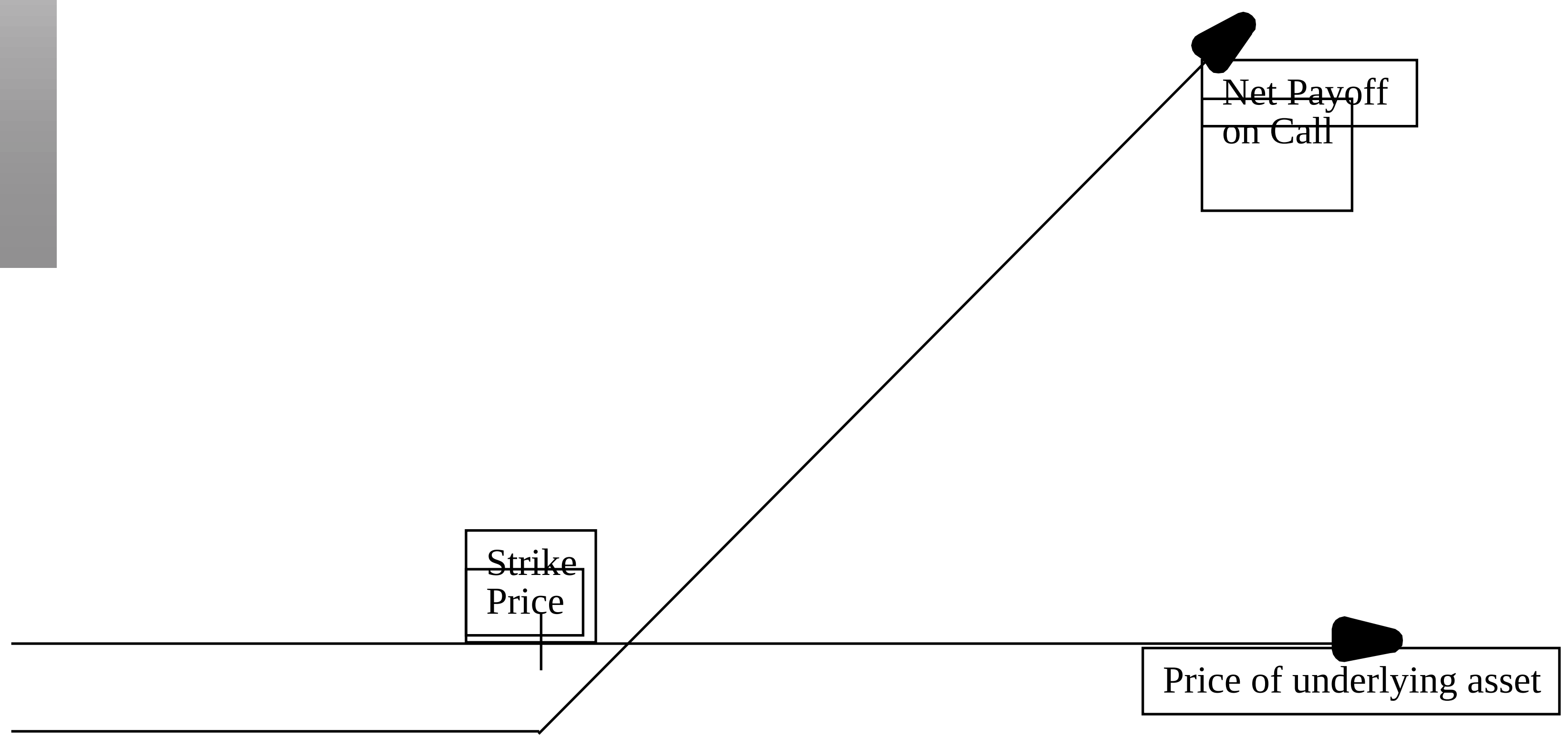
What is an option?

- An option provides the holder with the **right** to buy or sell a specified quantity of an underlying asset at a fixed price (called a strike price or an exercise price) at or before the expiration date of the option.
- Since it is a right and **not an obligation**, the holder can choose not to exercise the right and allow the option to expire.
- There are two types of options - **call** options (right to buy) and **put** options (right to sell).

Call Options

- A call option gives the buyer of the option the right to buy the underlying asset at a fixed price (strike price or K) at any time prior to the expiration date of the option. The buyer pays a price for this right.
- At expiration,
  - If the value of the underlying asset (S) > Strike Price(K)
    - Buyer makes the difference: S - K
  - If the value of the underlying asset (S) < Strike Price (K)
    - Buyer does not exercise
- More generally,
  - the value of a call increases as the value of the underlying asset increases
  - the value of a call decreases as the value of the underlying asset decreases

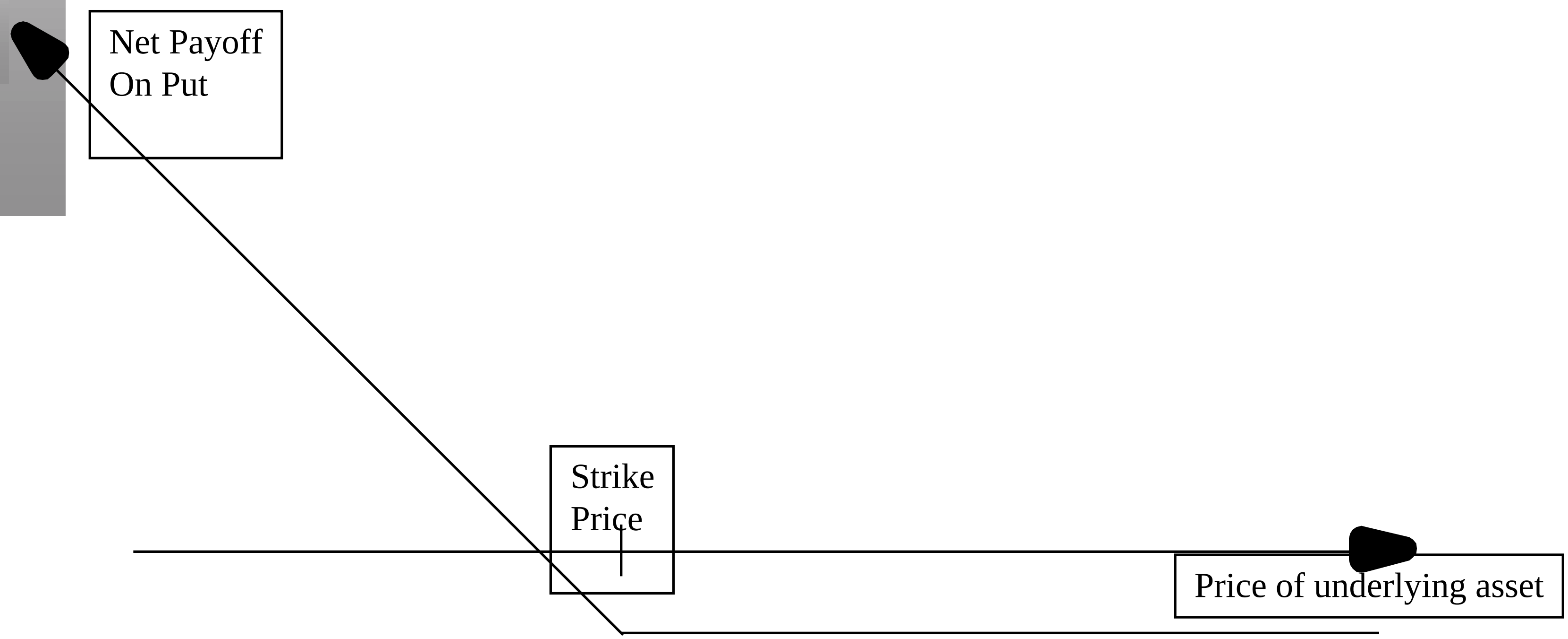
Payoff Diagram on a Call



Put Options

- A put option gives the buyer of the option the right to sell the underlying asset at a fixed price at any time prior to the expiration date of the option. The buyer pays a price for this right.
- At expiration,
  - If the value of the underlying asset (S) < Strike Price(K)
    - Buyer makes the difference: K-S
  - If the value of the underlying asset (S) > Strike Price (K)
    - Buyer does not exercise
- More generally,
  - the value of a put decreases as the value of the underlying asset increases
  - the value of a put increases as the value of the underlying asset decreases

Payoff Diagram on Put Option



Determinants of option value

- Variables Relating to Underlying Asset
  - Value of Underlying Asset; as this value increases, the right to buy at a fixed price (calls) will become more valuable and the right to sell at a fixed price (puts) will become less valuable.
  - Variance in that value; as the variance increases, both calls and puts will become more valuable because all options have limited downside and depend upon price volatility for upside.
  - Expected dividends on the asset, which are likely to reduce the price appreciation component of the asset, reducing the value of calls and increasing the value of puts.
- Variables Relating to Option
  - Strike Price of Options; the right to buy (sell) at a fixed price becomes more (less) valuable at a lower price.
  - Life of the Option; both calls and puts benefit from a longer life.
- Level of Interest Rates; as rates increase, the right to buy (sell) at a fixed price in the future becomes more (less) valuable.

American versus European options: Variables relating to early exercise

- An American option **can be exercised at any time prior** to its expiration, while a European option can be exercised only at expiration.
  - The possibility of early exercise makes **American options more valuable** than otherwise similar European options.
  - However, in most cases, the **time premium** associated with the remaining life of an option makes **early exercise sub-optimal**.
- While early exercise is generally not optimal, there are two exceptions:
  - One is where the underlying asset **pays large dividends**, thus reducing the value of the asset, and of call options on it. In these cases, call options may be exercised just before an ex-dividend date, if the time premium on the options is less than the expected decline in asset value.
  - The other is when an investor holds both the underlying asset and deep in-the-money puts on that asset, at a time when **interest rates are high**. The time premium on the put may be less than the potential gain from exercising the put early and earning interest on the exercise price.

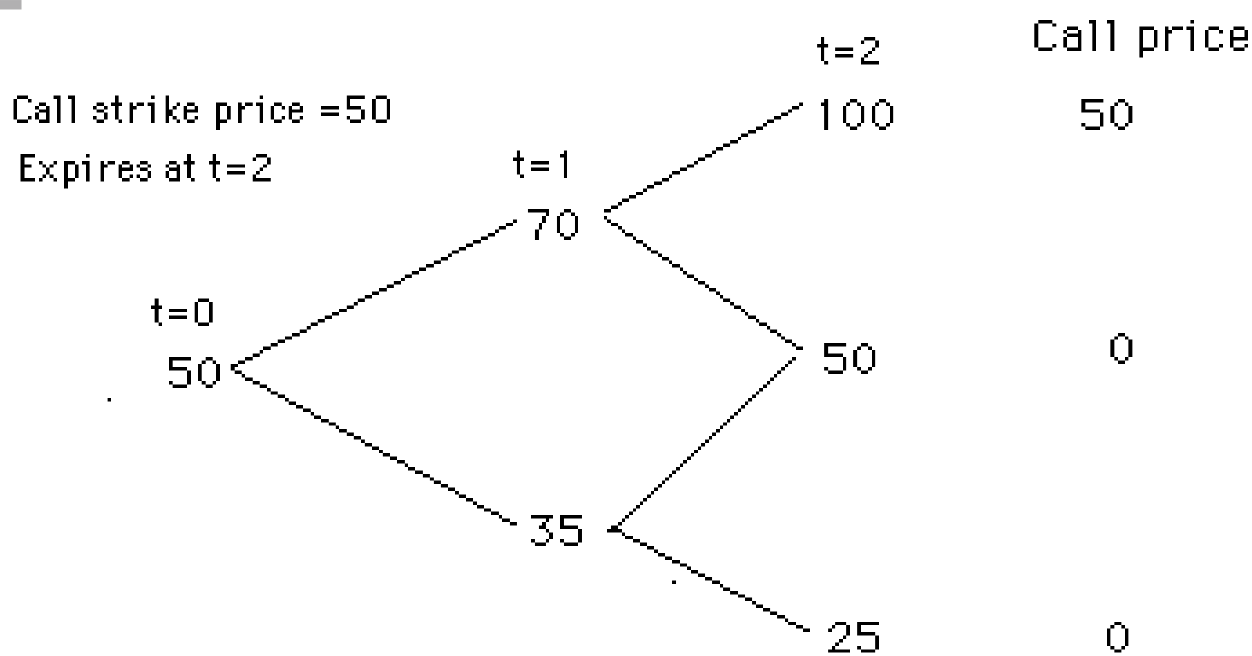
A Summary of the Determinants of Option Value

Factor	*Call Value*	*Put Value*
Increase in Stock Price	Increases	Decreases
Increase in Strike Price	Decreases	Increases
Increase in variance of underlying asset	Increases	Increases
Increase in time to expiration	Increases	Increases
Increase in interest rates	Increases	Decreases
Increase in dividends paid	Decreases	Increases

Creating a replicating portfolio

- The objective in creating a replicating portfolio is to use a combination of riskfree borrowing/lending and the underlying asset to create the same cashflows as the option being valued.
  - Call = Borrowing + Buying Δ of the Underlying Stock
  - Put = Selling Short Δ on Underlying Asset + Lending
  - The number of shares bought or sold is called the **option delta**.
- The principles of arbitrage then apply, and the value of the option has to be equal to the value of the replicating portfolio.

The Binomial Option Pricing Model



The Limiting Distributions….

- As the time interval is shortened, the limiting distribution, as t -> 0, can take one of two forms.
  - If as t -> 0, **price changes become smaller**, the limiting distribution is the normal distribution and the **price process is a continuous one**.
  - If as t->0, **price changes remain large**, the limiting distribution is the poisson distribution, i.e., a **distribution that allows for price jumps**.
- **The Black-Scholes model** applies when the **limiting distribution is the normal distribution** , and explicitly assumes that the price process is continuous and that there are no jumps in asset prices.

The Black-Scholes Model

- The version of the model presented by Black and Scholes was designed to value European options, which were dividend-protected.
- The value of a call option in the Black-Scholes model can be written as a function of the following variables:

S = Current value of the underlying asset

K = Strike price of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the ln(value) of the underlying asset

The Black Scholes Model

Value of call = $S\,N(d_1) - K\,e^{-rt}\,N(d_2)$

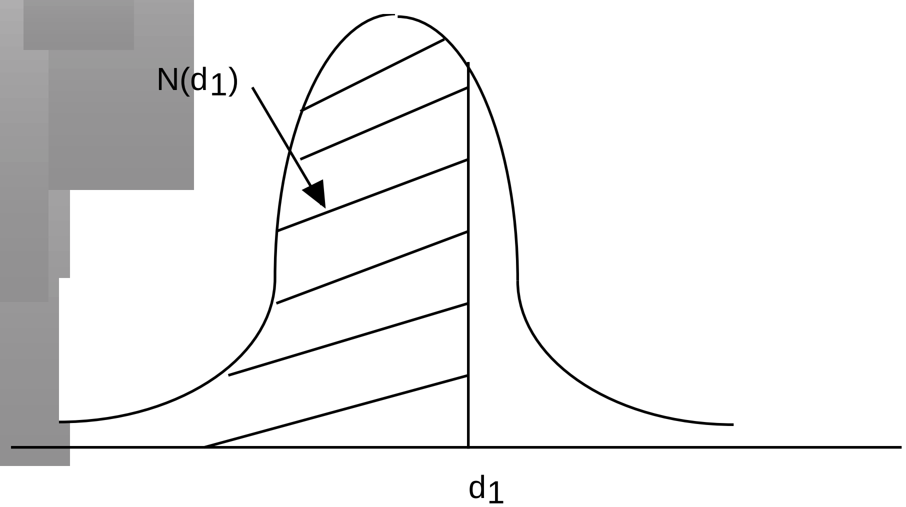
where,

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

- $d_2 = d_1 - \sigma\sqrt{t}$

- The replicating portfolio is embedded in the Black-Scholes model. To replicate this call, you would need to
  - Buy N(d1) shares of stock; N(d1) is called the option delta
  - Borrow $K\,e^{-rt}\,N(d_2)$

The Normal Distribution



d	N(d)	d	N(d)	d	N(d)
-3.00	0.0013	-1.00	0.1587	1.05	0.8531
-2.95	0.0016	-0.95	0.1711	1.10	0.8643
-2.90	0.0019	-0.90	0.1841	1.15	0.8749
-2.85	0.0022	-0.85	0.1977	1.20	0.8849
-2.80	0.0026	-0.80	0.2119	1.25	0.8944
-2.75	0.0030	-0.75	0.2266	1.30	0.9032
-2.70	0.0035	-0.70	0.2420	1.35	0.9115
-2.65	0.0040	-0.65	0.2578	1.40	0.9192
-2.60	0.0047	-0.60	0.2743	1.45	0.9265
-2.55	0.0054	-0.55	0.2912	1.50	0.9332
-2.50	0.0062	-0.50	0.3085	1.55	0.9394
-2.45	0.0071	-0.45	0.3264	1.60	0.9452
-2.40	0.0082	-0.40	0.3446	1.65	0.9505
-2.35	0.0094	-0.35	0.3632	1.70	0.9554
-2.30	0.0107	-0.30	0.3821	1.75	0.9599
-2.25	0.0122	-0.25	0.4013	1.80	0.9641
-2.20	0.0139	-0.20	0.4207	1.85	0.9678
-2.15	0.0158	-0.15	0.4404	1.90	0.9713
-2.10	0.0179	-0.10	0.4602	1.95	0.9744
-2.05	0.0202	-0.05	0.4801	2.00	0.9772
-2.00	0.0228	0.00	0.5000	2.05	0.9798
-1.95	0.0256	0.05	0.5199	2.10	0.9821
-1.90	0.0287	0.10	0.5398	2.15	0.9842
-1.85	0.0322	0.15	0.5596	2.20	0.9861
-1.80	0.0359	0.20	0.5793	2.25	0.9878
-1.75	0.0401	0.25	0.5987	2.30	0.9893
-1.70	0.0446	0.30	0.6179	2.35	0.9906
-1.65	0.0495	0.35	0.6368	2.40	0.9918
-1.60	0.0548	0.40	0.6554	2.45	0.9929
-1.55	0.0606	0.45	0.6736	2.50	0.9938
-1.50	0.0668	0.50	0.6915	2.55	0.9946
-1.45	0.0735	0.55	0.7088	2.60	0.9953
-1.40	0.0808	0.60	0.7257	2.65	0.9960
-1.35	0.0885	0.65	0.7422	2.70	0.9965
-1.30	0.0968	0.70	0.7580	2.75	0.9970
-1.25	0.1056	0.75	0.7734	2.80	0.9974
-1.20	0.1151	0.80	0.7881	2.85	0.9978
-1.15	0.1251	0.85	0.8023	2.90	0.9981
-1.10	0.1357	0.90	0.8159	2.95	0.9984
-1.05	0.1469	0.95	0.8289	3.00	0.9987
-1.00	0.1587	1.00	0.8413		

Adjusting for Dividends

- If the dividend yield (y = dividends/ Current value of the asset) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S\, e^{-yt}\, N(d_1) - K\, e^{-rt}\, N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The value of a put can also be derived:

$$P = K\, e^{-rt}\, (1-N(d_2)) - S\, e^{-yt}\, (1-N(d_1))$$

Problems with Real Option Pricing Models

1. The underlying asset may not be traded, which makes it difficult to estimate value and variance for the underlying asset.
2. The price of the asset may not follow a continuous process, which makes it difficult to apply option pricing models (like the Black Scholes) that use this assumption.
3. The variance may not be known and may change over the life of the option, which can make the option valuation more complex.
4. Exercise may not be instantaneous, which will affect the value of the option.
5. Some real options are complex and their exercise creates other options (compound) or involve learning (learning options)

Option Pricing Applications in Investment/Strategic Analysis

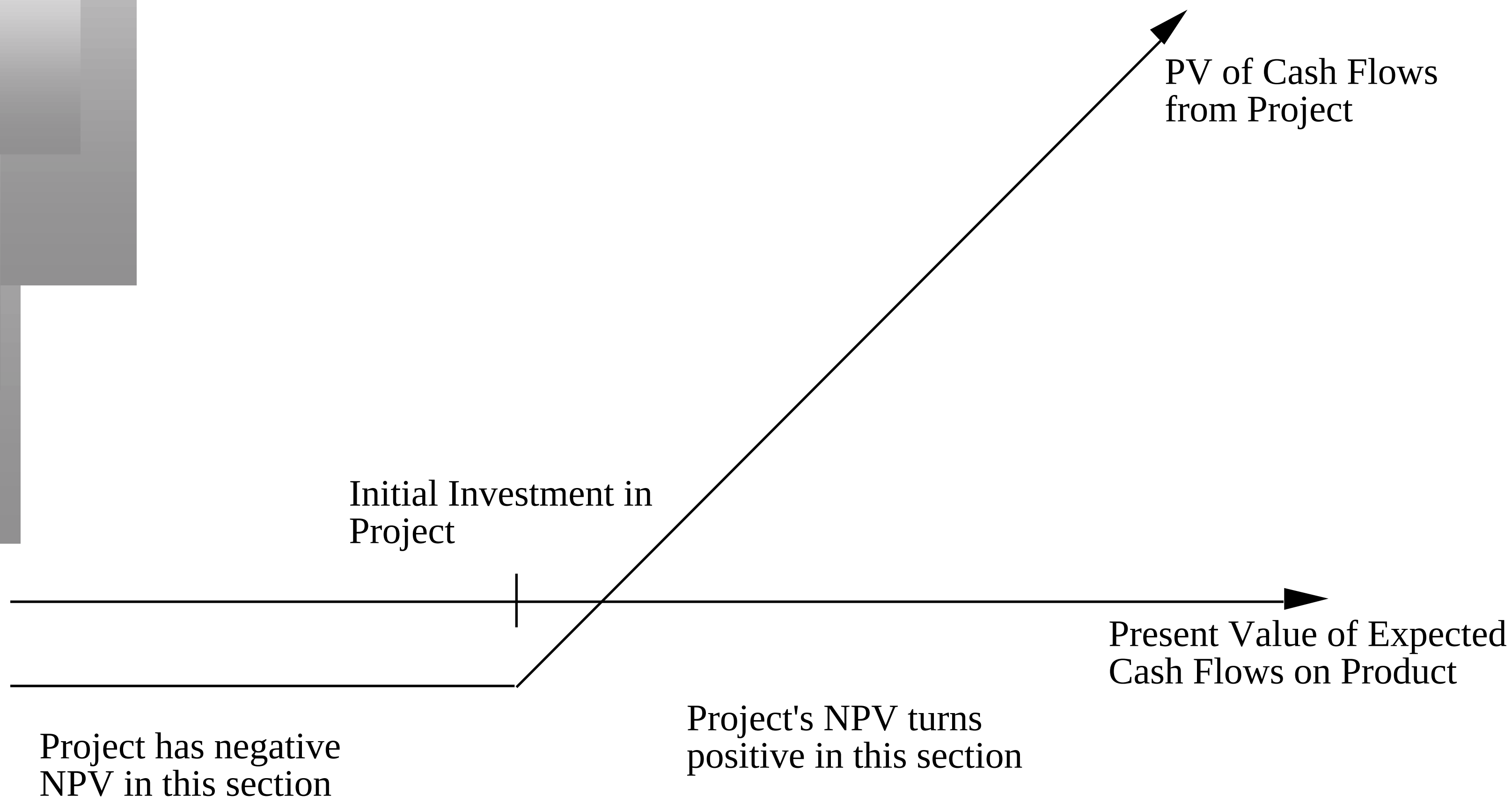
Options in Projects/Investments/Acquisitions

- One of the limitations of traditional investment analysis is that it is static and does not do a good job of capturing the options embedded in investment.
  - The first of these options is the option to delay taking a investment, when a firm has exclusive rights to it, until a later date.
  - The second of these options is taking one investment may allow us to take advantage of other opportunities (investments) in the future
  - The last option that is embedded in projects is the option to abandon a investment, if the cash flows do not measure up.
- These options all add value to projects and may make a “bad” investment (from traditional analysis) into a good one.

The Option to Delay

- When a firm has exclusive rights to a project or product for a specific period, it can delay taking this project or product until a later date.
- A traditional investment analysis just answers the question of whether the project is a “good” one if taken today.
- Thus, the fact that a project does not pass muster today (because its NPV is negative, or its IRR is less than its hurdle rate) does not mean that the rights to this project are not valuable.

Valuing the Option to Delay a Project



Insights for Investment Analyses

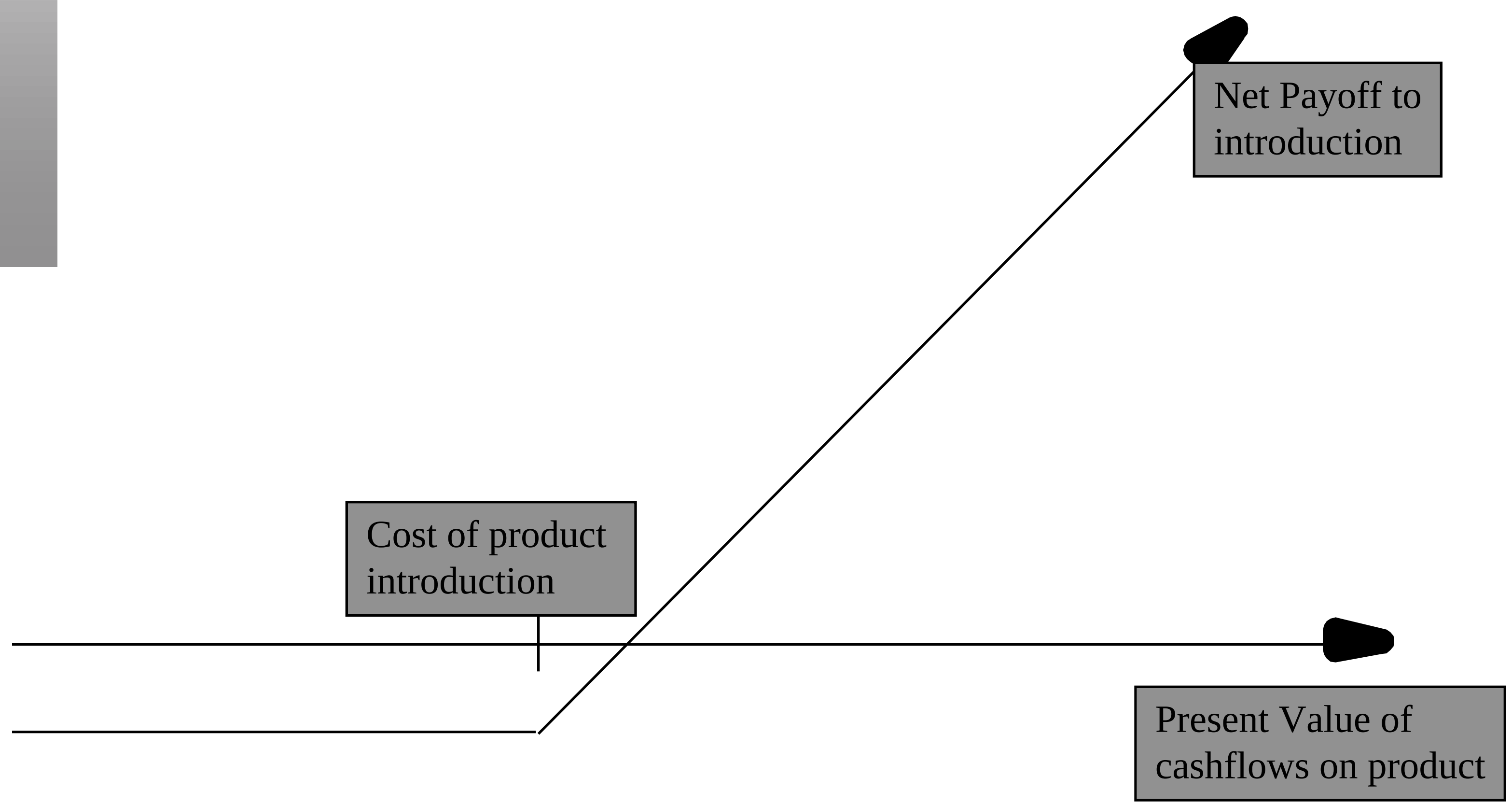
- Having the exclusive rights to a product or project is valuable, even if the product or project is not viable today.
- The value of these rights increases with the volatility of the underlying business.
- The cost of acquiring these rights (by buying them or spending money on development, for instance) has to be weighed off against these benefits.

Example 1: Valuing product patents as options

- A product patent provides the firm with the right to develop the product and market it.
- It will do so only if the present value of the expected cash flows from the product sales exceed the cost of development.
- If this does not occur, the firm can shelve the patent and not incur any further costs.
- If I is the present value of the costs of developing the product, and V is the present value of the expected cashflows from development, the payoffs from owning a product patent can be written as:

Payoff from owning a product patent	$= V - I$	if $V > I$
	$= 0$	if $V \le I$

Payoff on Product Option



Obtaining Inputs for Patent Valuation

Input	Estimation Process
1. Value of the Underlying Asset	· Present Value of Cash Inflows from taking project now · This will be noisy, but that adds value.
2. Variance in value of underlying asset	· Variance in cash flows of similar assets or firms · Variance in present value from capital budgeting simulation.
3. Exercise Price on Option	· Option is exercised when investment is made. · Cost of making investment on the project ; assumed to be constant in present value dollars.
4. Expiration of the Option	· Life of the patent
5. Dividend Yield	· Cost of delay · Each year of delay translates into one less year of value-creating cashflows $\text{Annual cost of delay} = \frac{1}{n}$

Valuing a Product Patent: Avonex

- Biogen, a bio-technology firm, has a patent on Avonex, a drug to treat multiple sclerosis, for the next 17 years, and it plans to produce and sell the drug by itself. The key inputs on the drug are as follows:

  PV of Cash Flows from Introducing the Drug Now = S = \$ 3.422 billion

  PV of Cost of Developing Drug for Commercial Use = K = \$ 2.875 billion

  Patent Life = t = 17 years   Riskless Rate = r = 6.7% (17-year T.Bond rate)

  Variance in Expected Present Values = σ^2 = 0.224 (Industry average firm variance for bio-tech firms)

  Expected Cost of Delay = y = 1/17 = 5.89%

  d1 = 1.1362   N(d1) = 0.8720

  d2 = -0.8512   N(d2) = 0.2076

Call Value= 3,422 $\exp^{(-0.0589)(17)}$ (0.8720) - 2,875 ($\exp^{(-0.067)(17)}$ (0.2076)= \$ 907 million

Valuing a firm with patents

- The value of a firm with a substantial number of patents can be derived using the option pricing model.

Value of Firm = Value of commercial products (using DCF value

+ Value of existing patents (using option pricing)

+ (Value of New patents that will be obtained in the future – Cost of obtaining these patents)

- The last input measures the efficiency of the firm in converting its R&D into commercial products. If we assume that a firm earns its cost of capital from research, this term will become zero.
- If we use this approach, we should be careful not to double count and allow for a high growth rate in cash flows (in the DCF valuation).

Value of Biogen's existing products

- Biogen had two commercial products (a drug to treat Hepatitis B and Intron) at the time of this valuation that it had licensed to other pharmaceutical firms.
- The license fees on these products were expected to generate $ 50 million in after-tax cash flows each year for the next 12 years. To value these cash flows, which were guaranteed contractually, the riskless rate of 6.7% was used:

Present Value of License Fees = $ 50 million $(1 – (1.067)^{-12})/.067$

= $ 403.56 million

Value of Biogen's Future R&D

- Biogen continued to fund research into new products, spending about $ 100 million on R&D in the most recent year. These R&D expenses were expected to grow 20% a year for the next 10 years, and 5% thereafter.
- It was assumed that every dollar invested in research would create $ 1.25 in value in patents (valued using the option pricing model described above) for the next 10 years, and break even after that (i.e., generate $ 1 in patent value for every $ 1 invested in R&D).
- There was a significant amount of risk associated with this component and the cost of capital was estimated to be 15%.

Value of Future R&D

Yr	*Value of Patents*		*R&D Cost*		*Excess Value*		*Present Value (at 15%)*	
1	$	150.00	$	120.00	$	30.00	$	26.09
2	$	180.00	$	144.00	$	36.00	$	27.22
3	$	216.00	$	172.80	$	43.20	$	28.40
4	$	259.20	$	207.36	$	51.84	$	29.64
5	$	311.04	$	248.83	$	62.21	$	30.93
6	$	373.25	$	298.60	$	74.65	$	32.27
7	$	447.90	$	358.32	$	89.58	$	33.68
8	$	537.48	$	429.98	$	107.50	$	35.14
9	$	644.97	$	515.98	$	128.99	$	36.67
10	$	773.97	$	619.17	$	154.79	$	38.26
							$	318.30

Value of Biogen

- The value of Biogen as a firm is the sum of all three components – the present value of cash flows from existing products, the value of Avonex (as an option) and the value created by new research:

Value = Existing products + Existing Patents + Value: Future R&D

= \$ 403.56 million + \$ 907 million + \$ 318.30 million

= \$1628.86 million

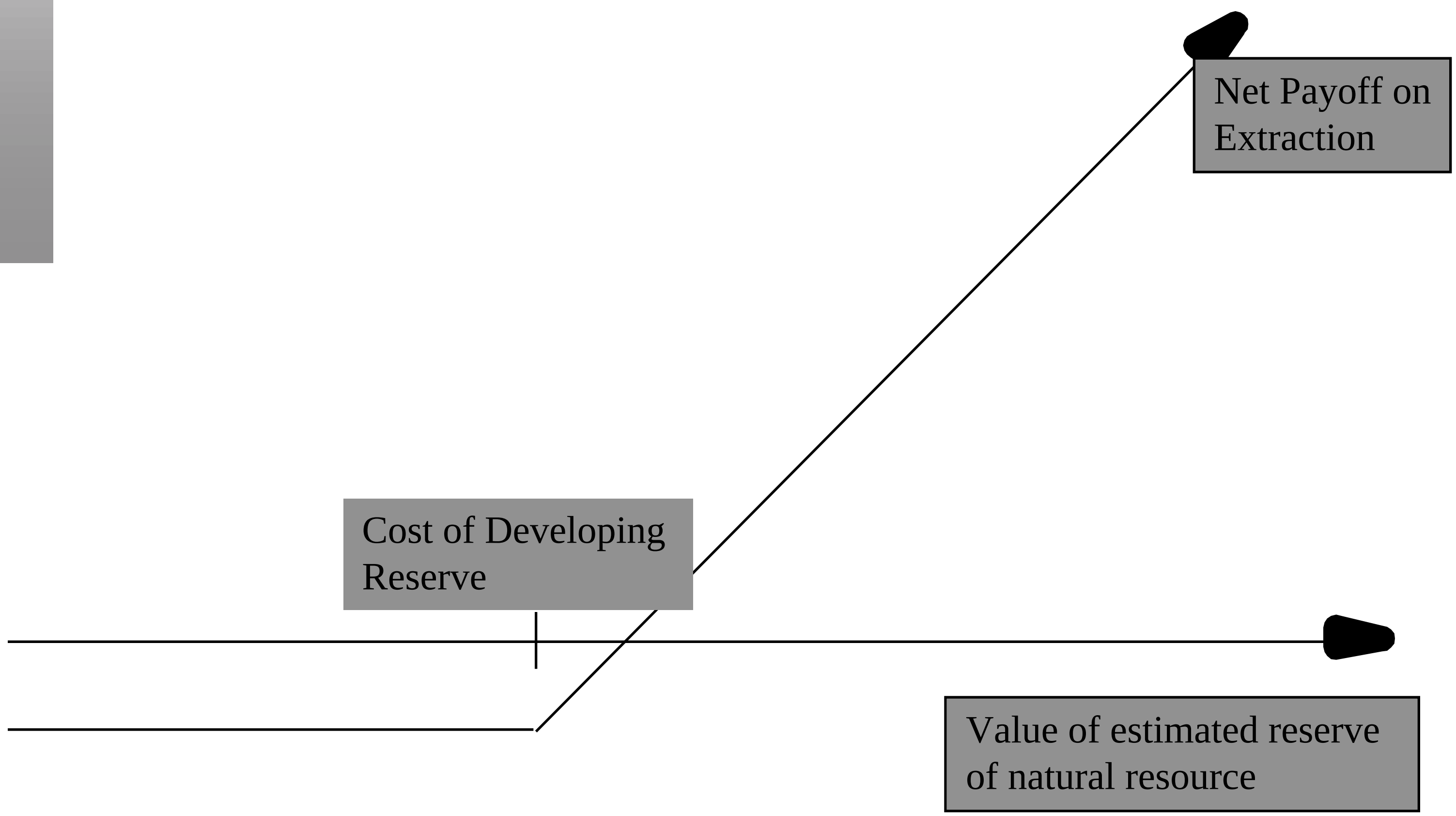
- Since Biogen had no debt outstanding, this value was divided by the number of shares outstanding (35.50 million) to arrive at a value per share:

Value per share = \$ 1,628.86 million / 35.5 = \$ 45.88

Example 2: Valuing Natural Resource Options

- In a natural resource investment, the underlying asset is the resource and the value of the asset is based upon two variables - the quantity of the resource that is available in the investment and the price of the resource.
- In most such investments, there is a cost associated with developing the resource, and the difference between the value of the asset extracted and the cost of the development is the profit to the owner of the resource.
- Defining the cost of development as X, and the estimated value of the resource as V, the potential payoffs on a natural resource option can be written as follows:
  - Payoff on natural resource investment $= V - X$ if $V > X$
  - $= 0$ if $V \leq X$

Payoff Diagram on Natural Resource Firms



Estimating Inputs for Natural Resource Options

Input	Estimation Process
1. Value of Available Reserves of the Resource	· Expert estimates (Geologists for oil..); The present value of the after-tax cash flows from the resource are then estimated.
2. Cost of Developing Reserve (Strike Price)	· Past costs and the specifics of the investment
3. Time to Expiration	· Relinqushment Period: if asset has to be relinquished at a point in time. · Time to exhaust inventory - based upon inventory and capacity output.
4. Variance in value of underlying asset	· based upon variability of the price of the resources and variability of available reserves.
5. Net Production Revenue (Dividend Yield)	· Net production revenue every year as percent of market value.
6. Development Lag	· Calculate present value of reserve based upon the lag.

Valuing an Oil Reserve

- Consider an offshore oil property with an estimated oil reserve of 50 million barrels of oil, where the present value of the development cost is $12 per barrel and the development lag is two years.
- The firm has the rights to exploit this reserve for the next twenty years and the marginal value per barrel of oil is $12 per barrel currently (Price per barrel - marginal cost per barrel).
- Once developed, the net production revenue each year will be 5% of the value of the reserves.
- The riskless rate is 8% and the variance in ln(oil prices) is 0.03.

Inputs to Option Pricing Model

- Current Value of the asset = S = Value of the developed reserve discounted back the length of the development lag at the dividend yield = $\$12 * 50 /(1.05)^2 = \$ 544.22$
- (If development is started today, the oil will not be available for sale until two years from now. The estimated opportunity cost of this delay is the lost production revenue over the delay period. Hence, the discounting of the reserve back at the dividend yield)
- Exercise Price = Present Value of development cost = $12 * 50 = $600 million
- Time to expiration on the option = 20 years
- Variance in the value of the underlying asset = 0.03
- Riskless rate =8%
- Dividend Yield = Net production revenue / Value of reserve = 5%

Valuing the Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = 1.0359   N(d1) = 0.8498

  d2 = 0.2613   N(d2) = 0.6030
- Call Value= 544 .22 $\exp^{(-0.05)(20)}$ (0.8498) -600 ($\exp^{(-0.08)(20)}$ (0.6030)= $ 97.08 million
- This oil reserve, though not viable at current prices, still is a valuable property because of its potential to create value if oil prices go up.

Extending the option pricing approach to value natural resource firms

- Since the assets owned by a natural resource firm can be viewed primarily as options, **the firm itself can be valued using option pricing** models.
- The preferred approach would be to **consider each option separately**, value it and cumulate the values of the options to get the firm value.
- Since this information is likely to be **difficult to obtain** for large natural resource firms, such as oil companies, which own hundreds of such assets, a variant is to value the entire firm as one option.
- A purist would probably disagree, arguing that **valuing an option on a portfolio of assets (as in this approach) will provide a lower value than valuing a portfolio of options** (which is what the natural resource firm really own). Nevertheless, the value obtained from the model still provides an interesting perspective on the determinants of the value of natural resource firms.

Inputs to the Model

Input to model	*Corresponding input for valuing firm*
Value of underlying asset	Value of cumulated estimated reserves of the resource owned by the firm, discounted back at the dividend yield for the development lag.
Exercise Price	Estimated cumulated cost of developing estimated reserves
Time to expiration on option	Average relinquishment period across all reserves owned by firm (if known) or estimate of when reserves will be exhausted, given current production rates.
Riskless rate	Riskless rate corresponding to life of the option
Variance in value of asset	Variance in the price of the natural resource
Dividend yield	Estimated annual net production revenue as percentage of value of the reserve.

Valuing Gulf Oil

- Gulf Oil was the target of a takeover in early 1984 at $70 per share (It had 165.30 million shares outstanding, and total debt of $9.9 billion).
- It had estimated reserves of 3038 million barrels of oil and the average cost of developing these reserves was estimated to be $10 a barrel in present value dollars (The development lag is approximately two years).
- The average relinquishment life of the reserves is 12 years.
- The price of oil was $22.38 per barrel, and the production cost, taxes and royalties were estimated at $7 per barrel.
- The bond rate at the time of the analysis was 9.00%.
- Gulf was expected to have net production revenues each year of approximately 5% of the value of the developed reserves. The variance in oil prices is 0.03.

Valuing Undeveloped Reserves

- Value of underlying asset = Value of estimated reserves discounted back for period of development lag= 3038 * ($ 22.38 **-** $7) / 1.05^2 = **$42,380.44**
- Exercise price = Estimated development cost of reserves = 3038 * $10 = **$30,380 million**
- Time to expiration = Average length of relinquishment option = **12 years**
- Variance in value of asset = Variance in oil prices = **0.03**
- Riskless interest rate = **9%**
- Dividend yield = Net production revenue/ Value of developed reserves = **5%**

■ Based upon these inputs, the Black-Scholes model provides the following value for the call:

d1 = 1.6548 N(d1) = 0.9510

d2 = 1.0548 N(d2) = 0.8542

■ Call Value= 42,380.44 $\exp^{(-0.05)(12)}$ (0.9510) -30,380 ($\exp^{(-0.09)(12)}$ (0.8542)= **$ 13,306 million**

Valuing Gulf Oil

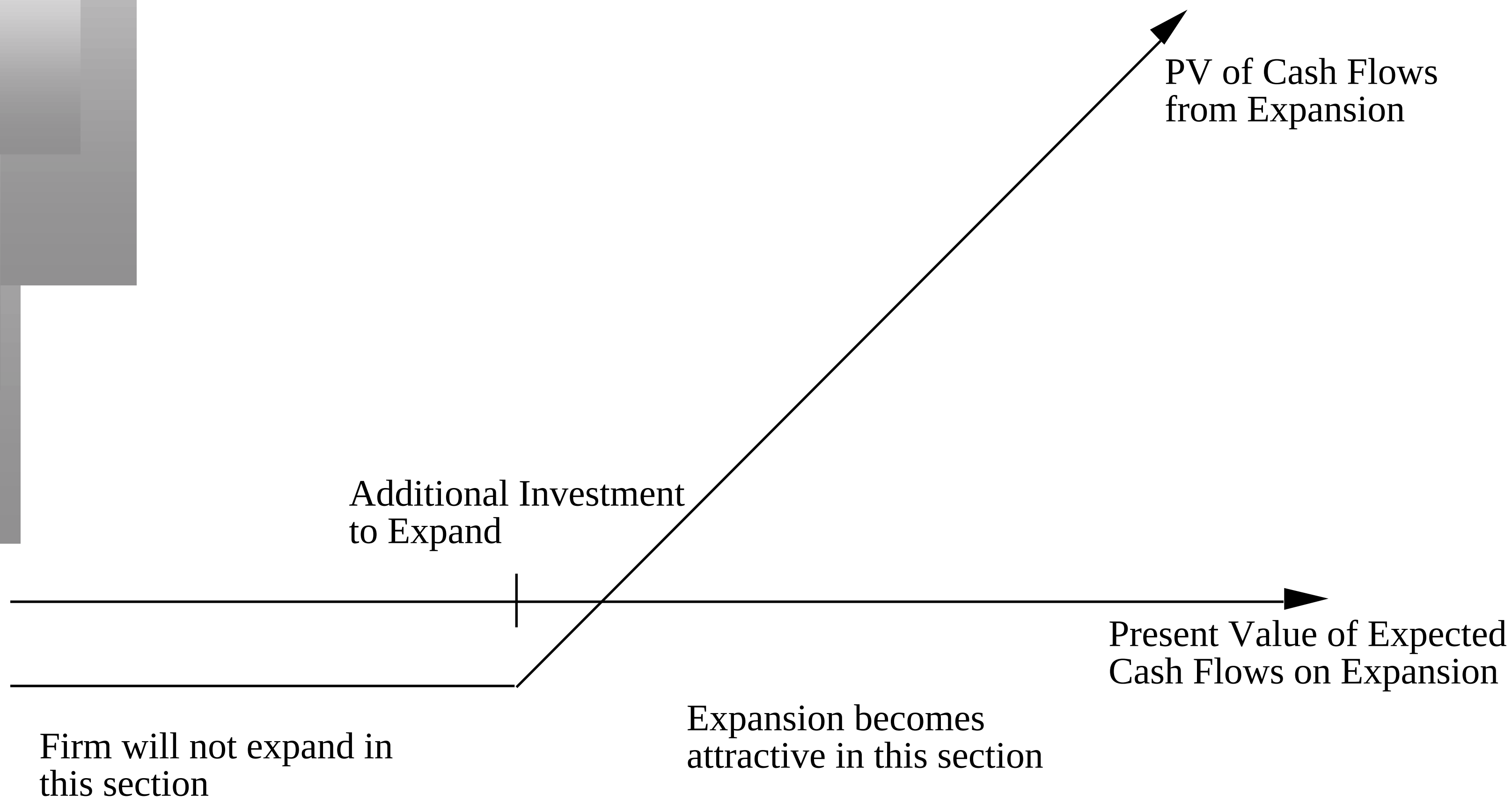
- In addition, Gulf Oil had free cashflows to the firm from its oil and gas production of $915 million from already developed reserves and these cashflows are likely to continue for ten years (the remaining lifetime of developed reserves).
- The present value of these developed reserves, discounted at the weighted average cost of capital of 12.5%, yields:
  - Value of already developed reserves = 915 $(1 - 1.125^{-10})/.125$ = $5065.83
- Adding the value of the developed and undeveloped reserves

Value of undeveloped reserves	= $ 13,306 million	
Value of production in place	= $ 5,066 million	
Total value of firm	= $ 18,372 million	
Less Outstanding Debt	= $ 9,900 million	
Value of Equity	= $ 8,472 million	
Value per share	= $ 8,472/165.3	= $51.25

The Option to Expand/Take Other Projects

- Taking a project today may allow a firm to consider and take other valuable projects in the future.
- Thus, even though a project may have a negative NPV, it may be a project worth taking if the option it provides the firm (to take other projects in the future) provides a more-than-compensating value.
- These are the options that firms often call "strategic options" and use as a rationale for taking on "negative NPV" or even "negative return" projects.

The Option to Expand



An Example of an Expansion Option

- Ambev is considering introducing a soft drink to the U.S. market. The drink will initially be introduced only in the metropolitan areas of the U.S. and the cost of this "limited introduction" is $ 500 million.
- A financial analysis of the cash flows from this investment suggests that the present value of the cash flows from this investment to Ambev will be only $ 400 million. Thus, by itself, the new investment has a **negative NPV of $ 100 million**.
- If the initial introduction works out well, Ambev **could go ahead with a full-scale introduction to the entire market** with **an additional investment of $ 1 billion** any time over the next 5 years. While the current expectation is that the cash flows from having this investment is only $ 750 million, there is considerable uncertainty about both the potential for the drink, leading to significant variance in this estimate.

Valuing the Expansion Option

- Value of the Underlying Asset (S) = PV of Cash Flows from Expansion to entire U.S. market, if done now =$ 750 Million
- Strike Price (K) = Cost of Expansion into entire U.S market = $ 1000 Million
- We estimate the standard deviation in the estimate of the project value by using the annualized standard deviation in firm value of publicly traded firms in the beverage markets, which is approximately 34.25%.
  - Standard Deviation in Underlying Asset's Value = 34.25%
- Time to expiration = Period for which expansion option applies = 5 years

Call Value= $ 234 Million

Considering the Project with Expansion Option

- NPV of Limited Introduction = $ 400 Million - $ 500 Million = - $ 100 Million
- Value of Option to Expand to full market= $ 234 Million
- NPV of Project with option to expand
  = - $ 100 million + $ 234 million
  = $ 134 million
- **Invest in the project**

The Link to Strategy

- In many investments, especially acquisitions, strategic options or considerations are used to take investments that otherwise do not meet financial standards.
- These strategic options or considerations are usually related to the expansion option described here. The key differences are as follows:
  - Unlike “strategic options” which are usually qualitative and not valued, expansion options can be assigned a quantitative value and can be brought into the investment analysis.
  - Not all “strategic considerations” have option value. For an expansion option to have value, the first investment (acquisition) must be necessary for the later expansion (investment). If it is not, there is no option value that can be added on to the first investment.

The Exclusivity Requirement in Option Value

Is the first investment necessary for the second investment?

Not necessary → Pre-Requisit

A Zero competitive advantage on Second Investment → An Exclusive Right to Second Investment

No option value → **100% of option value**

Option has no value → **Option has high value**

Second Investment has zero excess returns → Second investment has large sustainable excess return

First-Mover → Technological Edge → Brand Name → Telecom Licenses → Pharmaceutical patents

Increasing competitive advantage/ barriers to entry

The Determinants of Real Option Value

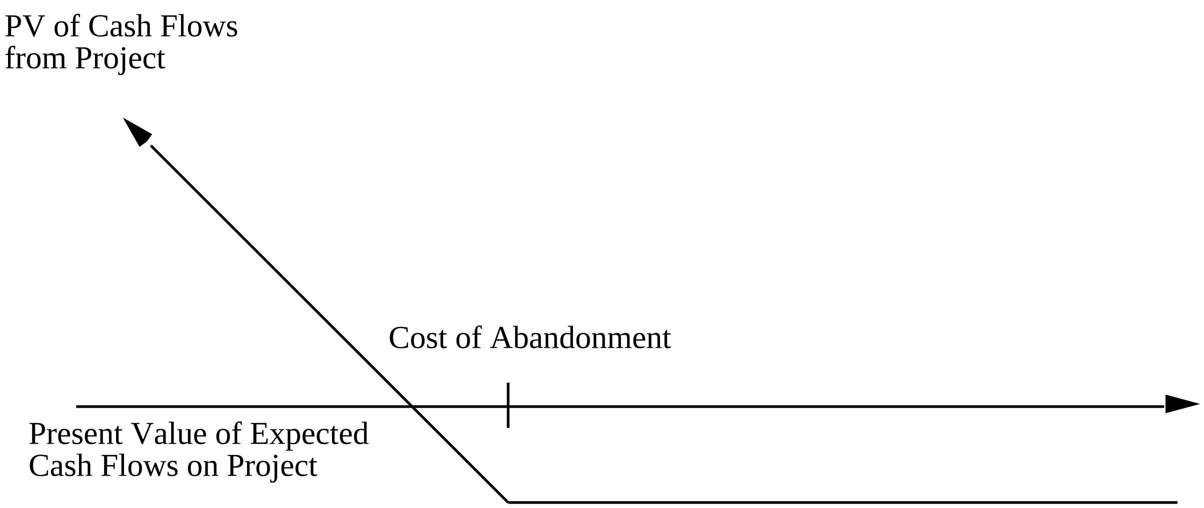
- Does taking on the first investment/expenditure provide the firm with an exclusive advantage on taking on the second investment?
  - If yes, the firm is entitled to consider 100% of the value of the real option
  - If no, the firm is entitled to only a portion of the value of the real option, with the proportion determined by the degree of exclusivity provided by the first investment?
- Is there a possibility of earning significant and sustainable excess returns on the second investment?
  - If yes, the real option will have significant value
  - If no, the real option has no value

Internet Firms as Options

- Some analysts have justified the valuation of internet firms on the basis that you are buying the option to expand into a very large market. What do you think of this argument?
  - Is there an option to expand embedded in these firms?
  - Is it a valuable option?

The Option to Abandon

- A firm may sometimes have the option to abandon a project, if the cash flows do not measure up to expectations.
- If abandoning the project allows the firm to save itself from further losses, this option can make a project more valuable.



Valuing the Option to Abandon

- Airbus is considering a joint venture with Lear Aircraft to produce a small commercial airplane (capable of carrying 40-50 passengers on short haul flights)
  - Airbus will have to invest $ 500 million for a 50% share of the venture
  - Its share of the present value of expected cash flows is 480 million.
- Lear Aircraft, which is eager to enter into the deal, offers to buy Airbus's 50% share of the investment anytime over the next five years for $ 400 million, if Airbus decides to get out of the venture.
- A simulation of the cash flows on this time share investment yields a variance in the present value of the cash flows from being in the partnership is 0.16.
- The project has a life of 30 years.

Project with Option to Abandon

- Value of the Underlying Asset (S) = PV of Cash Flows from Project = $ 480 million
- Strike Price (K) = Salvage Value from Abandonment = $ 400 million
- Variance in Underlying Asset's Value = 0.16
- Time to expiration = Life of the Project =5 years
- Dividend Yield = 1/Life of the Project = 1/30 = 0.033 (We are assuming that the project's present value will drop by roughly 1/n each year into the project)
- Assume that the five-year riskless rate is 6%. The value of the put option can be estimated as follows:

Should Airbus enter into the joint venture?

- Value of Put $=Ke^{-rt}(1-N(d2)) - Se^{-yt}(1-N(d1))$

  $=400\ (\exp^{(-0.06)(5)}\ (1-0.7496) - 480\ \exp^{(-0.033)(5)}\ (1-0.9105)$

  = $ 73.23 million
- The value of this abandonment option has to be added on to the net present value of the project of -$ 20 million, yielding a total net present value with the abandonment option of $ 53.23 million.

Implications for Investment Analysis

- Having a option to abandon a project can make otherwise unacceptable projects acceptable.
- Actions that increase the value of the abandonment option include
  - More cost flexibility, that is, making more of the costs of the projects into variable costs as opposed to fixed costs.
  - Fewer long-term contracts/obligations with employees and customers, since these add to the cost of abandoning a project
  - Finding partners in the investment, who are willing to acquire your investment in the future
- These actions will undoubtedly cost the firm some value, but this has to be weighed off against the increase in the value of the abandonment option.

Option Pricing Applications in the Capital Structure Decision

Options in Capital Structure

- The most direct applications of option pricing in capital structure decisions is in the design of securities. In fact, most complex financial instruments can be broken down into some combination of a simple bond/common stock and a variety of options.
  - If these securities are to be issued to the public, and traded, the options have to be priced.
  - If these are non-traded instruments (bank loans, for instance), they still have to be priced into the interest rate on the instrument.
- The other application of option pricing is in valuing flexibility. Often, firms preserve debt capacity or hold back on issuing debt because they want to maintain flexibility.

The Value of Flexibility

- Firms maintain excess debt capacity or larger cash balances than are warranted by current needs, to meet unexpected future requirements.
- While maintaining this financing flexibility has value to firms, it also has a cost; the excess debt capacity implies that the firm is giving up some value and has a higher cost of capital.
- The value of flexibility can be analyzed using the option pricing framework; a firm maintains large cash balances and excess debt capacity <u>in order to have the option to take projects that might arise in the future</u>.

Determinants of Value of Flexibility Option

- *Quality of the Firm's Projects*: It is the excess return that the firm earns on its projects that provides the value to flexibility. Other things remaining equal, firms operating in businesses where projects earn substantially higher returns than their hurdle rates should value flexibility more than those that operate in stable businesses where excess returns are small.
- *Uncertainty about Future Projects*: If flexibility is viewed as an option, its value will increase when there is greater uncertainty about future projects; thus, firms with predictable capital expenditures and excess returns should value flexibility less than those with high variability in both of those variables.

Value of Flexibility as an Option

- Consider a firm that has expected reinvestment needs of X each year, with a standard deviation in that value of σ_X. These external reinvestments include both internal projects and acquisitions.
- Assume that the firm can raise L from internal cash flows and its normal access to capital markets. (Normal access refers to the external financing that is used by a firm each year)
- Excess debt capacity becomes useful if external reinvestment needs exceed the firm's internal funds.

If X > L: Excess debt capacity can be used to cover the difference and invest in projects

If X<L: Excess debt capacity remains unused (with an associated cost)

What happens when you make the investment?

- If the investment earns excess returns, the firm's value will increase by the present value of these excess returns over time. If we assume that the excess return each year is constant and perpetual, the present value of the excess returns that would be earned can be written as:

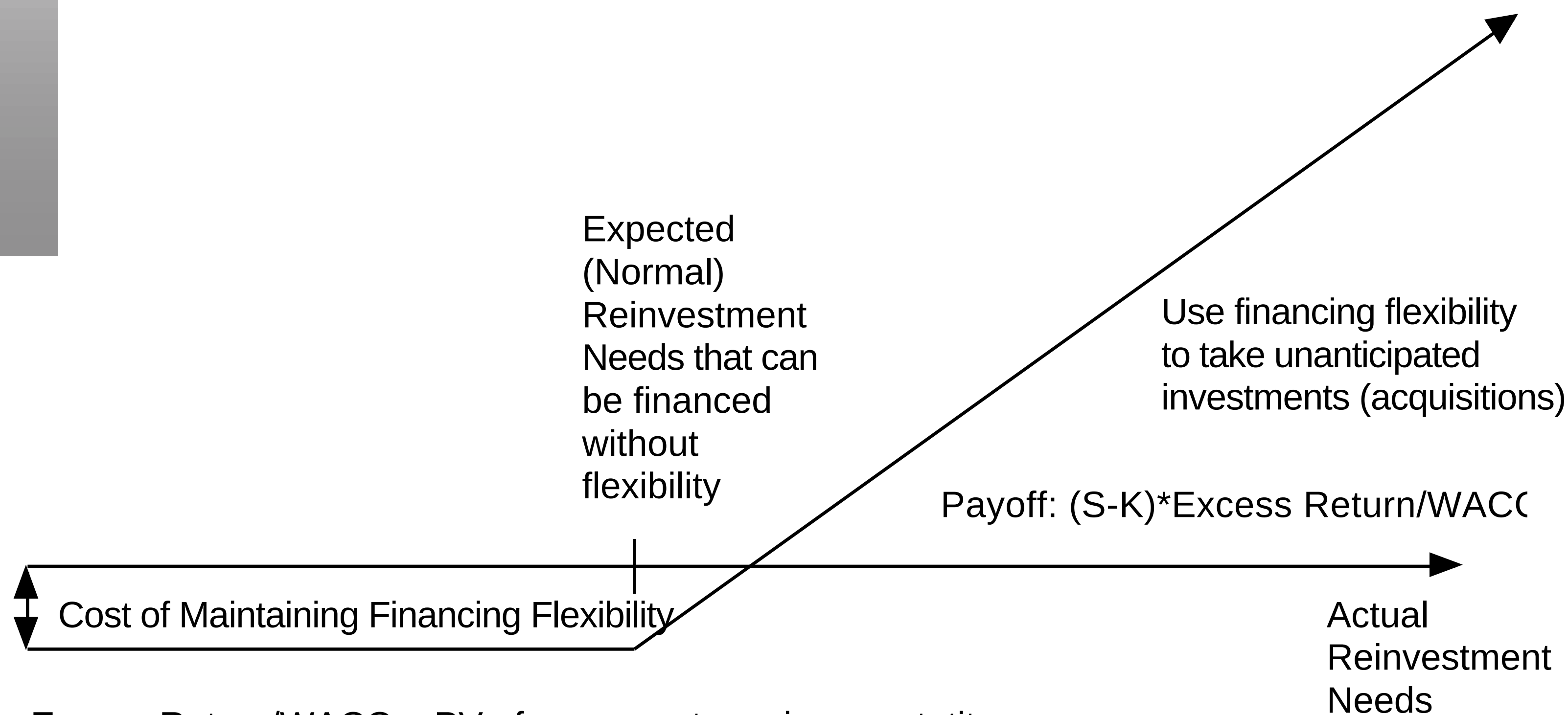
  Value of investment = (ROC - Cost of capital)/ Cost of capital

- The value of the investments that you can take because you have excess debt capacity becomes the payoff to maintaining excess debt capacity.

If X > L: [(ROC - Cost of capital)/ Cost of capital] New investments

If X<L: 0

The Value of Flexibility



Excess Return/WACC = PV of excess returns in perpetutity

Disney's Optimal Debt Ratio

Debt Ratio	Cost of Equity	Cost of Debt	Cost of Capital
0.00%	13.00%	4.61%	13.00%
10.00%	13.43%	4.61%	12.55%
Current:18%	**13.85%**	**4.80%**	**12.22%**
20.00%	13.96%	4.99%	12.17%
30.00%	14.65%	5.28%	11.84%
40.00%	*15.56%*	*5.76%*	*11.64%*
50.00%	16.85%	6.56%	11.70%
60.00%	18.77%	7.68%	12.11%
70.00%	21.97%	7.68%	11.97%
80.00%	28.95%	7.97%	12.17%
90.00%	52.14%	9.42%	13.69%

Inputs to Option Valuation Model

- One way to think about firms that preserve debt capacity because they want flexibility is that they are foregoing use this debt to invest in existing projects at existing excess returns because they think that they might have an increase in either investment needs or excess returns.
- To value flexibility as a percent of firm value (as an annual cost), these would be the inputs to the model:
  - S = Expected Reinvestment needs as percent of Firm Value
  - K = Expected Reinvestment needs that can be financed without financing flexibility
  - t = 1 year

  σ^2 = Variance in ln(Net Capital Expenditures)
- Once this option has been valued, estimate the present value of the excess returns that will be gained by taking the additional investments by multiplying by (ROC - WACC)/WACC

The Inputs for Disney

- Expected reinvestment needs as a percent of firm value:
  - Over the last 5 years, reinvestment (net cap ex, acquisitions and changes in working capital) has been approximately 5.3% of firm value
  - I am assuming that this is the expected reinvestment need; the variance in ln(reinvestment) over the last 5 years is 0.375
- Reinvestment needs that can be financed without flexibility.
  - We looked at *internal funds*, after debt payments but before reinvestment needs, as a percent of firm value over the last 5 years. (Internal funds = (Net Income + Depreciation)/Market Value of the Firm)
  - We looked at *net debt financing* each period, as a percent of firm value (as a measure of access to external financing each year). (New Debt - Debt Repaid)/Market Value of Firm)
  - Reinvestment needs that can be financed without flexibility = (Net Income + Depreciation + Net Debt Issued)/Market Value of Firm
  - This number has averaged 4.8%, over the last 5 years

Valuing Flexibility at Disney

The value of flexibility as a percentage of firm value can be estimated as follows:

- S = 5.3%
- K = 4.8%
- t = 1 year
- σ^2 = 0.375 (Variance in ln(Reinvestment Needs/Firm Value))

The value of an option with these characteristics is 1.6092%

- Disney earns 18.69% on its projects has a cost of capital of 12.22%. *The excess return (annually) is 6.47%.*

Value of Flexibility (annual)= 1.6092%(.0647/.1222) = 0.85 % of value

- Disney's cost of capital at its optimal debt ratio is 11.64%. The cost it incurs to maintain flexibility is therefore 0.58% annually (12.22%-11.64%). It therefore pays to maintain flexibility.

Determinants of the Value of Flexibility

- Capacity to raise funds to meet financing needs: The greater the capacity to raise funds, either internally or externally, the less the value of flexibility.
  - 1.1: Firms with significant internal operating cash flows should value flexibility less than firms with small or negative operating cash flows.
  - 1.2: Firms with easy access to financial markets should have a lower value for flexibility than firms without that access.
- Unpredictability of reinvestment needs: The more unpredictable the reinvestment needs of a firm, the greater the value of flexibility.
- Capacity to earn excess returns: The greater the capacity to earn excess returns, the greater the value of flexibility.
  - 1.3: Firms that do not have the capacity to earn or sustain excess returns get no value from flexibility.

Option Pricing Applications in Valuation

Equity Value in Deeply Troubled Firms

Value of Undeveloped Reserves for Natural Resource Firm

Value of Patent/License

Option Pricing Applications in Equity Valuation

- Equity in a troubled firm (i.e. a firm with high leverage, negative earnings and a significant chance of bankruptcy) can be viewed as a call option, which is the option to liquidate the firm.
- Natural resource companies, where the undeveloped reserves can be viewed as options on the natural resource.
- Start-up firms or high growth firms which derive the bulk of their value from the rights to a product or a service (eg. a patent)

Valuing Equity as an option

- The equity in a firm is a **residual claim**, i.e., equity holders lay claim to all cashflows left over after other financial claim-holders (debt, preferred stock etc.) have been satisfied.
- If a firm is liquidated, the same principle applies, with equity investors **receiving whatever is left over in the firm** after all outstanding debts and other financial claims are paid off.
- The **principle of limited liability**, however, protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm.

Equity as a call option

- The payoff to equity investors, on liquidation, can therefore be written as:

  Payoff to equity on liquidation $= V - D$ if $V > D$

  $= 0$ if $V \leq D$

  where,

  V = Value of the firm

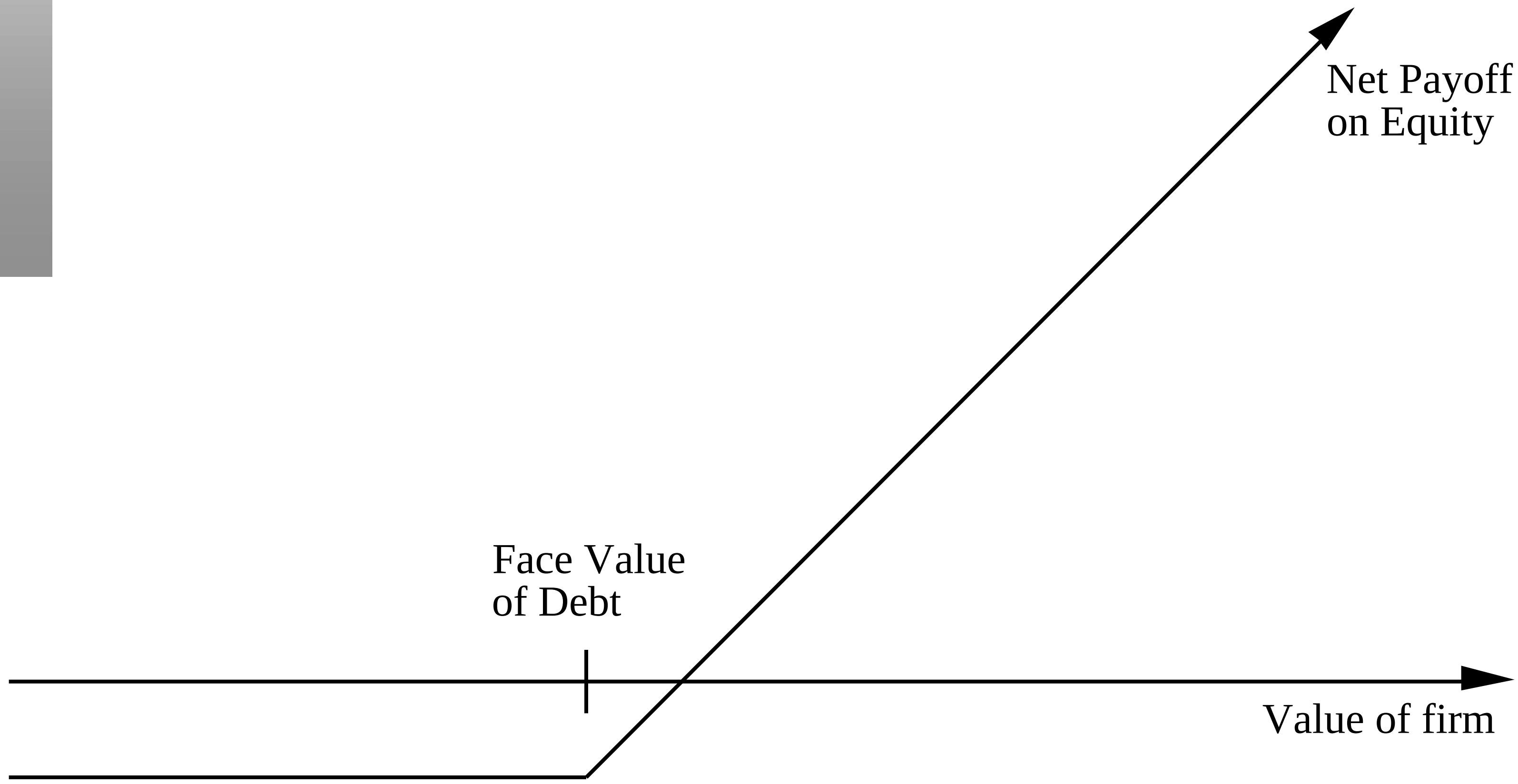
  D = Face Value of the outstanding debt and other external claims

- A call option, with a strike price of K, on an asset with a current value of S, has the following payoffs:

  Payoff on exercise $= S - K$ if $S > K$

  $= 0$ if $S \leq K$

Payoff Diagram for Liquidation Option



Application to valuation: A simple example

- Assume that you have a firm whose assets are currently valued at $100 million and that the standard deviation in this asset value is 40%.
- Further, assume that the face value of debt is $80 million (It is zero coupon debt with 10 years left to maturity).
- If the ten-year treasury bond rate is 10%,
  - how much is the equity worth?
  - What should the interest rate on debt be?

Model Parameters

- Value of the underlying asset = S = Value of the firm = $ 100 million
- Exercise price = K = Face Value of outstanding debt = $ 80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
- Riskless rate = r = Treasury bond rate corresponding to option life = 10%

Valuing Equity as a Call Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
  - d1 = 1.5994 N(d1) = 0.9451
  - d2 = 0.3345 N(d2) = 0.6310
- Value of the call = 100 (0.9451) - 80 $\exp^{(-0.10)(10)}$ (0.6310) = \$75.94 million
- Value of the outstanding debt = \$100 - \$75.94 = \$24.06 million
- Interest rate on debt = $(\$ 80 / \$24.06)^{1/10}$ -1 = 12.77%

The Effect of Catastrophic Drops in Value

- Assume now that a catastrophe wipes out half the value of this firm (the value drops to $ 50 million), while the face value of the debt remains at $ 80 million. What will happen to the equity value of this firm?
  - ❑ It will drop in value to $ 25.94 million [$ 50 million - market value of debt from previous page]
  - ❑ It will be worth nothing since debt outstanding > Firm Value
  - ❑ It will be worth more than $ 25.94 million

Illustration : Value of a troubled firm

- Assume now that, in the previous example, the value of the firm were reduced to $ 50 million while keeping the face value of the debt at $80 million.
- This firm could be viewed as troubled, since it owes (at least in face value terms) more than it owns.
- The equity in the firm will still have value, however.

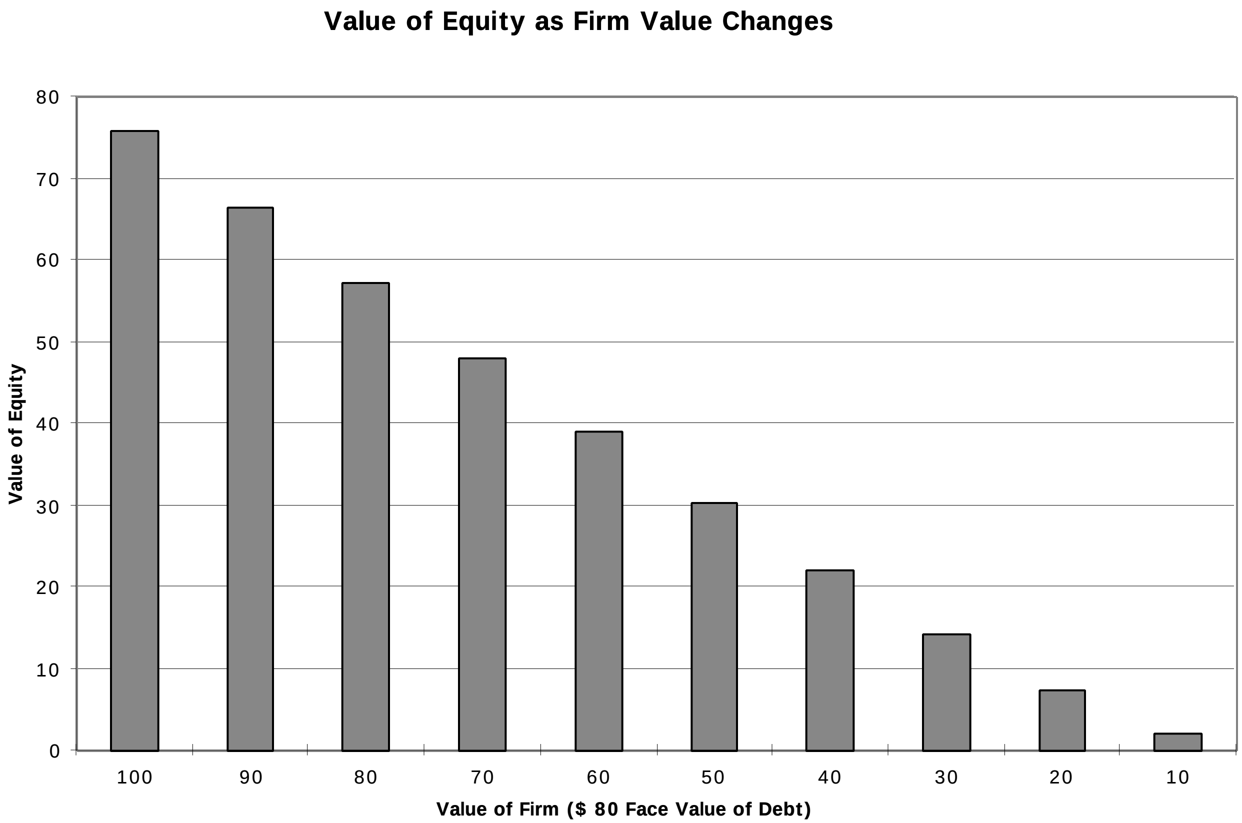
Valuing Equity in the Troubled Firm

- Value of the underlying asset = S = Value of the firm = $ 50 million
- Exercise price = K = Face Value of outstanding debt = $ 80 million
- Life of the option = t = Life of zero-coupon debt = 10 years
- Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.16
- Riskless rate = r = Treasury bond rate corresponding to option life = 10%

The Value of Equity as an Option

- Based upon these inputs, the Black-Scholes model provides the following value for the call:
  - d1 = 1.0515 N(d1) = 0.8534
  - d2 = -0.2135 N(d2) = 0.4155
- Value of the call = 50 (0.8534) - 80 $\exp^{(-0.10)(10)}$ (0.4155) = $30.44 million
- Value of the bond= $50 - $30.44 = $19.56 million
- The equity in this firm drops by, because of the option characteristics of equity.
- This might explain why stock in firms, which are in Chapter 11 and essentially bankrupt, still has value.

Equity value persists ..



Valuing equity in a troubled firm

- The first implication is that **equity will have value**, even if the **value of the firm** falls well **below the face value of the outstanding debt**.
- Such a firm will be viewed as **troubled** by investors, accountants and analysts, but that **does not mean that its equity is worthless**.
- Just as deep out-of-the-money traded options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, **equity will command value because of the time premium on the option** (the time until the bonds mature and come due) and the possibility that the value of the assets may increase above the face value of the bonds before they come due.

Obtaining option pricing inputs - Some real world problems

- The examples that have been used to illustrate the use of option pricing theory to value equity have made some simplifying assumptions. Among them are the following:
  - (1) There were only two claim holders in the firm - debt and equity.
  - (2) There is only one issue of debt outstanding and it can be retired at face value.
  - (3) The debt has a zero coupon and no special features (convertibility, put clauses etc.)
  - (4) The value of the firm and the variance in that value can be estimated.

Real World Approaches to Getting inputs

Input	Estimation Process
Value of the Firm	· Cumulate market values of equity and debt (or) · Value the assets in place using FCFF and WACC (or) · Use cumulated market value of assets, if traded.
Variance in Firm Value	· If stocks and bonds are traded, $\sigma^2_{firm} = w_e^2 \sigma_e^2 + w_d^2 \sigma_d^2 + 2 w_e w_d \rho_{ed} \sigma_e \sigma_d$ where σ_e^2 = variance in the stock price w_e = MV weight of Equity σ_d^2 = the variance in the bond price w_d = MV weight of debt · If not traded, use variances of similarly rated bonds. · Use average firm value variance from the industry in which company operates.
Value of the Debt	· If the debt is short term, you can use only the face or book value of the debt. · If the debt is long term and coupon bearing, add the cumulated nominal value of these coupons to the face value of the debt.
Maturity of the Debt	· Face value weighted duration of bonds outstanding (or) · If not available, use weighted maturity

Valuing Equity as an option - Eurotunnel in early 1998

- Eurotunnel has been a financial disaster since its opening
  - In 1997, Eurotunnel had earnings before interest and taxes of -£56 million and net income of -£685 million
  - At the end of 1997, its book value of equity was -£117 million
- It had £8,865 million in face value of debt outstanding
  - The weighted average duration of this debt was 10.93 years

Debt Type	Face Value	Duration
Short term	935	0.50
10 year	2435	6.7
20 year	3555	12.6
Longer	1940	18.2
Total	*£8,865 mil*	*10.93 years*

The Basic DCF Valuation

- The value of the firm estimated using projected cashflows to the firm, discounted at the weighted average cost of capital was £2,312 million.
- This was based upon the following assumptions –
  - Revenues will grow 5% a year in perpetuity.
  - The COGS which is currently 85% of revenues will drop to 65% of revenues in yr 5 and stay at that level.
  - Capital spending and depreciation will grow 5% a year in perpetuity.
  - There are no working capital requirements.
  - The debt ratio, which is currently 95.35%, will drop to 70% after year 5. The cost of debt is 10% in high growth period and 8% after that.
  - The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years.
  - The long term bond rate is 6%.

Other Inputs

- The stock has been traded on the London Exchange, and the annualized std deviation based upon ln (prices) is 41%.
- There are Eurotunnel bonds, that have been traded; the annualized std deviation in ln(price) for the bonds is 17%.
  - The correlation between stock price and bond price changes has been 0.5. The proportion of debt in the capital structure during the period (1992-1996) was 85%.
  - Annualized variance in firm value

$= (0.15)^2 (0.41)^2 + (0.85)^2 (0.17)^2 + 2 (0.15) (0.85)(0.5)(0.41)(0.17)= 0.0335$

- The 15-year bond rate is 6%. (I used a bond with a duration of roughly 11 years to match the life of my option)

Valuing Eurotunnel Equity and Debt

- Inputs to Model
  - Value of the underlying asset = S = Value of the firm = £2,312 million
  - Exercise price = K = Face Value of outstanding debt = £8,865 million
  - Life of the option = t = Weighted average duration of debt = 10.93 years
  - Variance in the value of the underlying asset = σ^2 = Variance in firm value = 0.0335
  - Riskless rate = r = Treasury bond rate corresponding to option life = 6%
- Based upon these inputs, the Black-Scholes model provides the following value for the call:

  d1 = -0.8337 N(d1) = 0.2023

  d2 = -1.4392 N(d2) = 0.0751
- Value of the call = 2312 (0.2023) - 8,865 $\exp^{(-0.06)(10.93)}$ (0.0751) = £122 million
- Appropriate interest rate on debt = $(8865/2190)^{(1/10.93)}$-1= 13.65%

Industry Name	*Std Dev(Equity)*	*Std Dev(Firm)*	*Industry Name*	*Std Dev(Equity)*	*Std Dev(Firm)*
Advertising	35.48%	27.11%	Household Products	29.40%	24.91%
Aerospace/Defense	37.40%	33.13%	Industrial Services	43.95%	39.62%
Air Transport	44.52%	33.80%	Insurance (Diversified)	28.46%	26.99%
Aluminum	29.20%	22.05%	Insurance (Life)	30.61%	29.15%
Apparel	45.25%	37.34%	Insurance (Prop/Casualty)	26.98%	25.68%
Auto & Truck	31.01%	23.90%	Investment Co. (Domestic)	23.40%	22.28%
Auto Parts (OEM)	31.21%	26.63%	Investment Co. (Foreign)	28.01%	27.91%
Auto Parts (Replacement)	33.28%	25.71%	Investment Co. (Income)	10.95%	10.95%
Bank	24.44%	22.44%	Machinery	35.25%	30.94%
Bank (Canadian)	21.18%	19.12%	Manuf. Housing/Rec Veh	41.09%	36.00%
Bank (Foreign)	23.12%	22.39%	Maritime	33.85%	24.38%
Bank (Midwest)	20.13%	19.15%	Medical Services	63.58%	55.77%
Beverage (Alcoholic)	22.21%	20.24%	Medical Supplies	54.33%	50.44%
Beverage (Soft Drink)	37.59%	32.50%	Metal Fabricating	35.61%	32.85%
Building Materials	35.68%	31.08%	Metals & Mining (Div.)	55.48%	50.20%
Cable TV	41.41%	21.67%	Natural Gas (Distrib.)	19.35%	15.23%
Canadian Energy	25.24%	21.41%	Natural Gas (Diversified)	33.69%	28.21%
Cement & Aggregates	32.83%	29.86%	Newspaper	23.54%	19.99%
Chemical (Basic)	29.43%	25.16%	Office Equip & Supplies	34.40%	29.32%
Chemical (Diversified)	30.87%	27.01%	Oilfield Services/Equip.	43.25%	39.70%
Chemical (Specialty)	33.74%	29.34%	Packaging & Container	37.44%	30.32%
Coal/Alternate Energy	40.48%	34.85%	Paper & Forest Products	28.41%	17.50%
Computer & Peripherals	64.64%	59.54%	Petroleum (Integrated)	25.66%	20.98%
Computer Software & Svcs	52.88%	50.35%	Petroleum (Producing)	49.32%	42.47%
Copper	30.41%	12.62%	Precision Instrument	47.36%	44.21%
Diversified Co.	42.82%	35.20%	Publishing	35.89%	30.75%
Drug	59.77%	58.50%	R.E.I.T.	25.06%	24.52%
Drugstore	47.64%	36.63%	Railroad	23.73%	19.37%
Electric Util. (Central)	14.93%	11.38%	Recreation	50.25%	39.58%
Electric Utility (East)	16.56%	11.67%	Restaurant	40.12%	35.55%
Electric Utility (West)	18.18%	13.80%	Retail (Special Lines)	51.20%	39.98%
Electrical Equipment	43.70%	39.49%	Retail Building Supply	40.55%	33.95%
Electronics	53.39%	48.39%	Retail Store	40.14%	29.46%
Entertainment	36.01%	28.95%	Securities Brokerage	33.42%	22.74%
Environmental	53.98%	43.74%	Semiconductor	54.64%	52.72%
Financial Services	36.16%	27.68%	Semiconductor Cap Equip	53.41%	52.50%
Food Processing	33.13%	26.83%	Shoe	44.63%	40.08%
Food Wholesalers	27.60%	22.11%	Steel (General)	33.73%	28.96%
Foreign Diversified	91.01%	44.08%	Steel (Integrated)	40.34%	27.69%
Foreign Electron/Entertn	34.03%	29.17%	Telecom. Equipment	61.61%	56.72%
Foreign Telecom.	36.18%	32.99%	Telecom. Services	42.29%	35.05%
Furn./Home Furnishings	34.62%	30.90%	Textile	31.60%	24.12%
Gold/Silver Mining	49.57%	46.46%	Thrift	28.94%	26.42%
Grocery	31.64%	21.84%	Tire & Rubber	26.39%	23.60%
Healthcare Info Systems	57.80%	54.69%	Tobacco	33.85%	25.31%
Home Appliance	34.82%	29.48%	Toiletries/Cosmetics	42.97%	36.82%
Homebuilding	43.66%	27.13%	Trucking/Transp. Leasing	38.09%	29.21%
Hotel/Gaming	45.01%	29.76%	Utility (Foreign)	23.17%	18.34%
			Water Utility	18.53%	14.16%