Chapter 3: The Basics of Risk

1. If we were restricted to invest in only one of the three securities, A would be most preferable, since it has the highest expected return and lowest standard deviation of returns of the three.

2. The average annual return is computed as follows:

Year	Return
1995	42.1
1994	-10.9
1993	20.4
1992	12.5
1991	10.3
1990	45.8
1989	-30.5
1988	11.4
1987	10.2
1986	-2.2
Average	10.91

a. The average annual return is 10.91%, while the standard deviation is 22.72%.
b. If the company pays no dividends for the last ten years, the average return also estimates the percentage price change. Hence, if the stock price at the end of 1985 was 25.6, the stock price at the end of 1986 would be computed at 25.6(1 -.022) = $25.04. Working in this fashion, we find that the price at the end of 1995 is $58.91.
c. The annual compounding growth rate is computed as the geometric average of the annual growth rates, i.e. $[(0.978)(1.102)(1.114)\ldots(1.0421)]^{1/10}$ = 8.69%. A short cut to the same answer is:
Geometric Mean = $(58.91/25.6)^{(1/10)} - 1$ = .0869 or 8.69%
This is less than the answer in part (a). The arithmetic mean is always greater than the geometric mean.

3 a. Using the formula for the expected return on a portfolio, we get:
$E(R_p) = 0.5(12) + 0.5(18) = 15\%$
$Var(R_p) = (0.5)^2(25)^2 + (0.5)^2(40)^2 + 2(0.5)(0.5)(25)(40)0.8) = 956.25.$

$\sigma(R_p) = (956.25)^{0.5} = 30.92\%$

b. The expected return on this portfolio is halfway between the expected returns on the two individual securities; however, the standard deviation is less than halfway between the standard deviations on the two securities. Hence, unless the investor were very risk averse (in which case, he might choose to invest only in security A), he would invest in the portfolio rather than invest in the two securities separately.

4 a. If we constructed a portfolio with weights, $w_A = 45/(45+25) = 64.29\%$ and $w_B = 25/(45+25) = 35.71\%$, this portfolio would have a variance equal to $(0.6429)^2(25)^2 + (0.3571)^2(45)^2 - 2(0.3571)(0.6429)(25)(45) = 0$.

b. The expected return on this portfolio would be (0.6429)(12) + (0.3571)(15) = 13.07%

c. If it were possible to borrow from and lend to the local bank at 8%, we could arrange to borrow funds at 8%, invest in our portfolio, earn a guaranteed return of 13.07%, and repay the funds borrowed, leaving over an excess of 5.07% on any amount borrowed.

5. a. Since B is riskfree, the correlation coefficient would be zero.

b. If we constructed a portfolio with weights of w and 1-w in A and B respectively, the standard deviation of the portfolio would be equal to 40w. Equating this to 20, we see that w = 0.5; hence 1-w = 0.5 as well.

c. The expected return on this portfolio would be 0.5(15) + 0.5(5) = 10%.

6. The variance of returns on the portfolio is computed as $(.30)^2(20)^2 + (.40)^2(40)^2 + (.30)^2(70)^2 + 2(0.3)(0.4)(20)(40)(0.5) + 2(0.4)(0.3)(40)(70)(0.9) + 2(0.3)(0.3)(20)(70)(0.7) = 1610.2$

The standard deviation = $(1610.2)^{0.5} = 40.13\%$

The expected return equals (0.3)(15) + (0.4)(20) + (0.3)(35) = 23%

7. Using the CAPM, we get the expected risk premium = 1.5(10) = 15%.

8. The expected return on the stock equals 5 + 0.9(12.5 - 5) = 11.75%

9. If the analysts estimate that the return on the stock market is 2.5% higher than the historical average, estimated return on the stock market is 12.5 + 2.5 = 15%. Using the CAPM formula again, we get 5 + 0.9(15-5) = 14%.

(We are using 20% of 12.5% to get 2.5%)

10. We set up the equation $15 = 5 + \beta(12-5)$; solving, we get $\beta = 10/7 = 1.43$

11. According to the CAPM, given the beta of the mutual fund, it should have, on average, earned 5 + 1.4(12-5) = 14.8% over a period when the S&P Index grew by 12%. If the actual mutual fund actually earned only 14%, on a risk-adjusted basis, it actually underperformed the market by 0.8%. Consequently, if we believed that the

CAPM is valid, we would reject the claim that the fund outperformed the market. On the other hand, the 0.8% underperformance might have been solely due to chance, and not due to an inherent weakness in the performance of the fund.

12. The beta of the portfolio is a weighted average of the betas of the stocks making up the portfolio, i.e. 0.4(1.2) + 0.3(0.9) + 0.3(1.8) = 1.29. The CAPM would, therefore, predict an expected return on this portfolio equal to 5 + 1.29(12 - 5) = 14.03%.

13. a. The equation to set up is 10 = w(5) + (1-w)(12), where w is the proportion in Treasury bills. Solving, we get w = 2/7 = 0.2857.

b. The beta of this portfolio would be 1 - 0.2857 = 0.7143.

14. a. This mutual fund is diversified, and hence its beta would be close to 1.

b. Hence its average annual return should approximate the S&P 500 Index return of 12%.

c. If the market prices stocks correctly, on average, the only service that a diversified mutual fund is providing its investors is the saving in transactions costs if they were to construct such a diversified portfolio themselves. However, a cost of as much as 1% to 3% annually for this service is excessive. It is, therefore, not surprising that these funds underperform the market, after expenses.

15. a. The expected return on this stock would be 5 + 1.2(6.5 - 5) + 0.5(4.3 - 5) + 0.8(8.0 - 5) + 1.6(7.5 - 5) = 12.85%

b. If the actual parameters, turn out to be as given, our estimate of the return on the stock (except for the idiosyncratic component) would be 5 + 1.2(7.2 - 5) + 0.5(5.2 - 5) + 0.8(6.3- 5) + 1.6(10 - 5) = 16.78%. If we define the surprise as the actual reutrn on the stock less the expected return, we would estimate this surprise as 16.78 - 12.85 = 3.93%

16. The expected difference on the average monthly return between these two groups of stocks can be computed as 0.35(ln(0.3) - ln(1.2)) = -0.4852%, i.e. the second group of stocks would earn about 0.5% more than the first group.

17.

A. False. Variance measures total risk, while beta measures market risk. A company can have high total risk and low market risk. Empirically, however, the two tend to be correlated.

B. True. A firm with no unsystematic risk has to be perfectly correlated with the market portfolio, and thus has to be one of the efficient portfolios (a combination of the market portfolio and the riskfree assset)

C. False. The beta for stock *i* is defined as correlation(*i*,m)*σ(*i*)/ σ(m). Hence a stock with a higher correlation can still have a lower σ(*i*) leading to a lower beta.

D. False. Since the average beta is 1, this is impossible.

E. False. If the firm is well managed, its stock will have a higher price, because its mean cashflows per share will be higher than if it were badly managed. It's beta need not be affected at all.
F. False. The market portfolio contains all stocks in the market.
G. False. Both of them will hold the market portfolio. The risk lover will leverage his portfolio by borrowing at the riskfree rate.
H. False, since the expected returns (from which the efficient portfolio is extracted) will all change as the riskfree rate changes.

18. a. You'd pick the market portfolio, since it dominates gold on both average return and standard deviation.
b. The higher possible returns on gold are balanced by the lower possible returns at other times. Note that the average return on gold is much less than that on the stock market.
c. The expected return on this portfolio would be (8+20)/2 = 14%. The variance would equal $(0.5)^2(25)^2 + (0.5)^2(22)^2 - 2(0.5)(0.5)(25)(22)(0.4) = 167.25$; the standard deviation equals 12.93%
d. If the supply of gold is negatively correlated with the level of the market, and the price of gold is inversely related to the supply of gold, we have a positive correlation between the return on the market and the return on gold. This would make gold less desirable, since it does not help as much in reducing portfolio variance. The optimal amount to invest in gold would drop.

19.The variance of a portfolio consisting of N securities can be estimated as (1/N)(average variance) + (1-1/N)(average covariance) = 10 + (50-10)/N.

Number of securities in portfolio (N)	Estimated portfolio variance
5	18
10	14
20	12
50	10.8
100	10.4

We must solve 10 + 40/N = 1.1(10) = 11, or N = 40

20. Some of the criticisms of the CAPM are:
- The CAPM makes some stringent assumptions, such as, e.g. that investors only care about expected return and risk.
- It cannot be tested
- It only allows for one factor
- It has not fared well in empirical testing.

Although the criticisms are true, the intuitive appeal of the CAPM and the fact that it can be easily estimated make it worthwhile. Furthermore, the empirical shortcomings of the CAPM have been shown to result at least partly from data problems and nonstationarities in the beta factors.

21. a. The similarities are:
They are both based on measures of non-diversifiable risk.
They are both linear.

The differences are:
The APM has more than one factor
The factor in the CAPM is conceptually well-defined, whereas the APM factors are statistical factors, that in theory, at least, come from broader macro economic risks.

b. If there is only one factor driving returns and that factor happens to be measured by the market portfolio, the CAPM and the APM will converge The market then does not require a risk premium for any factor risk other than the market risk.Alternatively, the different factors measured by the APM might simply be a decomposition of the market risk.

In practice, the estimated expected returns would be somewhat different. However, the differences may not be statistically significant. If the differences are significant, it might be that there are different sources of factor risk, such as inflation factor risk, GNP factor risk, etc. which command different risk premiums in such a way that they cannot be summarized by a single market beta risk factor.