Intuition behind the Present Value Rule

There are three reasons why a dollar tomorrow is worth less than a dollar today

- Individuals prefer present consumption to future consumption. To induce peo consumption you have to offer them more in the future.
- When there is monetary inflation, the value of currency decreases over time. T greater the difference in value between a dollar today and a dollar tomorrow.
- If there is any uncertainty (risk) associated with the cash flow in the future, the valued.

Other things remaining equal, the value of cash flows in future time periods wil

- the preference for current consumption increases.

- expected inflation increases.

- the uncertainty in the cash flow increases.

Discounting and Compounding: The Mechanism For Factoring In Th

The mechanism for factoring in these elements is the discount rate.

Discount Rate: The discount rate is a rate at which present and future cash flows

incorporates -

(1) the preference for current consumption (Greater preference .... Highe

(2) expected inflation (Higher inflation .... Higher Discount Rate)

(3) the uncertainty in the future cash flows (Higher Risk .... Higher Disc

A higher discount rate will lead to a lower value for cash flows in the future.

The discount rate is also an opportunity cost, since it captures the returns that a

made on the next best opportunity.

- Discounting future cash flows converts them into cash flows in present value do discounting converts future cash flows into present cash flows,
- Compounding converts present cash flows into future cash flows.

Present Value Principle 1

Cash flows at different points in time cannot be compared and aggregated. All c
brought to the same point in time, before comparisons and aggregations are made.

Cashflow Types and the Mechanics of Discounting

There are four types of cash flows -

- simple cash flows,
- annuities,
- growing annuities
- perpetuities and
- growing perpetuities.

A. Simple Cash Flows: A simple cash flow is a single cash flow in a specified fut

Cash Flow: CF_t

____|

Time Period: t

The present value of this cash flow is-

PV of Simple Cash Flow = $CF_t / (1+r)^t$

· Thus, the present value of $5,000 in five years, assuming a 10% discount rate,

PV of $5,000 in five years = $\$5,000 / 1.10^5 = \$3,105$

The future value of a cash flow is -

FV of Simple Cash Flow = $CF_0 (1+ r)^t$

· Thus, the future value, in five years, of $10,000 today, assuming a 10% discoun

FV of $10,000 = $\$10,000 * 1.10^5 = \$16,105$

Application 1: The power of compounding - *Stocks, Bonds and Bills*

Ibbotson and Sinquefield, in a study of returns on stocks and bonds between 192(on the average made 12.4%, treasury bonds made 5.2% and treasury bills mad table provides the future values of $ 100 invested in each category at the end of periods - 1, 5 , 10 , 20 , 30 and 40 years.

Holding Period	Stocks	T. Bonds	T.Bills
1	$112.40	$105.20	$103.60
5	$179.40	$128.85	$119.34
10	$321.86	$166.02	$142.43
20	$1,035.92	$275.62	$202.86
30	$3,334.18	$457.59	$288.93
40	$10,731.30	$759.68	$411.52

Concept Check: Most pension plans allow individuals to decide where their pensi
invested - stocks, bonds or money market accounts. Where would you choose to i
funds? Will your allocation change as you get older? Why?

Application 2: The Frequency of Compounding

The frequency of compounding affects the future and present values of cash

interest rate can deviate significantly from the true interest rate –

For instance, a 10% annual interest rate, if there is semiannual compounding, wor

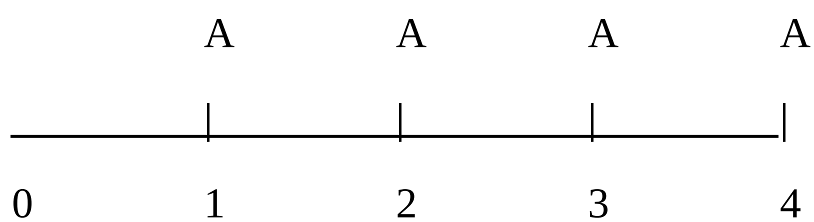
$$\text{Effective Interest Rate} = 1.05^2 - 1 = .10125 \text{ or } 10.25\%$$

The following table provides the effective rates as a function of the compounding f

Frequency	Rate	t	Formula	Effective Annual Rate
Annual	10%	1	r	10%
Semi-Annual	10%	2	$(1+r/2)^2-1$	10.25%
Monthly	10%	12	$(1+r/12)^{12}-1$	10.47%
Daily	10%	365	$(1+r/365)^{365}-1$	10.5156%
Continuous	10%		$\exp^r-1$	10.5171%

B. Annuities: An annuity is a constant cash flow that occurs at regular intervals fc

time. Defining A to be the annuity,



The present value of an annuity can be calculated by taking each cash flow and di

the present, and adding up the present values. Alternatively, there is a short cut tha

calculation -

$$\text{PV of an Annuity} = \text{PV}(A, r, n) = A \left[\frac{1 - \frac{1}{(1+r)^n}}{r} \right]$$

where,

A = Annuity

r = Discount Rate

n = Number of years

Thus, the present value of an annuity of $1,000 for the next five years, assuming a

is -

$$\text{PV of \$1000 each year for next 5 years} = \$1000 \left[\frac{1 - \frac{1}{(1.10)^5}}{.10} \right]$$

The notation that will be used in the rest of these lecture notes for the present valu

PV(A,r,n).

The reverse of this problem, is when the present value is known and the annuity is

A(PV,r,n).

$$\text{Annuity given Present Value} = A(PV, r, n) = PV \left[\frac{\quad}{1 - \quad (} \right.$$

The future value of an end-of-the-period annuity can also be calculated as follows-

$$\text{FV of an Annuity} = \text{FV(A, r, n)} = A \left[\frac{(1+r)^n - 1}{r}\right.$$

Thus, the future value of $1,000 each year for the next five years, at the end of the

a 10% discount rate) -

$$\text{FV of \$1,000 each year for next 5 years} = \$1000 \left[\frac{(1.10)^5 - 1}{.10}\right.$$

The notation that will be used for the future value of an annuity will be FV(A,r,n).

are given the future value and you are looking for an annuity - A(FV,r,n) in terms

$$\text{Annuity given Future Value} = \text{A(FV, r, n)} = FV \left[\frac{r}{(1+r)}\right.$$

Application 2: Saving for college tuition

Assume that you want to send your newborn child to a private college (whe years old). The tuition costs are $ 16000/year now and that these costs are expect for the next 18 years. Assume that you can invest, after taxes, at 8%.

Expected tuition cost/year 18 years from now = $16000*(1.05)^{18}$ = $38,506

PV of four years of tuition costs at $38,506/year = $38,506 * PV(A ,8%,4 years)=

[For simplicity, I have assumed that tuition costs are frozen at these levels for four can be allowed to keep growing at 5% a year and the present value can be compu

If you need to set aside a lump sum now, the amount you would need to set aside

Amount one needs to set apart now = $127,357/$(1.08)^{18}$ = $31,916

If set aside as an annuity each year, starting one year from now -

If set apart as an annuity = $127,537 * A(FV,8%,18 years) = $3,405

Application 3: How much is your full-time MBA worth?

Assume that you were earning $40,000/year before entering program and that tuit

Expected salary is $ 54,000/year after graduation. You can invest money at 8%.

For simplicity, assume that the first payment of $16,000 has to be made at the sta

second payment one year later.

PV Of Cost Of MBA = $16,000+16,000/1.08 + 40000 * PV(A,8%,2 years) = $1(

Assume that you will work 30 years after graduation, and that the salary differen

$40000) will continue through this period.

PV of Benefits Before Taxes = $14,000 * PV(A,8%,30 years) = $157,609

This has to be discounted back two years - $157,609/1.08^2 = $135,124

The present value of getting an MBA is = $135,124 - $102,145 = $32,979

Some Interesting Questions:

1. How much would your salary increment have to be for you to break even on yo

2. Keeping the increment constant, how many years would you have to work to br

Application 4: Refinancing your mortgage - Is it worth it?

Assume that you have a thirty-year mortgage for $200,000 that carries an int
The mortgage was taken three years ago. Since then, assume that interest rates hav
7.50%, and that you are thinking of refinancing. The cost of refinancing is expecte
loan. (This cost includes the points on the loan.) Assume also that you can invest y

Monthly payment based upon 9% mortgage rate (0.75% monthly rate)

= $200,000 * A(PV,0.75%,360 months)

= $1,609

Monthly payment based upon 7.50% mortgage rate (0.625% monthly rate)

= $200,000 * A(PV,0.625%,360 months)

= $1,398

Monthly Savings from refinancing = $1,609 - $1,398 = $211

If you plan to remain in this house indefinitely,

Present Value of Savings (at 6% annually; 0.5% a month)

= $211 * PV(A,0.5%,324 months)

= $33,815

The savings will last for 27 years - the remaining life of the existing mortgage.

You will need to make payments for three additional years as a consequence of th

Present Value of Additional Mortgage payments - years 28,29 and 30

= $1,398 * PV(A,0.5%,36 months)/1.06^{27}

= $9,532

Refinancing Cost = 2.5% of $200,000 = $5,000

Total Refinancing Cost = $9,532 + $5,000 = $14,532

Follow-up Questions:

1. How many years would you have to live in this house for you break even on thi

2. We've ignored taxes in this analysis. How would it impact your decision?

Application 5: The Value of a Straight Bond

You are trying to value a straight bond with a fifteen year maturity and a 10.

current interest rate on bonds of this risk level is 8.5%.

PV of cash flows on bond = 107.50* PV(A,8.5%,15 years) + $1000/1.085^{15}$ = $ 1

If interest rates rise to 10%,

PV of cash flows on bond = 107.50* PV(A,10%,15 years)+ $1000/1.10^{15}$ = $1,05

Percentage change in price = -10.94%

If interest rate fall to 7%,

PV of cash flows on bond = 107.50* PV(A,7%,15 years)+ $1000/1.07^{15}$ = $1,341.

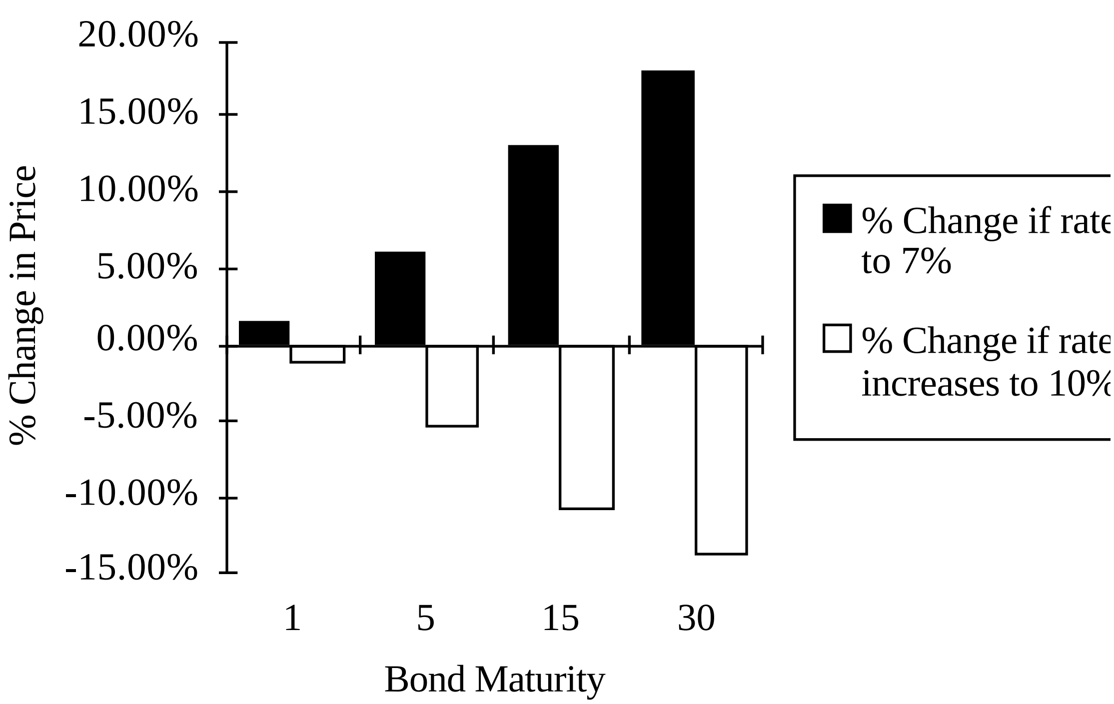
Percentage change in price = +13.03%

This asymmetric response to interest rate changes is called **convexity**.

Application 6: Contrasting short term versus long term bonds

You are valuing four bonds - 1 year, 5 year, 15 year and 30 year- with a coupon r

Price Changes as a function of Bond Maturities



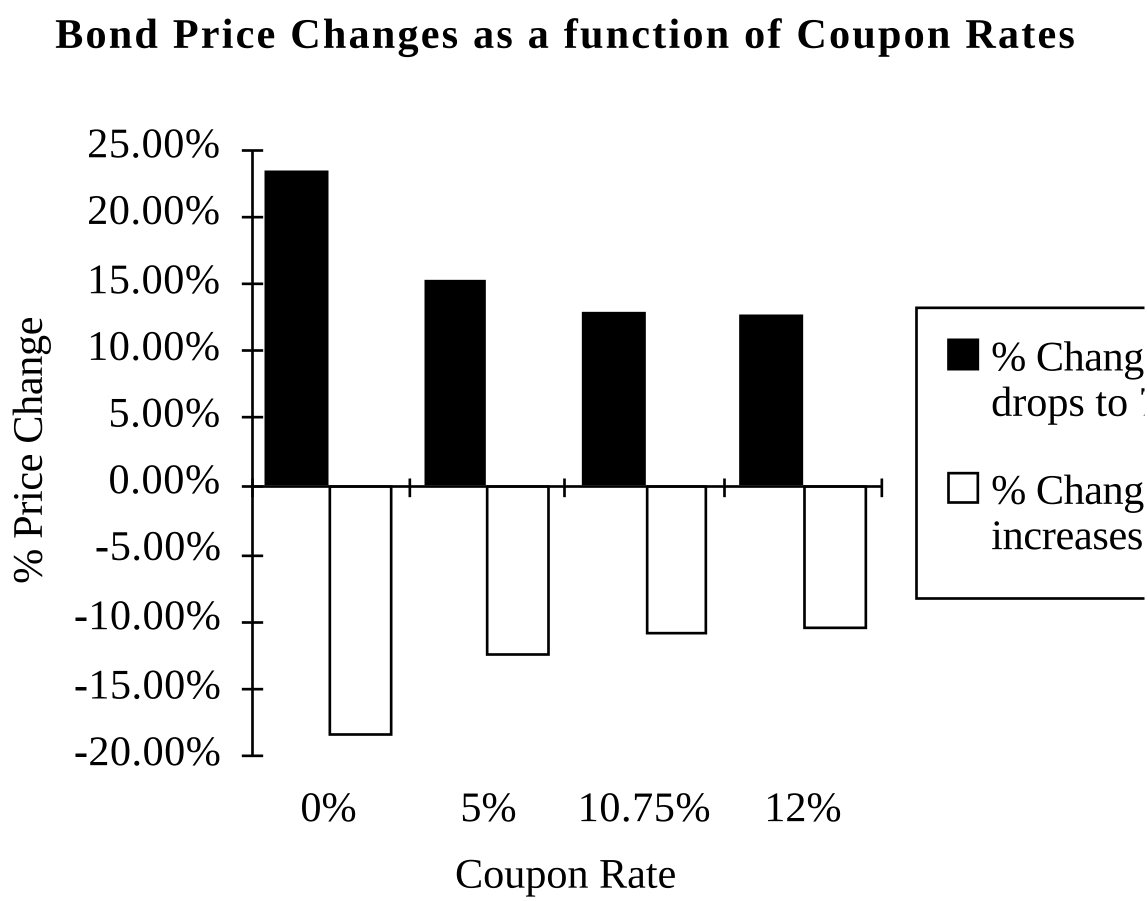
Bond Pricing Proposition 1: The longer the maturity of a bond, the more sensiti

interest rates.

Application 7: Contrasting low coupon and high coupon bonds

You are valuing four different bonds all with the same maturity - 15 years, bu

rates - 0%, 5%, 10.75% and 12%.


Bond Price Changes as a function of Coupon Rates
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
-10.00%
-15.00%
-20.00%
% Price Change
0%
5%
10.75%
12%
Coupon Rate
% Chang
drops to
% Chang
increases

Bond Pricing Proposition 2: The lower the coupon rate on the bond, the more se

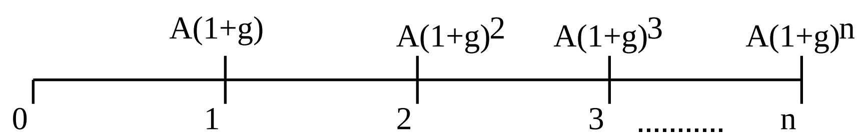
changes in interest rates.

C. Growing Annuity: A growing annuity is a cash flow growing at a consta

period of time. If A is the current cash flow, and g is the expected growth rate, the

growing annuity looks as follows –

Figure 3.8: A Growing Annuity



Note that to qualify as a growing annuity, the growth rate in each period has to be

growth rate in the prior period.

PRESENT VALUE OF GROWING ANNUITY

$$PV \text{ of a Growing Annuity} = A(1+g)\left[\frac{1 - \frac{(1+g)^n}{(1+r)^n}}{r-g}\right]$$

- The present value of a growing annuity can be estimated in all cases, but one - is equal to the discount rate.
- In that specific case, the present value is equal to the nominal sums of the annui without the growth effect.

PV of a Growing Annuity for n years (when r=g) = n A

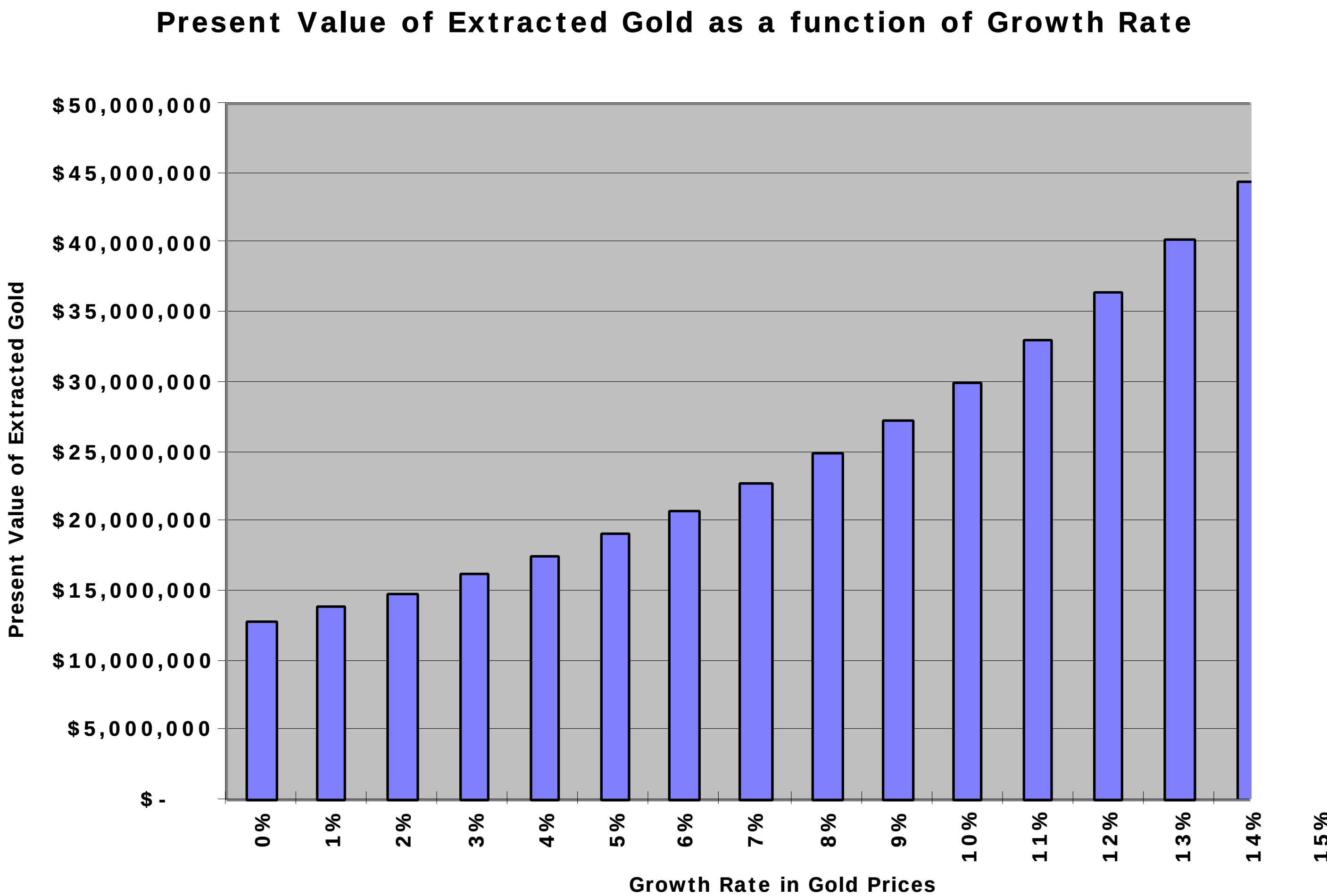
Note also that this formulation works even when the growth rate is greater than th

[1] Both the denominator and the numerator in the formula will be negative, yielding a positive present value.

Application 8: The value of a gold mine

Consider the example of a gold mine, where you have the rights to the mine over which period you plan to extract 5,000 ounces of gold every year. The price currently, but it is expected to increase 3% a year. The appropriate discount rate is value of the gold that will be extracted from this mine can be estimated as follows

$$PV \text{ of extracted gold} = \$300 * 5000 * (1.03) \left[\frac{1 - \frac{(1.03)^{20}}{(1.10)^{20}}}{.10 - .03} \right] =$$


Present Value of Extracted Gold as a function of Growth Rate
$50,000,000
$45,000,000
$40,000,000
$35,000,000
$30,000,000
$25,000,000
$20,000,000
$15,000,000
$10,000,000
$5,000,000
$ -
Present Value of Extracted Gold
0%
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
11%
12%
13%
14%
15%
Growth Rate in Gold Prices

Concept Check: If both the growth rate and the discount rate go up by 1%, will th

gold to be extracted from this mine increase or decrease?

D. Perpetuities: A perpetuity is a constant cash flow at regular intervals forever. T
perpetuity is-

$$\text{PV of Perpetuity} = \frac{A}{r}$$

where A is the perpetuity. The future value of a perpetuity is infinite.

Application 9: Valuing a Console Bond

A console bond is a bond that has no maturity and pays a fixed coupon. Ass

6% coupon console bond. The value of this bond, if the interest rate is 9%, is as fo

Value of Console Bond = \$60 / .09 = \$667

E. Growing Perpetuities: A growing perpetuity is a cash flow that is expected tc rate forever. The present value of a growing perpetuity is -

$$\text{PV of Growing Perpetuity} = \frac{CF_1}{(r - g)}$$

where

CF_1 is the expected cash flow next year,

g is the constant growth rate and

r is the discount rate.

Application 10: Valuing a Stock with Stable Growth in Dividends

Southwestern Bell paid dividends per share of $2.73 in 1992. Its earnings ar grown at 6% a year between 1988 and 1992, and are expected to grow at the sam term. The rate of return required by investors on stocks of equivalent risk is 12.23

Current Dividends per share = $2.73

Expected Growth Rate in Earnings and Dividends = 6%

Discount Rate = 12.23%

Value of Stock = $2.73 *1.06 / (.1223 -.06) = $46.45