Diversification, Control & Liquidity: The Discount Trifecta

Aswath Damodaran

www.damodran.com

Fundamental Assumptions

- The Diversified Investor: Investors are rational and attempt to maximize expected returns, given risk taken. In the process, they end up with diversified portfolios and use information to make reasoned judgments on value.
- The Liquid Market: Investments are liquid. Trading is easy, instantaneous and costless.
- The Powerful Stockholder: As the owners of companies, stockholders exercise power over managers, who seek mightily to maximize stockholder wealth.

And the real world…

- To the extent that the buyer or buyers of a business is (are) not diversified, they may incorporate some or all firm-specific risk into their discount rates, thus reducing value. Let's call this the lack of diversification discount.
- If markets are illiquid, the buyers of an investment will incorporate the expected cost of that illiquidity (over time) into the value of the asset today, thus reducing value. Let's term this the illiquidity discount.
- If a firm is not optimally run (and what firm is?), running the firm differently (and optimally) will generate higher cash flows and/or lower discount rates. The inability to make these changes will thus result in the value being lower. Let's term this the lack of control (or minority) discount.

Why value them individually?

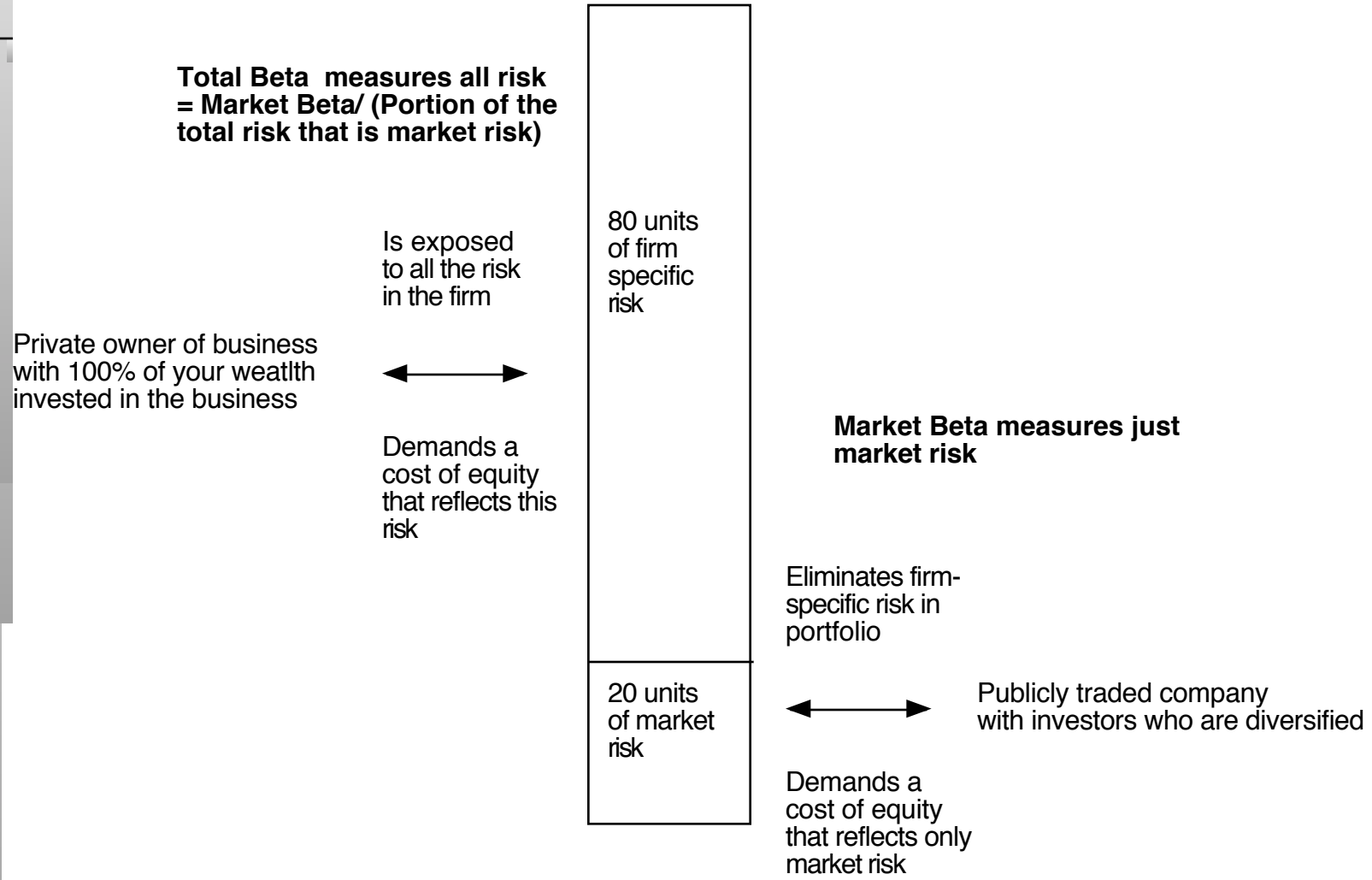
- They are separable: The degree and magnitude of each discount will vary not only across firms but also for the same firms, across time and for different transactions. Without valuing each one separately, you cannot estimate the correct discount.
- Prevent double counting: Trying to consolidate these discounts into one number is a dangerous exercise and can lead to miscounting and double counting of risks.
- Each is negotiable: The fact that you can value something (lack of diversification, lack of control or lack of liquidity) does not mean that you will pay for it…

I. The Undiversified Investor Implications for Valuation

Diversified Investors and the Cost of Equity

- The assumption that the marginal investor in a company is diversified is central to how we measure risk in finance.
- Since we assume that the marginal investor is diversified, we assume that the only risk that will be priced into the cost of equity is the risk that cannot be diversified away.
- When we use a beta to measure risk, we are measuring only that portion of the risk that cannot be diversified away. We are assuming that the remaining risk is ignored because it can be diversified.
- Is it possible that the marginal investor is not diversified? If so, how should we measure risk?

Private Owner versus Publicly Traded Company Perceptions of Risk in an Investment



Use bottom-up betas of publicly traded firms to get the unlevered beta of the busines

- Kristin Kandy is a privately owned, candy manufacturer, in the United States. The owner of the company has all of her wealth tied up in the company and wants to assess its value (to her).
- The average unlevered beta across publicly traded candy companies in the United States is 0.78. We will assume that this is a fair measure of the market risk in the candy business.

Estimating a total beta

- To get from the market beta to the total beta, we need a measure of how much of the risk in the firm comes from the market and how much is firm-specific.
- Looking at the regressions of publicly traded firms that yield the bottom-up beta should provide an answer.
  - The average R-squared across the regressions is 10.89%.
  - Since betas are based on standard deviations (rather than variances), we will take the correlation coefficient (the square root of the R-squared) as our measure of the proportion of the risk that is market risk.

Correlation of candy companies with market = $\sqrt{.1089} = 0.33$

Total Unlevered Beta = Market Beta/ Correlation with the market

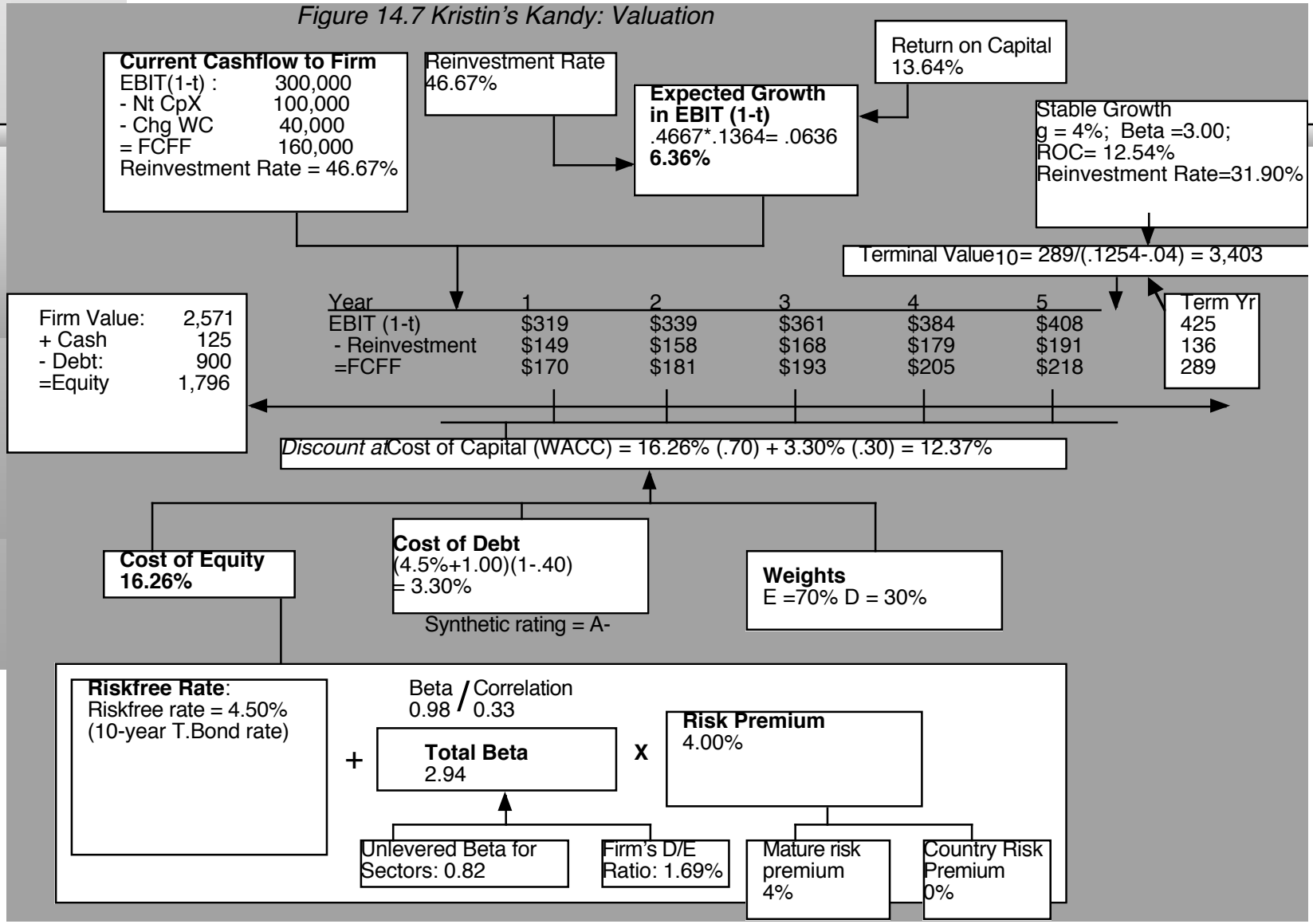
$= 0.78/0.333 = 2.34$

The final step in the beta computation: Estimate a Debt to equity ratio and cost of equity

- With publicly traded firms, we re-lever the beta using the market D/E ratio for the firm. With private firms, this option is not feasible. We have two alternatives:
  - Assume that the debt to equity ratio for the firm is similar to the average market debt to equity ratio for publicly traded firms in the sector.
  - Use your estimates of the value of debt and equity as the weights in the computation. (There will be a circular reasoning problem: you need the cost of capital to get the values and the values to get the cost of capital.)
- We will assume that this privately owned candy company will have a debt to equity ratio (42%) similar to the average publicly traded restaurant (even though we used retailers to the unlevered beta).
  - Levered beta = 2.34 (1 + (1-.4) (.42)) = 2.94
  - Cost of equity =4.5% + 2.94 (4%) = 16.26%

  (T Bond rate was 4.5% at the time; 4% is the equity risk premium)

Figure 14.7 Kristin's Kandy: Valuation



Bottom line on diversification…

- A diversified investor will see less risk in the same investment than an undiversified investor looking at that investment.
- If these investors have to face the same market price per risk, the diversified investor will demand a lower expected return (and discount rate) for the same investment as an undiversified investor.
- If the investors have the same expectations of cash flows from the asset, the diversified investor will pay a higher price for the same asset than an undiversified investor.

Implication 1: When selling a private business or asset, the best potential buyer, other things remaining equal, will be a diversified investor or an entity with diversified investors (a publicly traded firm).

Implication 2: Private business owners who are fully invested in their own businesses are holding on to these businesses at a discount, especially if going public or selling to a publicly traded company is an option.

A diversification continuum..

- Assume that you have a private business operating in a sector, where publicly traded companies have an average beta of 1 and where the average correlation of firms with the market is 0.25. Consider the cost of equity at three stages in the process (Riskfree rate = 4%; ERP = 5%):

Stage 1: The nascent business, with a private owner, who is fully invested in that business.

Perceived Beta = 1/ 0.25 = 4

Cost of Equity = 4% + 4 (5%) = 24%

Stage 2: Angel financing provided by specialized venture capitalist, who holds multiple investments, in high technology companies. (Correlation of portfolio with market is 0.5)

Perceived Beta = 1/0.5 = 2

Cost of Equity = 4% + 2 (5%) = 14%

Stage 3: Public offering, where investors are retail and institutional investors, with diversified portfolios:

Perceived Beta = 1

Cost of Equity = 4% + 1 (5%) = 9%

To value this company…

Assume that this company will be fully owned by its current owner for two years, will access the technology venture capitalist at the start of year 3 and that is expected to either go public or be sold to a publicly traded firm at the end of year 5.

Growth rate 2% forever after year 5

	1	2	3	4	5	Terminal year
E(Cash flow)	$100	$125	$150	$165	$170	$175
Market beta	1	1	1	1	1	1
Correlation	0.25	0.25	0.5	0.5	0.5	1
Beta used	4	4	2	2	2	1
Cost of equity	24.00%	24.00%	14.00%	14.00%	14.00%	9.00%
Terminal value					$2,500	
Cumulated COE	1.2400	1.5376	1.7529	1.9983	2.2780	2.4830
PV	$80.65	$81.30	$85.57	$82.57	$1,172.07	

175/ (.09-.02)

Value of firm	$1,502	(Correct value, using changing costs of equity)
Value of firm	$1,221	(using 24% as cost of equity forever. You will undervalue firm)
Value of firm	$2,165	(Using 9% as cost of equity forever. You will overvalue firm)

Total beta… notes of caution…

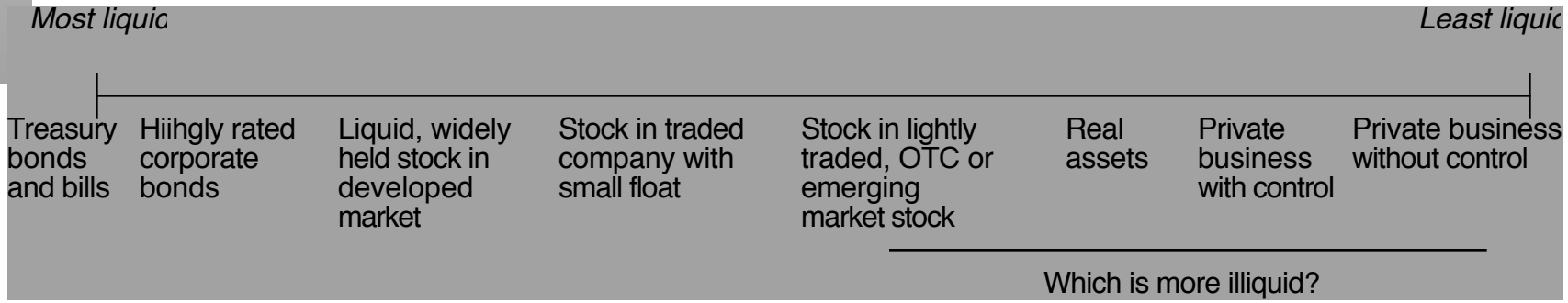
- Total beta should provide little explanatory power for expected returns at publicly traded firms, especially those that are widely held by institutions and have large market cap.
- It is not the appropriate measure of risk if an asset is being valued to a potential buyer, who is partially or mostly diversified. Thus, when valuing a private business for sale to a publicly traded company or even to a partially diversified investor, it is not appropriate to use total beta (and cost of equity).
- If asked to assess fair value, where fair value is the value to the best potential buyer of a business, using total beta is unlikely to provide the answer, unless you happen to be in a business where all of the potential buyers are undiversified.

II. The bane of illiquidity…

Aswath Damodaran

What is illiquidity?

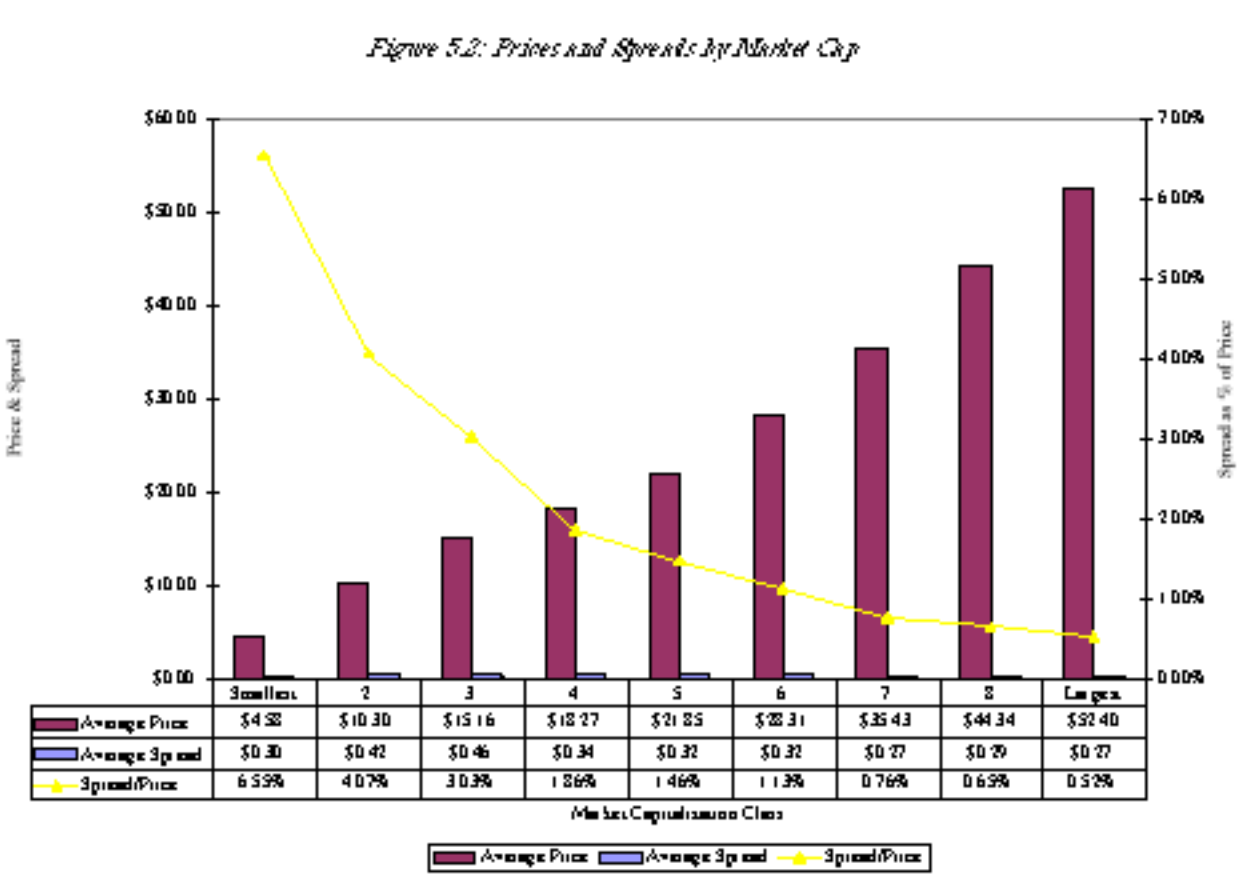
- The simplest way to think about illiquidity is to consider it the cost of buyer's remorse: it is the cost of reversing an asset trade almost instantaneously after you make the trade.
- Defined thus, all assets are illiquid. The difference is really a continuum, with some assets being more liquid than others.
- The notion that publicly traded firms are liquid and private businesses are not is too simplistic.



The Components of Trading Costs for an asset

- *Brokerage Cost*: This is the most explicit of the costs that any investor pays but it is by far the smallest component.
- *Bid-Ask Spread*: The spread between the price at which you can buy an asset (the dealer's ask price) and the price at which you can sell the same asset at the same point in time (the dealer's bid price).
- *Price Impact*: The price impact that an investor can create by trading on an asset, pushing the price up when buying the asset and pushing it down while selling.
- *Opportunity Cost*: There is the opportunity cost associated with waiting to trade. While being a patient trader may reduce the previous two components of trading cost, the waiting can cost profits both on trades that are made and in terms of trades that would have been profitable if made instantaneously but which became unprofitable as a result of the waiting.

The Magnitude of the Spread



Round-Trip Costs (including Price Impact) as a Function of Market Cap and Trade Size

	Dollar Value of Block ($ thoustands)								
Sector	*5*	*25*	*250*	*500*	*1000*	*2500*	*5000*	*10000*	*20000*
Smallest	17.30%	27.30%	43.80%						
2	8.90%	12.00%	23.80%	33.40%					
3	5.00%	7.60%	18.80%	25.90%	30.00%				
4	4.30%	5.80%	9.60%	16.90%	25.40%	31.50%			
5	2.80%	3.90%	5.90%	8.10%	11.50%	15.70%	25.70%		
6	1.80%	2.10%	3.20%	4.40%	5.60%	7.90%	11.00%	16.20%	
7	1.90%	2.00%	3.10%	4.00%	5.60%	7.70%	10.40%	14.30%	20.00%
8	1.90%	1.90%	2.70%	3.30%	4.60%	6.20%	8.90%	13.60%	18.10%
Largest	1.10%	1.20%	1.30%	1.71%	2.10%	2.80%	4.10%	5.90%	8.00%

The Theory on Illiquidity Discounts

- Illiquidity discount on value: You should reduce the value of an asset by the expected cost of trading that asset over its lifetime.
  - The illiquidity discount should be greater for assets with higher trading costs
  - The illiquidity discount should be decrease as the time horizon of the investor holding the asset increases
- Illiquid assets should be valued using higher discount rates
  - Risk-Return model: Some illiquidity risk is systematic. In other words, the illiquidity increases when the market is down. This risk should be built into the discount rate.
  - Empirical: Assets that are less liquid have historically earned higher returns. Relating returns to measures of illiquidity (turnover rates, spreads etc.) should allow us to estimate the discount rate for less liquid assets.
- Illiqudiity can be valued as an option: When you are not allowed to trade an asset, you lose the option to sell it if the price goes up (and you want to get out).

a. Illiquidity Discount in Value

- Amihud and Mendelson make the interesting argument that when you pay for an asset today will incorporate the present value of all expected future transactions costs on that asset. For instance, assume that the transactions costts are 2% of the price and that the average holding period is 1 year. The illiquidity discount can be computed as follows:

  Illiquidity discount = $\frac{2\%}{(1.10)} + \frac{2\%}{(1.10)^2} + \frac{2\%}{(1.10)^3} \ldots = \frac{2\%}{.10} = 20\%$

  With a holding period of 3 years, the illiqudity discount will be much smaller (about 6.67%)

- It follows then that the illiquidity discount will be
  - An increasing function of transactions costs
  - A decreasing function of the average holding period

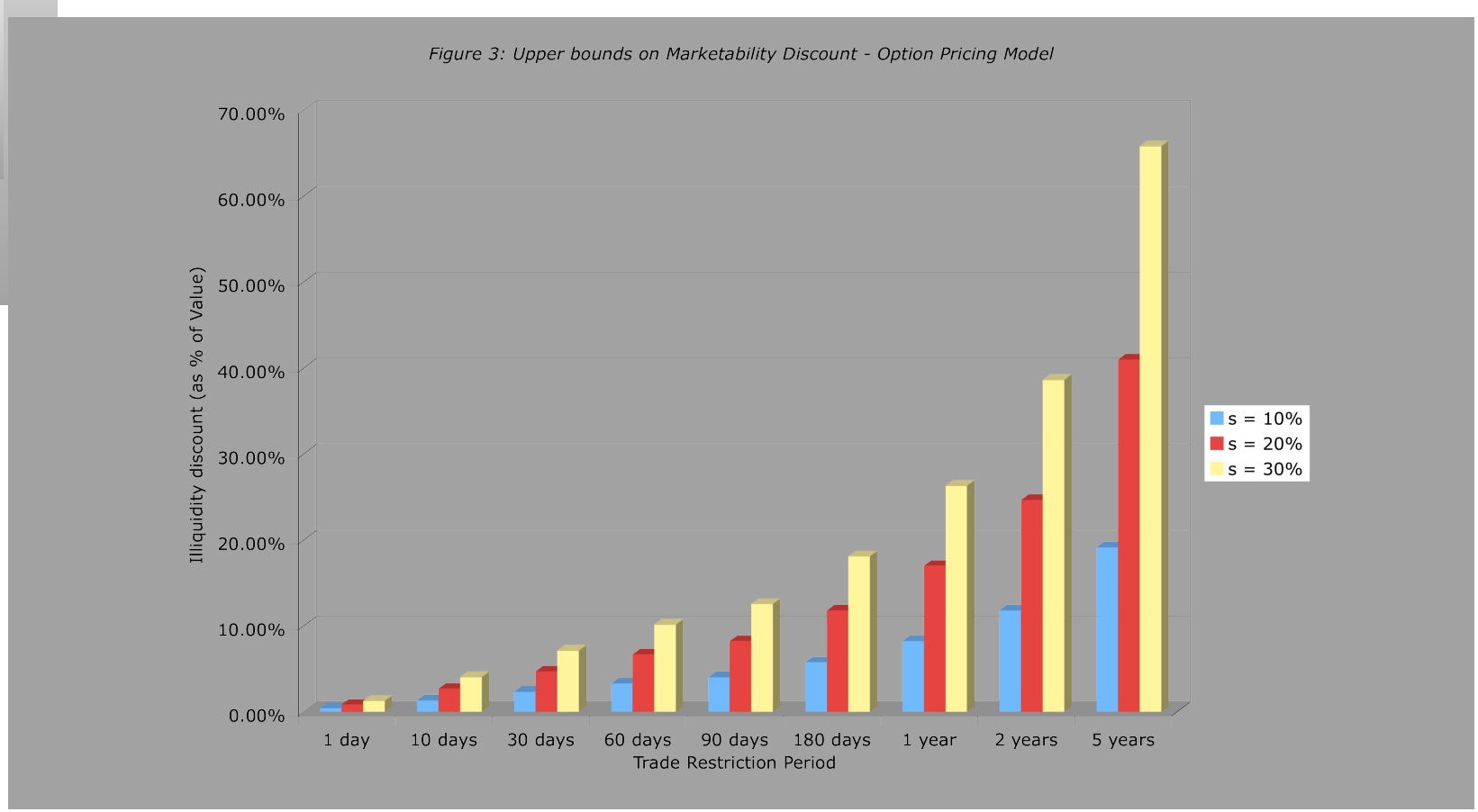
b. Adjusting discount rates for illiquidity

- Liquidity as a systematic risk factor
  - If liquidity is correlated with overall market conditions, less liquid stocks should have more market risk than more liquid stocks
  - To estimate the cost of equity for stocks, we would then need to estimate a "liquidity beta" for every stock and multiply this liquidity beta by a liquidity risk premium.
  - The liquidity beta is not a measure of liquidity, per se, but a measure of liquidity that is correlated with market conditions.
- Liquidity premiums
  - You can always add liquidity premiums to conventional risk and return models to reflect the higher risk of less liquid stocks.
  - These premiums are usually based upon historical data and reflect what you would have earned on less liquid investments historically (usually smaller stocks with lower trading volume) relative to more liquid investments. Amihud and Mendelson estimate that the expected return increases about 0.25% for every 1% increase in the bid-ask spread.

c. Illiquidity as a lookback option

- Longstaff (1995) presents an upper bound for the option by considering an investor with perfect market timing abilities who owns an asset on which she is not allowed to trade for a period.
- In the absence of trading restrictions, this investor would sell at the maximum price that an asset reaches during the time period and the value of the look-back option estimated using this maximum price should be the outer bound for the value of illiquidity. Using this approach,

Valuing the Lookback Option



Dealing with illiquidity in valuation

- If we accept that illiquidity affects value, and both the theory and empirical evidence suggest that it does, the question becomes how best to bring it into the value.
- There are three choices:
  - Estimate the value of the asset as if it were a liquid asset and then discount that value for illiquidity
  - Adjust the discount rates and use a higher discount rate for illiquid companies
  - Estimate the illiquidity discount by looking at comparable companies and seeing how much their values are impacted by illiquidity

Conclusion

- All assets are illiquid, but there are differences in the degree of illiquidity.
- Illiquidity matters to investors. They pay lower prices and demand higher returns from less liquid assets than from otherwise similar more liquid assets

III. The value of control

Aswath Damodaran

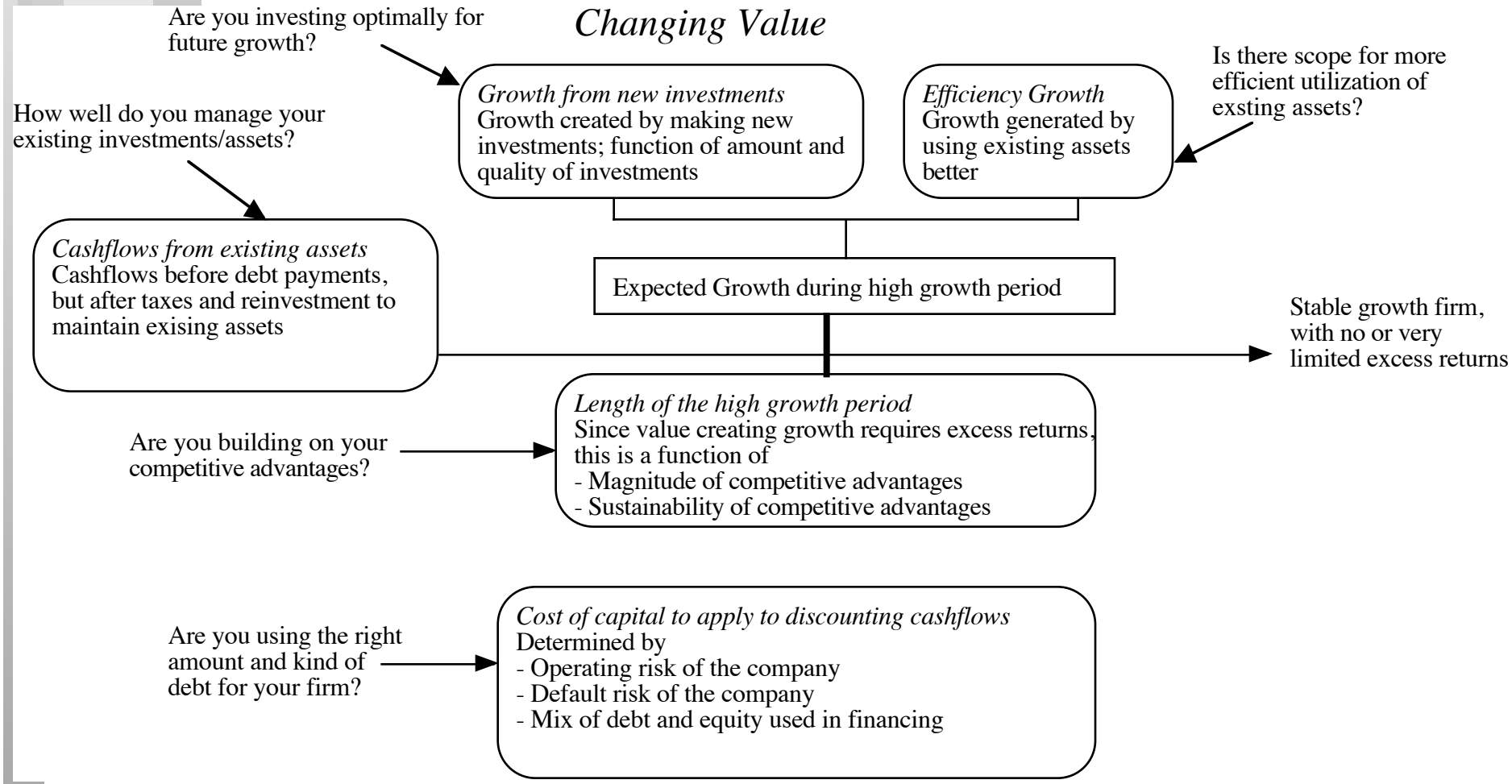
Home Page: www.damodaran.com

E-Mail: adamodar@stern.nyu.edu

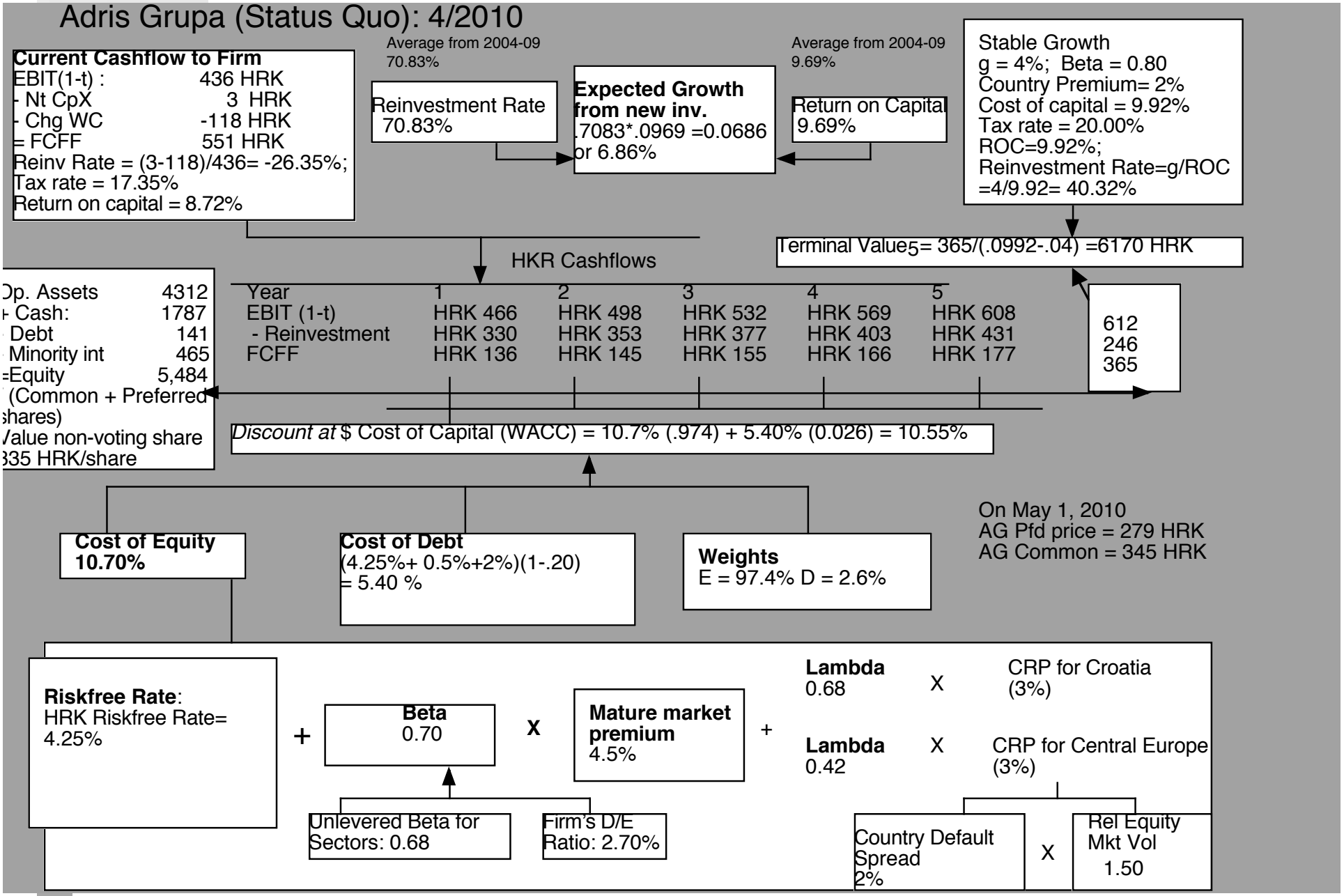
What is the value of control?

- The value of controlling a firm derives from the fact that you believe that you or someone else would operate the firm differently (and better) from the way it is operated currently.
- The expected value of control is the product of two variables:
  - the change in value from changing the way a firm is operated
  - the probability that this change will occur

Value of Gaining Control



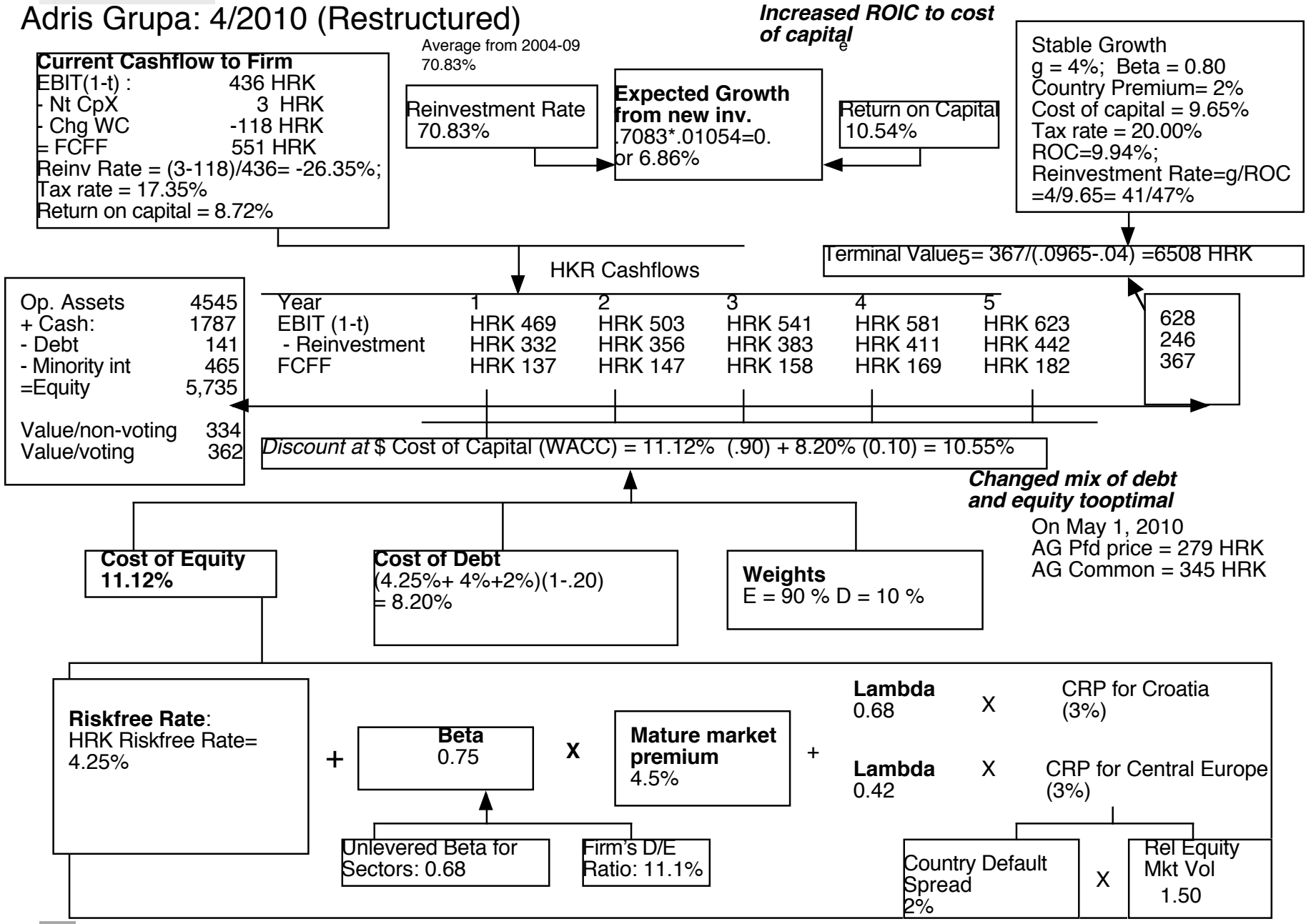
Adris Grupa (Status Quo): 4/2010



Ardis : Optimal Capital Structure

Debt Ratio	Beta	Cost of Equity	Bond Rating	Interest rate on debt	Tax Rate	Cost of Debt (after-tax)	WACC	Firm Value (G)
0%	0.69	10.56%	AAA	6.75%	20.00%	5.40%	10.56%	HRK 5,346
10%	0.75	11.12%	AAA	6.75%	20.00%	5.40%	10.55%	HRK 5,353
20%	0.82	11.83%	A	7.50%	20.00%	6.00%	10.66%	HRK 5,280
30%	0.92	12.73%	BB	10.25%	20.00%	8.20%	11.37%	HRK 4,846
40%	1.05	13.93%	B	11.50%	20.00%	9.20%	12.04%	HRK 4,496
50%	1.23	15.61%	B-	11.75%	20.00%	9.40%	12.51%	HRK 4,279
60%	1.51	18.14%	CC	16.25%	20.00%	13.00%	15.06%	HRK 3,381
70%	2.01	22.74%	CC	16.25%	17.36%	13.43%	16.22%	HRK 3,082
80%	3.06	32.40%	C	18.25%	13.52%	15.78%	19.11%	HRK 2,522
90%	6.11	60.56%	C	18.25%	12.02%	16.06%	20.51%	HRK 2,315

Adris Grupa: 4/2010 (Restructured)



Voting and Non-voting Shares: An Example

- The value of a voting share derives entirely from the capacity you have to change the way the firm is run.
- In this case, we have two values for Adris Grupa's Equity.

Status Quo Value of Equity = 5,469 million HKR

All shareholders, common and preferred, get an equal share of the status quo value.

Value for a non-voting share = 5469/(9.616+6.748) = 334 HKR/share

Optimal value of Equity = 5,735 million HKR

Value of control at Adris Grupa = 5,735 – 5469 = 266 million HKR

Only voting shares get a share of this value of control

Value per voting share =334 HKR + 266/9.616 = 362 HKR

Using “expected value of control” to derive a minority discount…

- Assume that you are valuing Kristin Kandy, a privately owned candy business for sale in a private transaction. You have estimated a value of $ 1.6 million for the equity in this firm, assuming that the existing management of the firm continues into the future and a value of $ 2 million for the equity with new and more creative management in place.
  - Value of 51% of the firm = 51% of optimal value = 0.51* $ 2 million = $1.02 million
  - Value of 49% of the firm = 49% of status quo value = 0.49 * $1.6 million = $784,000
- Note that a 2% difference in ownership translates into a large difference in value because one stake ensures control and the other does not.

Closing thoughts on "discounting" practice…

- Don't discount multiple times for the same factor. Thus, if you increased the discount rate for a firm, because it is illiquid, you canot discount the value of liquidity. (Hint: You may be doing this if you incorporate a small cap premium into your discount rate and then proceed to reduce the value by an illiquidity discount)
- Be aware of your valuation assumptions: If you value a firm, be aware of how you are estimating cash flows and what assumptions you are making about how the firm will be run. If you have already incorporated the "sub-optimal" practices into your cash flows, you cannot apply a minority (control) discount to your estimated value.
- Be wary of build up approaches, where each "add on" to the discount rate is estimated separately: a small cap premium from studies that look at small cap stocks, an illiquidity premium from studies that looking at illiquid investments etc.